

Chapter 1

1.1

$$f(t) = 5 \sin(10t - 2.5) \quad (1)$$

Eq. (1) can be rewritten as

$$\begin{aligned} f(t) &= 5 [\sin \omega t \cos 2.5 - \cos \omega t \sin 2.5] \\ &= C_1 \sin \omega t + C_2 \cos \omega t \quad (2) \end{aligned}$$

$$\text{where } C_1 = 5 \cos 2.5 = 5 (-0.8011) = -4.0057$$

$$\text{and } C_2 = -5 \sin 2.5 = -5 (0.5985) = -2.9925$$

$f(t)$ is expressed as a sum of sine and cosine functions with $C_1 = -4.0057$ and $C_2 = -2.9925$.

1.2

(a)

$$x_1(t) = 5 \sin 20t, \quad x_2(t) = 8 \cos \left(20t + \frac{\pi}{3} \right) \quad (1)$$

$$x(t) = x_1(t) + x_2(t) = 5 \sin 20t + 8 \cos \left(20t + \frac{\pi}{3} \right) \quad (2)$$

$x(t)$ is a cosine function with a phase angle.

Eq. (2) can be rewritten as

$$\begin{aligned} x(t) &= 5 \sin 20t + 8 \left(\cos 20t \cdot \cos \frac{\pi}{3} - \sin 20t \cdot \sin \frac{\pi}{3} \right) \\ &= \cos 20t \left(8 \cos \frac{\pi}{3} \right) - \sin 20t \left(-5 + 8 \sin \frac{\pi}{3} \right) \quad (3) \end{aligned}$$

or

$$\begin{aligned} x(t) &= A \cos (20t + \phi) \\ &= A \cos 20t \cdot \cos \phi - A \sin 20t \cdot \sin \phi \end{aligned}$$

By equating Eqs. (3) and (4), we obtain

$$A \cos \phi = 8 \cos \frac{\pi}{3} = 8(0.5) = 4.0$$

$$A \sin \phi = -5 + 8 \sin \frac{\pi}{3} = -5 + 8(0.8660) = 1.9282$$

$$\begin{aligned} \text{Thus } A &= \sqrt{(A \cos \phi)^2 + (A \sin \phi)^2} = \sqrt{4.0^2 + 1.9282^2} \\ &= \sqrt{16.0 + 3.7179} = \sqrt{19.7179} = 4.4405 \end{aligned}$$

$$\begin{aligned} \text{and } \phi &= \tan^{-1} \left(\frac{A \sin \phi}{A \cos \phi} \right) = \tan^{-1} \left(\frac{1.9282}{4.0} \right) = \tan^{-1}(0.48205) \\ &= 25.7364^\circ \text{ or } 0.4492 \text{ rad} \end{aligned}$$

$$\begin{aligned} \therefore x(t) &= 4.4405 \cos (20t + 0.4492) \\ &= 4.4405 \cos (20t + 25.7364^\circ) \quad (5) \end{aligned}$$

Problem 1.2 p.1/2

(b) $x(t)$ is a sine function with a phase angle.

Eg. (2) can be rewritten as

$$x(t) = 5 \sin 20t + 8 \left(\cos 20t \cdot \cos \frac{\pi}{3} - \sin 20t \cdot \sin \frac{\pi}{3} \right)$$

$$= \sin 20t \left(5 - 8 \sin \frac{\pi}{3} \right) + \cos 20t \left(8 \cos \frac{\pi}{3} \right) \quad (6)$$

or

$$x(t) = A \sin 20t \cos \phi + A \cos 20t \sin \phi \quad (7)$$

We obtain, by comparing Eqs. (6) and (7),

$$A \cos \phi = 5 - 8 \sin \frac{\pi}{3} = -1.9282 \quad (8)$$

$$A \sin \phi = 8 \cos \frac{\pi}{3} = 4.0 \quad (9)$$

So that

$$A = \sqrt{(A \cos \phi)^2 + (A \sin \phi)^2}$$

$$= \sqrt{(-1.9282)^2 + (4.0)^2} = 4.4405$$

and

$$\phi = \tan^{-1} \left(\frac{A \sin \phi}{A \cos \phi} \right) = \tan^{-1} \left(\frac{4.0}{-1.9282} \right) = \tan^{-1} (-2.0745)$$

