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1.1. Since the plastic strain is one quarter of the elastic strain, the total strain at the yield point is

$$\varepsilon = \varepsilon^e + \varepsilon^p = \frac{5}{4}\varepsilon^e = \frac{5Y}{4E}$$

and the stress/strain equation gives

$$Y = \frac{E}{180}\left(\frac{5Y}{4E}\right)^{0.25}$$

or $$\left(\frac{Y}{E}\right)^{0.75} = \frac{1}{180}\left(\frac{5}{4}\right)^{0.25}$$

or $$\frac{Y}{E} = \left(\frac{1}{180}\right)^{4/3}\left(\frac{5}{4}\right)^{1/3} = \frac{1}{180}\left(\frac{1}{144}\right)^{1/3} \cong \frac{1}{943} .$$

The true strain at instability is $\varepsilon = 0.25$, while the true and nominal stresses at instability are given by

$$\frac{\sigma}{Y} = \frac{E}{Y}\frac{(0.25)^{0.25}}{180} = (144)^{1/3}(0.25)^{0.25} \cong 3.71$$

$$\frac{s}{Y} = \frac{\sigma}{Y}\left(\frac{A}{A_o}\right) = \frac{\sigma}{Y}e^{-\varepsilon} = 3.71\, e^{-0.25} \cong 2.89 .$$

1.2. The engineering strain at the onset of instability is given by the instability condition

$$\frac{d\sigma}{de} = \frac{d}{de}(C\, e^n) = \frac{n\,\sigma}{e} = \frac{\sigma}{1+e}$$

or $$(1+e)n = e , \quad \text{or} \quad e = n/(1-n)$$

and $$\varepsilon = \ell n(1+e) = \ell n\left(\frac{1}{1-n}\right),$$ which exceeds $\ell n\left(\frac{1}{1-e}\right)$ when $e < n$.

Since the magnitude of the total true strain at instability must be equal to $\ell n\{1/(1-n)\}$,

$$\ell n\frac{1}{1-e} + \ell n\frac{\ell_2}{\ell_1} = \ell n\left(\frac{1}{1-n}\right)$$

where ℓ_1 is the length of the bar at the end of compression, and ℓ_2 the length at instability. Then

$$\ell_2 = \ell_1\left(\frac{1-e}{1-n}\right) = \ell_o(1-e)\left(\frac{1-e}{1-n}\right) = \ell_o\frac{(1-e)^2}{1-n}$$

where ℓ_o is the initial length of the bar.

1.3. According to the Voce equation, the instability condition is

$$\frac{d\sigma}{d\varepsilon} = \frac{d}{d\varepsilon}\{C(1-m\,e^{-n\varepsilon})\} = C\,m\,n\,e^{-n\varepsilon} = n(C-\sigma)$$

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and the onset of instability corresponds to

$$n(C - \sigma) = \sigma, \quad \text{or} \quad \sigma = Cn/(1 + n) .$$

Hence, $$e^{n\varepsilon} = \frac{C\ m}{C - \sigma} = m\left(1 - \frac{n}{1 + n}\right) = m(1 + n)$$

and $$\varepsilon = \frac{1}{n}\ \ln[\,m(1 + n)\,] \ , \quad m(1 + n) \geqslant 0 .$$

When $m(1 + n) < 1$, the instability strain is zero for a rigid/plastic bar. Introducing ε^*,

$$\sigma = C(1 - m\ e^{-n\varepsilon}) = C\left\{1 - m\left(\frac{\ell_0}{\ell}\right)^n\right\}$$

$$= C\left\{(1 - m) + m\left[1 - \left(\frac{\ell_0}{\ell}\right)^n\right]\right\} = C(1 - m + m\,n\,\varepsilon^*) .$$

The stress-strain curve is thus linearized. Note that when $n=0$, $\varepsilon^*= n(\ell/\ell_0)=\varepsilon$.