$$= -64.2636^\circ = -1.1216 \text{ rad}$$

$$\text{or } 180^\circ - 64.2636^\circ$$

$$= 115.7372^\circ$$

Since cosine is negative and sine is positive,

$$\phi = 115.7372^\circ \text{ or } 2.0200 \text{ rad.}$$

$$\therefore x(t) = 4.4405 \sin (20t + 2.0200)$$

$$\text{or } 4.4405 \sin (20t + 115.7372^\circ)$$

Problem 1.2 p.2/2

1.3

$$x_1(t) = 6 \sin 30t$$

$$x_2(t) = 4 \cos \left(30t + \frac{\pi}{4} \right)$$

$$x(t) = x_1(t) - x_2(t) = 6 \sin 30t - 4 \cos \left(30t + \frac{\pi}{4} \right) \quad (1)$$

(a) $x(t)$ as a sine function: with a phase angle.

Eq. (1) can be rewritten as

$$x(t) = 6 \sin 30t - 4 \left(\cos 30t \cdot \cos \frac{\pi}{4} - \sin 30t \cdot \sin \frac{\pi}{4} \right) \quad (2)$$

$$= \sin 30t \left(6 + 4 \sin \frac{\pi}{4} \right) + \cos 30t \left(-4 \cos \frac{\pi}{4} \right) \quad (3)$$

By expressing Eq. (3) as

$$x(t) = A \sin(30t + \phi)$$

$$\Rightarrow = A \sin 30t \cdot \cos \phi + A \cos 30t \cdot \sin \phi \quad (4)$$

Eqs. (3) and (4) yield:

$$A \cos \phi = 6 + 4 \sin \frac{\pi}{4} = 6 + 4(0.7071) = 8.8284$$

$$A \sin \phi = -4 \cos \frac{\pi}{4} = -4(0.7071) = -2.8284$$

where

$$A = \sqrt{(A \cos \phi)^2 + (A \sin \phi)^2} = \sqrt{(8.8284)^2 + (-2.8284)^2}$$

$$= \sqrt{77.9406 + 7.9998} = \sqrt{85.9404} = 9.2704$$

$$\text{and } \phi = \tan^{-1} \left(\frac{A \sin \phi}{A \cos \phi} \right) = \tan^{-1} \left(\frac{-2.8284}{8.8284} \right) = \tan^{-1}(-0.3204)$$

$$= -17.7642^\circ = -0.3100 \text{ rad}$$

$$\therefore x(t) = 9.2704 \sin(30t - 0.31) = 9.2704 \sin(30t - 17.7642^\circ) \quad (5)$$

(b) $x(t)$ as a cosine function with a phase angle:

Eq. (2) can be rewritten as

$$x(t) = \cos 30t \left(-4 \cos \frac{\pi}{4} \right) - \sin 30t \left(-6 - 4 \sin \frac{\pi}{4} \right) \quad (6)$$

By expanding Eq. (6) as

$$x(t) = A \cos(30t - \phi) = A \cos 30t \cdot \cos \phi - A \sin 30t \cdot \sin \phi \quad (7)$$

By equating Eqs. (6) and (7), we find

$$A \cos \phi = -4 \cos \frac{\pi}{4} = -2.8284 \quad (8)$$

$$A \sin \phi = -6 - 4 \sin \frac{\pi}{4} = -8.8284 \quad (9)$$

So that

$$\begin{aligned} A &= \sqrt{(A \cos \phi)^2 + (A \sin \phi)^2} = \sqrt{(-2.8284)^2 + (-8.8284)^2} \\ &= \sqrt{7.9998 + 77.9406} = \sqrt{85.9404} = 9.2704 \end{aligned}$$

and

$$\begin{aligned} \phi &= \tan^{-1} \left(\frac{A \sin \phi}{A \cos \phi} \right) = \tan^{-1} \left(\frac{-8.8284}{-2.8284} \right) = \tan^{-1}(3.1213) \\ &= \tan^{-1}(178.8369^\circ) = 89.6796^\circ \text{ or } 180^\circ + 89.6796^\circ \\ &= 1.5652^{\text{rad}} \text{ or } 4.7068 \text{ rad} \end{aligned}$$