1.4. The total compressive load at any stage is

$$P = \sigma A = \sigma A_0\left(\frac{h_0}{h}\right) = \sigma A_0/(1 - e) .$$

Since $d\varepsilon = de/(1 - e)$ in compression, the differentiation of the above equation gives

$$\frac{dP}{de} = \frac{A_0}{(1 - e)^2}\ \frac{d\sigma}{d\varepsilon} + \frac{\sigma A_0}{(1 - e)^2} = \frac{A_0}{(1 - e)^2}\left(\frac{d\sigma}{d\varepsilon} + \sigma\right)$$

$$\frac{d^2P}{de} = \frac{A_0}{(1 - e)^3}\left(\frac{d^2\sigma}{d\varepsilon^2} + \frac{d\sigma}{d\varepsilon}\right) + \frac{2A_0}{(1 - e)^3}\left(\frac{d\sigma}{d\varepsilon} + \sigma\right) .$$

Hence, at the point of inflection ($d^2P/de^2 = 0$),

$$\frac{d^2\sigma}{d\varepsilon^2} + 3\ \frac{d\sigma}{d\varepsilon} + 2\sigma = 0 .$$

The empirical stress-strain equation $\sigma = C\varepsilon^n$ gives

$$\frac{d\sigma}{d\varepsilon} = \frac{n\ \sigma}{\varepsilon}\ , \quad \frac{d^2\sigma}{d\varepsilon^2} = \frac{n}{\varepsilon}\ \frac{d\sigma}{d\varepsilon} - \frac{n\ \sigma}{\varepsilon^2} = n(n - 1)\ \frac{\sigma}{\varepsilon^2} .$$

Hence the point of inflection corresponds

$$\left\{\frac{n(n - 1)}{\varepsilon^2} + \frac{3n}{\varepsilon} + 2\right\}\sigma = 0$$

or $$2\varepsilon^2 + 3\,n\,\varepsilon - n(1 - n) = 0$$

or $$\varepsilon = \frac{1}{4}\left[\,-3n + \sqrt{n(8 + n)}\,\right]$$

Thus, $\varepsilon > n$ corresponds to $\sqrt{n(8 + n)} > 7n$, or $8n + n^2 > 49n^2$, or $n < 1/6$.

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1.5. The nominal compressive stress at any stage is

$$s = \frac{P}{A_0} = \frac{P}{A}\left(\frac{h_0}{h}\right) = \sigma \exp(\varepsilon) .$$

By successive differentiation, we get

$$\frac{ds}{d\varepsilon} = \left(\frac{d\sigma}{d\varepsilon} + \sigma\right) \exp(\varepsilon)$$

$$\frac{d^2 s}{d\varepsilon^2} = \left(\frac{d^2\sigma}{d\varepsilon^2} + 2\frac{d\sigma}{d\varepsilon} + \sigma\right) \exp(\varepsilon) .$$

The point of inflection corresponds to $d^2s/d\varepsilon^2 = 0$, or

$$\frac{d^2\sigma}{d\varepsilon^2} + 2\frac{d\sigma}{d\varepsilon} + \sigma = \left(\frac{d}{d\varepsilon} + 1\right)^2 \sigma = 0$$

or $$\left\{\frac{n(n-1)}{\varepsilon^2} + \frac{2n}{\varepsilon} + 1\right\}\sigma = 0$$

or $$\varepsilon^2 + 2n\varepsilon - n(1-n) = 0, \quad \text{or} \quad \varepsilon = \sqrt{n} - n.$$

The inflection strain will exceed the uniaxial instability strain if $\sqrt{n} - n > n$, or $n < 0.25$.

1.6. The differentiation of the Ramberg-Osgood equation furnishes

$$\frac{d\varepsilon}{d\sigma} = \frac{1}{E} + \frac{3}{7nE}\left(\frac{\sigma}{\sigma_0}\right)^{(1-n)/n} = \frac{1}{\sigma}$$

at the onset of instability in simple tension.

Since $E >> \sigma$, we have

$$\frac{3\sigma_0}{7nE}\left(\frac{\sigma}{\sigma_0}\right)^{1/n} \cong 1, \quad \text{or} \quad \frac{\sigma}{\sigma_0} \cong \left(\frac{7nE}{3\sigma_0}\right)^n$$

and $$\varepsilon \cong \frac{\sigma_0}{E}\left(\frac{7nE}{3\sigma_0}\right)^n + \frac{3\sigma_0}{7E}\left(\frac{7nE}{3\sigma_0}\right)$$

or $$\varepsilon = n + \left(\frac{7n}{3}\right)^n \left(\frac{\sigma_0}{E}\right)^{1-n}$$

When $n = 0.05$ and $\sigma_0/E = 0.002$, the instability strain is

$$\varepsilon = 0.05 + \left(\frac{0.35}{3}\right)^{0.05} (0.002)^{0.95} \cong 0.0525$$

the percentage error being $0.25/(0.0525) \cong 4.8\%$.

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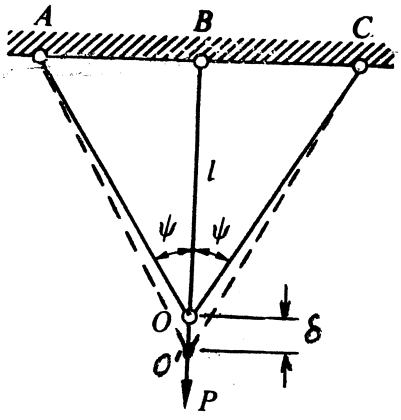
1.7.



If plastic instability occurs simultaneously in the three bars, the true strain is n_1 in the central bar and n_2 in the inclined bars at instability. If O is displaced to O' at the point of necking, then from geometry,

$$O'B = \ell e^{n_1},$$

$$O'A = O'C = \ell e^{n_2} \sec\psi .$$

From geometry,

$$\ell^2 \tan^2\psi = AB^2 = O'A^2 - O'B^2$$

$$= \ell^2 (e^{2n_2} \sec^2\psi - e^{2n_1})$$

or $$e^{2n_1} - 1 = (e^{2n_2} - 1)\sec^2\psi$$

or $$\cos\psi = \sqrt{\frac{\exp(2n_2) - 1}{\exp(2n_1) - 1}} .$$

1.8. During a small deformation produced by a vertical load P at O, the strains in the vertical and inclined bars are

$$\varepsilon_1 = \frac{\delta}{\ell}, \qquad \varepsilon_2 = \frac{\delta \cos\psi}{\ell \sec\psi} = \frac{\delta}{\ell}\cos^2\psi$$

respectively. When all the bars are plastic, the corresponding stresses are

$$\sigma_1 = Y\left(\frac{E\delta}{Y\ell}\right)^n, \qquad \sigma_2 = Y\left(\frac{E\delta}{Y\ell}\cos^2\psi\right)^n$$

Hence the applied load is

$$P = A(\sigma_1 + 2\sigma_2 \cos\psi)$$

or $$\frac{P}{AY} = \left(\frac{E\delta}{Y\ell}\right)^n (1 + 2\cos^{2n+1}\psi)$$

which holds for $\varepsilon_2 \geqslant Y/E$, or $E\delta/Y\ell \geqslant \sec^2\psi$. When only the vertical bar is plastic, the stresses are

$$\sigma_1 = Y\left(\frac{E\delta}{Y\ell}\right)^n, \qquad \sigma_2 = \frac{E\delta}{\ell}\cos^2\psi .$$

Hence $$\frac{P}{AY} = \left(\frac{E\delta}{Y\ell}\right)^n + 2\left(\frac{E\delta}{Y\ell}\right)\cos^3\psi$$

which holds for $\varepsilon_1 \geqslant Y/E$ and $\varepsilon_2 \leqslant Y/E$. Therefore, the restriction is $1 \leqslant E\delta/Y\ell \leqslant \sec^2\psi$. The results corresponding to $\psi = \pi/4$ and $n = 0.25$ are