Since both $\sin \phi$ and $\cos \phi$ are negative, the correct value of ϕ is 4.7068 rad or 269.6796°

$$\begin{aligned} \therefore x(t) &= 9.2704 \cos(30t - 4.7068) \\ &= 9.2704 \cos(30t - 269.6796^\circ) \quad (10) \end{aligned}$$

Problem 1.3 p.2/2

1.4

$$x_1(t) = 5 \cos \omega t, \quad x_2(t) = 10 \cos(\omega t + 1) \quad (1)$$

(a) Using trigonometric relations:

$$x(t) = x_1(t) + x_2(t) = 5 \cos \omega t + 10 \cos(\omega t + 1) \quad (2)$$

$$= 5 \cos \omega t + 10 (\cos \omega t \cdot \cos 1 - \sin \omega t \cdot \sin 1)$$

$$= \cos \omega t (5 + 10 \cos 1) - \sin \omega t (10 \sin 1) \quad (3)$$

By expressing Eq. (3) as

$$x(t) = A \cos(\omega t + \phi) = A(\cos \omega t \cdot \cos \phi - \sin \omega t \cdot \sin \phi)$$

$$= \cos \omega t (A \cos \phi) - \sin \omega t (A \sin \phi) \quad (4)$$

We obtain, by comparing Eqs. (3) and (4),

$$A \cos \phi = 5 + 10 \cos 1$$

$$A \sin \phi = +10 \sin 1$$

$$\therefore A = \sqrt{(A \cos \phi)^2 + (A \sin \phi)^2} = \sqrt{(5 + 10 \cos 1)^2 + (10 \sin 1)^2}$$

$$= \sqrt{[5 + 10 * 0.5403]^2 + [10 * 0.8417]^2}$$

$$= (108.2224 + 70.8459)^{\frac{1}{2}} = 13.3816$$

and

$$\phi = \tan^{-1} \left(\frac{10 \sin 1}{5 + 10 \cos 1} \right) = \tan^{-1} \left(\frac{8.417}{10.403} \right)$$

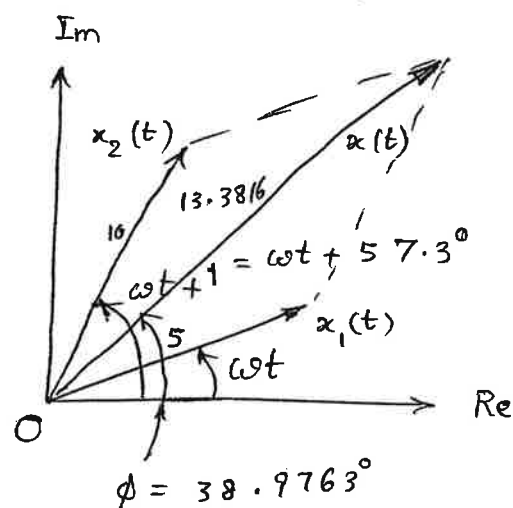
$$= \tan^{-1} (0.8091) = 0.6803 \text{ rad} = 38.9763^\circ$$

$$\therefore x(t) = x_1(t) + x_2(t) = 13.3816 \cos(\omega t + 0.6803)$$

Problem 1.4 p.1/2

(b) Using Vectors:

For an arbitrary value of ωt , the harmonic motions $x_1(t)$ and $x_2(t)$ can be denoted graphically as shown in the following figure.



(c) Using Complex number representation

Given $x_1(t)$ and $x_2(t)$ can be expressed as complex numbers:

$$x_1(t) = \text{Re} [A_1 e^{i\omega t}] \equiv \text{Re} [5 e^{i\omega t}]$$

$$x_2(t) = \text{Re} [A_2 e^{i(\omega t + 1)}] \equiv \text{Re} [10 e^{i(\omega t + 1)}]$$

where $A_1 = \sqrt{(5 + 10 \cos 1)^2 + 0} = 10.403$

and $A_2 = \sqrt{(0 + 10 \sin 1)^2} = 8.417$

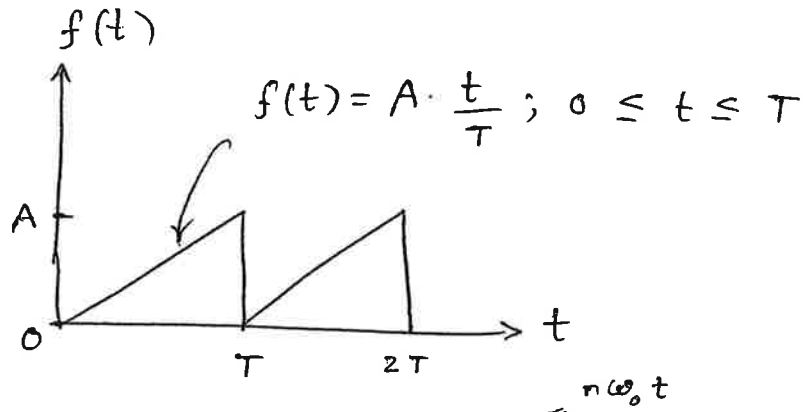
Thus $x(t) = \text{Re} [A e^{i(\omega t + \phi)}] = x_1 + x_2$

with $A = \sqrt{A_1^2 + A_2^2} = 13.3816$

and $\phi = \tan^{-1} \left(\frac{A_2}{A_1} \right) = 38.9763^\circ$ or 0.6803 rad

Problem 1.4 p.2/2

1.6



$$\begin{cases} T = \frac{2\pi}{\omega_0} \\ \omega_0 = \frac{2\pi}{T} \end{cases}$$

$$a_n = \frac{2}{T} \int_0^T A \cdot \frac{t}{T} \cos\left(\frac{2\pi n t}{T}\right) dt$$

$$= \frac{2A}{T^2} \int_0^T t \cdot \cos \frac{2\pi n t}{T} dt$$

$$= \frac{2A}{T^2} \left[\frac{T^2}{(2\pi n)^2} \cdot \cos \frac{2\pi n}{T} \cdot t + \frac{t \cdot T}{2\pi n} \cdot \sin \frac{2\pi n}{T} \cdot t \right]_0^T$$

$$= \frac{2A}{T^2} \left[\frac{T^2}{4\pi^2 n^2} \left(\cos \frac{2\pi n \cdot T}{T} - 1 \right) + \frac{T^2}{2\pi n} \left(\sin \frac{2\pi n \cdot T}{T} \right) \right]$$

$$= \frac{A}{2\pi^2 n^2} (\cos 2\pi n - 1) + \frac{A}{\pi n} \underbrace{(\sin 2\pi n)}_{=0}$$

$$b_n = \frac{2}{T} \int_0^T A \cdot \frac{t}{T} \sin \frac{2\pi n t}{T} dt$$

$$a_0 = A, \quad a_n = 0 ; n = 1, 2, 3, \dots ; \quad b_n = -\frac{A}{n\pi} ; n = 1, 2, 3, \dots$$

$$\therefore f(t) = A \left(\frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{2\pi n t}{T} \right)$$

1.7

$$x(t) = \begin{cases} A \sin \frac{2\pi t}{\tau} & , 0 \leq t \leq \frac{\tau}{2} \\ 0 & , \frac{\tau}{2} \leq t \leq \tau \end{cases}$$

$$a_0 = \frac{2}{\tau} \int_0^{\tau} x(t) dt = \frac{2A}{\tau} \int_0^{\tau/2} \sin \frac{2\pi t}{\tau} dt = \frac{2A}{\tau} \left(-\frac{\tau}{2\pi} \cos \frac{2\pi t}{\tau} \right) \Big|_0^{\tau/2} \\ = \frac{2A}{\pi}$$

$$a_n = \frac{2}{\tau} \int_0^{\tau} x(t) \cos n\omega t dt = \frac{2A}{\tau} \int_0^{\tau/2} \sin \frac{2\pi t}{\tau} \cdot \cos n\omega t dt \quad \text{---- (E}_1\text{)}$$