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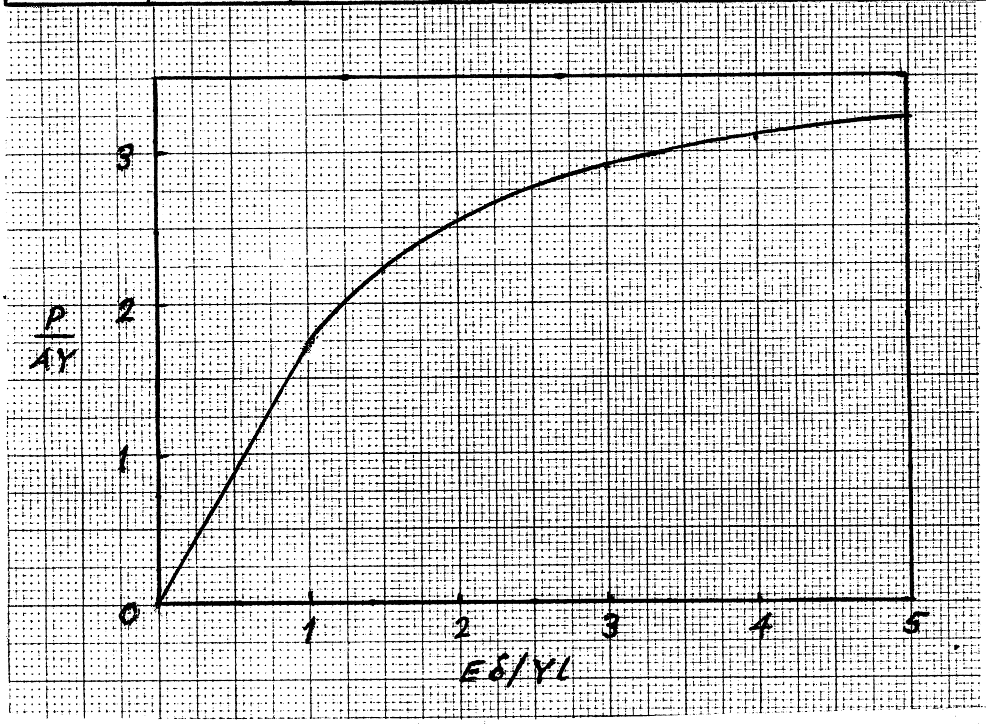
$E\delta/Y\ell$	1.0	1.5	2.0	3.0	4.0	5.0
P/AY	1.707	2.167	2.603	2.881	3.096	3.276

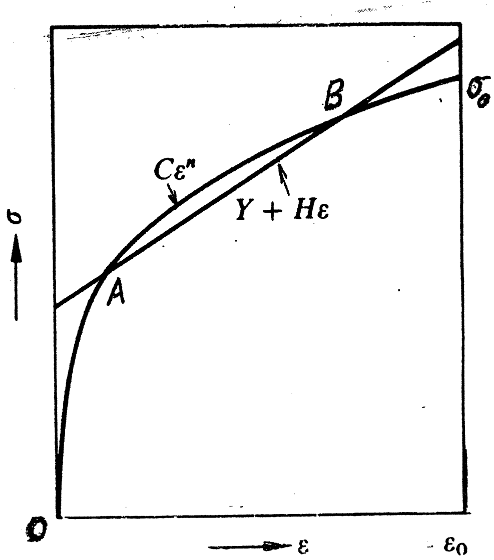


1.9. Equating the total work corresponding to the stress-strain equations,

$$\int_0^{\varepsilon_0} (Y + H\varepsilon)\,d\varepsilon = C \int_0^{\varepsilon_0} \varepsilon^n \,d\varepsilon$$

or $\quad Y\,\varepsilon_0 + \frac{1}{2} H\,\varepsilon_0 = \frac{C}{1+n}\,\varepsilon_0^{1+n} = \frac{\sigma_0\,\varepsilon_0}{1+n}$

or $\quad 2Y + H\,\varepsilon_0 = 2\,\sigma_0/(1+n)$ (a)



The second condition furnishes the relation

$$(Y + H\varepsilon_0) - C\,\varepsilon_0^n = 2\left\{ C\left(\frac{\varepsilon_0}{2}\right)^n - \left(Y + \frac{H\varepsilon_0}{2}\right)\right\}$$

or $\quad 3Y + 2H\,\varepsilon_0 = C\,\varepsilon_0^n\,(1 + 2^{1-n}) = \sigma_0(1 + 2^{1-n})$ (b)

Solving (a) and (b) for Y and H, we have

$$Y = \sigma_0\left(\frac{3-n}{1+n} - 2^{1-n}\right), \qquad H\,\varepsilon_0 = 2\,\sigma_0\left(2^{1-n} - \frac{2-n}{1+n}\right).$$

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The maximum percentage error in the linear approximation over the range AB corresponds to

$$\frac{d}{d\varepsilon}\left(1 - \frac{Y + H\varepsilon}{C\varepsilon^n}\right) = \frac{-H + (Y + H\varepsilon)(n/\varepsilon)}{C\varepsilon^n} = 0$$

or $$\varepsilon = \left(\frac{n}{1-n}\right)\frac{Y}{H}.$$

Hence $$Y + H\varepsilon = \frac{Y}{1-n} = \frac{\sigma_0}{1-n}\left(\frac{3-n}{1+n} - 2^{1-n}\right)$$

and $$C\varepsilon^n = \sigma_0\left(\frac{\varepsilon}{\varepsilon_0}\right)^n = \sigma_0\left(\frac{n}{1-n}\right)^n\left(\frac{Y}{H\ \varepsilon_0}\right)^n$$

$$= \sigma_0\left(\frac{n}{1-n}\right)^n\left(\frac{3-n}{1+n} - 2^{1-n}\right)^n \left\{2\left(2^{1-n} - \frac{2-n}{1+n}\right)\right\}^n .$$

Setting $n = 0.3$, we get

$$Y + H\varepsilon = \frac{\sigma_0}{0.7}\left(\frac{2.7}{1.3} - 2^{0.7}\right) = \frac{0.4524}{0.7}\sigma_0 = 0.6463\ \sigma_0$$

$$C\varepsilon^n = \sigma_0\left(\frac{3}{7}\right)^{0.3}(0.4524)^{0.3}\left\{2\left(2^{0.7} - \frac{1.7}{1.3}\right)\right\}^{0.3}$$

$$= \sigma_0\ (0.3060)^{0.3} = 0.7010\ \sigma_0$$

and the percentage error is $\left(1 - \frac{0.6463}{0.7010}\right)(100) = 7.8\%$.

1.10. Let r denote the current mean radius of the ring and t the current thickness. When the angular velocity is ω, the centrifugal force acting in an element of the ring is

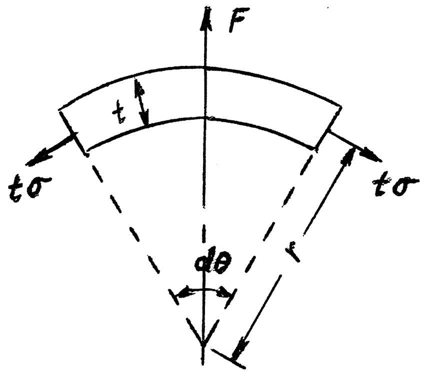
$$F = \rho\,\omega^2 r(t\,r\,d\theta) = \rho\,\omega^2 r^2 t\,d\theta .$$