Using the relation $\sin m\omega t \cdot \cos n\omega t = \frac{\sin(m+n)\omega t + \sin(m-n)\omega t}{2}$,

Eq. (E₁) can be rewritten as

$$a_n = \frac{A}{\tau} \int_0^{\pi/\omega} [\sin(1+n)\omega t + \sin(1-n)\omega t] dt$$

$$\text{When } n=1, \quad a_1 = \frac{A}{\tau} \int_0^{\pi/\omega} \sin 2\omega t \cdot dt = 0$$

$$\text{When } n=2, 3, 4, \dots, \quad a_n = \frac{A}{\tau} \left[-\frac{\cos(1+n)\omega t}{(1+n)\omega} - \frac{\cos(1-n)\omega t}{(1-n)\omega} \right] \Big|_0^{\pi/\omega} \\ = \frac{A}{2\pi} \left[\frac{1 - \cos(1+n)\pi}{1+n} + \frac{1 - \cos(1-n)\pi}{1-n} \right] \\ = \begin{cases} 0 & \text{if } n \text{ is odd} \\ -\frac{2A}{(n-1)(n+1)\pi} & \text{if } n \text{ is even} \end{cases}$$

$$\text{Similarly} \quad b_n = \frac{2}{\tau} \int_0^{\tau} x(t) \sin n\omega t dt = \frac{2A}{\tau} \int_0^{\tau/2} \sin \frac{2\pi t}{\tau} \cos n\omega t dt \\ = \frac{A}{\tau} \int_0^{\tau/2} [\cos(1-n)\omega t - \cos(1+n)\omega t] dt$$

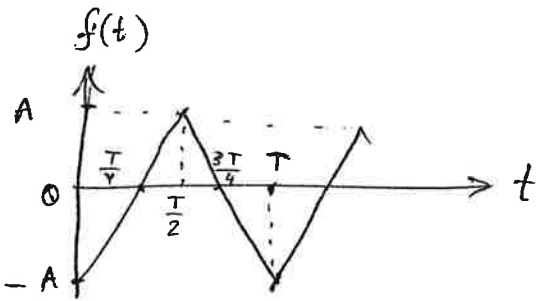
$$\text{When } n=1, \quad b_1 = \frac{A}{\tau} \int_0^{\pi/\omega} (dt - \cos 2\omega t) dt = \frac{A}{2}$$

$$\text{When } n=2, 3, 4, \dots, \quad b_n = \frac{A}{\tau} \left[\frac{\sin(1-n)\omega t}{(1-n)\omega} - \frac{\sin(1+n)\omega t}{(1+n)\omega} \right] \Big|_0^{\pi/\omega} = 0$$

$$\therefore x(t) = \frac{A}{\pi} + \frac{A}{2} \sin \omega t - \frac{2A}{\pi} \sum_{n=2, 4, 6, \dots}^{\infty} \frac{\cos n\omega t}{(n^2-1)}$$

1.9

$$f(t) = \begin{cases} \left(\frac{4A}{T}\right)t - A & ; 0 \leq t \leq \frac{T}{2} \\ A - \frac{4A}{T}\left(t - \frac{T}{2}\right) & ; \frac{T}{2} \leq t \leq T \end{cases}$$



$$\begin{aligned} a_0 &= \frac{2}{T} \int_0^T f(t) \cdot dt = \frac{2}{T} \left[\int_0^{T/2} \left\{ \left(\frac{4A}{T}\right)t - A \right\} dt \right. \\ &\quad \left. + \int_{T/2}^T \left\{ A - \frac{4A}{T}\left(t - \frac{T}{2}\right) \right\} dt \right] \\ &= \frac{2}{T} \left[\frac{4A}{T} \cdot \left(\frac{t^2}{2} \right) \Big|_0^{T/2} - A \cdot (t) \Big|_0^{T/2} \right. \\ &\quad \left. + A \cdot (t) \Big|_{T/2}^T - \frac{4A}{T} \cdot \left(\frac{t^2}{2} \right) \Big|_{T/2}^T + \left(\frac{4AT}{2T} \right) \cdot (t) \Big|_{T/2}^T \right] \\ &= \frac{2}{T} \left[\frac{4A}{2T} \cdot \left(\frac{T^2}{4} \right) - A \cdot \frac{T}{2} \right. \\ &\quad \left. + A \cdot \left(T - \frac{T}{2} \right) - \frac{4A}{2T} \left(T^2 - \frac{T^2}{4} \right) + 2A \left(T - \frac{T}{2} \right) \right] \\ &= \frac{2}{T} \left[\frac{AT}{2} - \cancel{\frac{AT}{2}} + \cancel{\frac{AT}{2}} - \frac{2A}{T} \left(\frac{3T^2}{4} \right) + 2A \cdot \frac{T}{2} \right] \\ &= \frac{2}{T} \left[\frac{AT}{2} - \frac{3AT}{2} + \frac{2AT}{2} \right] = 0 \end{aligned}$$

Problem 1.9 p.1/3

$$\begin{aligned}
 a_n &= \frac{2}{T} \int_0^{T/2} \left(\frac{4A}{T} \cdot t - A \right) \cos n\omega_0 t \, dt \\
 &+ \frac{2}{T} \int_{T/2}^T \left[3A - \frac{4A}{T} \cdot t \right] \cos n\omega_0 t \, dt \\
 &= \frac{2}{T} \cdot \frac{4A}{T} \int_0^{T/2} t \cos n\omega_0 t \, dt - \frac{2A}{T} \int_0^{T/2} \cos n\omega_0 t \, dt \\
 &+ \frac{2}{T} (3A) \int_{T/2}^T \cos n\omega_0 t \, dt \\
 &- \left(\frac{2}{T} \cdot \frac{4A}{T} \right) \int_{T/2}^T t \cos n\omega_0 t \, dt \\
 &- \left(\frac{8A}{T^2} \right) \\
 &= \frac{8A}{T^2} \left[\frac{1}{n^2\omega_0^2} \cos n\omega_0 t + \frac{t}{n\omega_0} \sin n\omega_0 t \right]_0^{T/2} \\
 &- \frac{2A}{T} \left[\frac{1}{n\omega_0} \sin n\omega_0 t \right]_0^{T/2} + \frac{6A}{T} \left[\frac{1}{n\omega_0} \sin n\omega_0 t \right]_0^{T/2} \\
 &- \frac{8A}{T^2} \left[\frac{1}{n^2\omega_0^2} \cos n\omega_0 t + \frac{t}{n\omega_0} \sin n\omega_0 t \right]_{T/2}^T \quad (A)
 \end{aligned}$$

where $\int \cos ax \, dx = \frac{1}{a} \sin ax$

and $\int x \cos ax \, dx = \frac{1}{a^2} \cos ax + \frac{x}{a} \sin ax$

with $a = n\omega_0$ were used.

Problem 1.9 p.2/3

$E_p(A)$ gives

$$\frac{8A}{T^2} \cdot \frac{1}{n^2 \omega_0^2} \left[\cos \frac{n \omega_0 T}{2} \right]$$

$$+ \frac{8A}{T^2} \cdot \frac{T}{2 n \omega_0} \left[\sin \frac{n \omega_0 T}{2} \right] - \frac{2A}{T n \omega_0} \cdot \frac{\sin \frac{n \omega_0 T}{2}}{2} + \frac{6A}{T n \omega_0} \cdot \frac{\sin \frac{n \omega_0 T}{2}}{2}$$

$$- \frac{8A}{T^2} \cdot \frac{1}{n^2 \omega_0^2} \cos n \omega_0 T - \frac{8A}{T n \omega_0} \cdot \frac{\sin n \omega_0 T}{2}$$

$$+ \frac{8A}{T^2 \cdot n^2 \omega_0^2} \cos \frac{n \omega_0 T}{2} + \frac{4A}{T n \omega_0} \cdot \frac{\sin \frac{n \omega_0 T}{2}}{2}$$

$$= \frac{16A}{T^2 \cdot n^2 \omega_0^2} \cos \frac{n \omega_0 T}{2} + \sin \frac{n \omega_0 T}{2} \left[\frac{4A}{T n \omega_0} - \frac{2A}{T n \omega_0} + \frac{6A}{T n \omega_0} + \frac{4A}{T n \omega_0} \right]$$

$$- \frac{8A}{T^2 n^2 \omega_0^2} \cos n \omega_0 T - \frac{8A}{T n \omega_0} \sin n \omega_0 T$$

$$= \frac{4A}{n^2 \pi^2} \cos n \pi + \frac{6A}{n \pi} \sin n \pi$$

$$- \frac{2A}{n^2 \pi^2} \cos 2 n \pi - \frac{4A}{n \pi} \sin 2 n \pi$$

If $n = \text{even} = 2, 4, 6, \dots$; $\cos n \pi = 1$

$$\cos 2 \pi n = 1$$

Problem 1.9 p. 3/3

1.11

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad (1)$$

Since $f(t) = 0$ for $t < 0$, Eq. (1) becomes

$$\begin{aligned} F(\omega) &= \int_0^{\infty} f(t) e^{-i\omega t} dt \\ &= \int_0^{\infty} e^{-t} e^{-i\omega t} dt = \int_0^{\infty} e^{-(1+i\omega)t} dt \\ &= \frac{1}{-(1+i\omega)} e^{-(1+i\omega)t} \Big|_0^{\infty} \\ &= \frac{1}{1+i\omega} \end{aligned}$$

1.12

$$\begin{aligned} F_s(\omega) &= \int_0^{\infty} f(t) \cdot \sin \omega t dt ; \omega > 0 \\ &= \int_0^{\infty} e^{-at} \sin \omega t dt ; a > 0 \\ &= \frac{e^{-at} \left\{ -a \sin \omega t - \omega \cos \omega t \right\}}{a^2 + \omega^2} \Bigg]_{t=0}^{\infty} \\ &= 0 - \frac{\left\{ e^0 [0 - \omega] \right\}}{a^2 + \omega^2} \\ &= \frac{\omega}{a^2 + \omega^2} \end{aligned}$$

1.13

$$F_s(\omega) = \int_0^{\infty} f(t) \cdot \sin \omega t \cdot dt \quad ; \quad \omega > 0$$

$$= \int_0^{\infty} t e^{-t} \sin \omega t \cdot dt \quad ; \quad t \geq 0 \quad (1)$$

From Tables of Integrals, Eq. (1) can be expressed as

$$F_s(\omega) = \left\{ \frac{t e^{-t}}{a^2 + \omega^2} (a \sin \omega t - \omega \cos \omega t) - \frac{e^{-t}}{(1 + \omega^2)^2} [(1 - \omega^2) \sin \omega t + 2\omega \cos \omega t] \right\}_{t=0}^{\infty}$$

With $a = -1$,

$$= \left\{ \frac{\infty \cdot e^{-\infty}}{1 + \omega^2} (-\sin \omega(\infty) - \omega \cos \omega(\infty)) - \frac{e^{-\infty}}{(1 + \omega^2)^2} [(1 - \omega^2) \sin \omega t + 2\omega \cos \omega t] - 0 + \frac{e^{-0}}{(1 + \omega^2)^2} [0 + 2\omega] \right\}$$

$$= \frac{2\omega}{(1 + \omega^2)^2}$$

1.14

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad (1)$$

For $f(t) = 0$ when $t < 0$, Eq. (1) becomes

$$\begin{aligned} F(\omega) &= \int_0^{\infty} f(t) e^{-i\omega t} dt \\ &= \int_0^{\infty} e^{-at} e^{-i\omega t} dt \\ &= \int_0^{\infty} e^{-(a+i\omega)t} dt \\ &= \frac{1}{-(a+i\omega)} e^{-(a+i\omega)t} \bigg|_{t=0}^{\infty} \\ &= \frac{1}{(a+i\omega)} \end{aligned}$$