For radial equilibrium, $F = t\,\sigma\,d\theta$, or

$$\sigma = \rho\,\omega^2 r^2 .$$

Hence $$\frac{d\sigma}{\sigma} = 2\left(\frac{d\omega}{\omega} + \frac{dr}{r}\right).$$

Since $d\omega = 0$ at the onset of instability, we have



$$\frac{d\sigma}{\sigma} = 2\,d\,\varepsilon \quad , \quad \text{or} \quad \frac{d\sigma}{d\varepsilon} = 2\sigma$$

where σ is the uniaxial stress acting in the ring circumferentially, and ε the corresponding strain. Since $d\sigma/d\varepsilon = n\sigma/\varepsilon$ by the given power law, $\varepsilon = n/2$

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at instability. The maximum angular velocity is given by

$$\rho\,\omega^2 r_0^2 = \sigma(r_0/r)^2 = C\,\varepsilon^n e^{-2\varepsilon} = C\left(\frac{n}{2}\right)^n e^{-n}$$

1.11. In a thin spherical shell under an internal pressure p, the non-zero principal stresses are each equal to σ. Thus

$$\sigma_\theta = \sigma_\phi = \sigma = \frac{pr}{2t}$$

where r is the current mean radius and t the current thickness. The components of the strain increment are

$$d\varepsilon_r = \frac{dt}{t} = -d\varepsilon, \qquad d\varepsilon_\theta = d\varepsilon_\phi = \frac{dr}{r} = \frac{1}{2}\,d\varepsilon \quad .$$

Since $dp = 0$ at the onset of instability,

$$\frac{d\sigma}{\sigma} = \frac{dp}{p} + \frac{dr}{r} - \frac{dt}{t} = \frac{3}{2}\,d\varepsilon$$

and the instability condition becomes

$$\frac{d\sigma}{d\varepsilon} = \frac{3}{2}\,\sigma \quad .$$

The empirical equation $\sigma = C\varepsilon^n$ gives $n\,\sigma/\varepsilon = 3\,\sigma/2$, or $\varepsilon = \frac{2}{3}\,n$ at instability, and the corresponding thickness and radius are

$$t = t_0 e^{-\varepsilon} = t_0 \exp\left(-\frac{2}{3}\,n\right)$$

$$r = r_0 e^{-\varepsilon/2} = r_0 \exp\left(\frac{1}{3}\,n\right).$$

Hence, the bursting pressure is given by

$$\frac{p}{C} = \frac{2\sigma t}{Cr} = \frac{2\sigma t_0}{Cr_0}\,e^{-n} = \frac{2\,t_0}{r_0}\left(\frac{2}{3}\,n\right)^n e^{-n} \quad .$$

1.12. Let the longitudinal tensile stresses existing in the inner and outer cylinders, having cross-sectional areas A_1 and A_2 respectively, be denoted by σ_1 and σ_2. Then the resultant a ial tension is

$$P = A_1\sigma_1 + A_2\sigma_2 \quad .$$

Hence
$$\frac{dP}{d\varepsilon} = \left(A_1\,\frac{d\sigma_1}{d\varepsilon} + \sigma_1\,\frac{dA_1}{d\varepsilon}\right) + \left(A_2\,\frac{d\sigma_2}{d} + \sigma_2\,\frac{dA_2}{d}\right)$$

where ε is the longitudinal strain in the composite bar of length ℓ. Since $A_1\ell$ and $A_2\ell$ are constants by the constancy of volume,

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$$\frac{dA_1}{A_1} = \frac{dA_2}{A_2} = -\frac{d\ell}{\ell} = -d\varepsilon$$

and A_2/A_1 remains constant during the deformation. At the load maximum, $dP = 0$, giving

$$0 = A_1\left(\frac{d\sigma_1}{d\varepsilon} - \sigma_1\right) + A_2\left(\frac{d\sigma_2}{d\varepsilon} - \sigma_2\right)$$

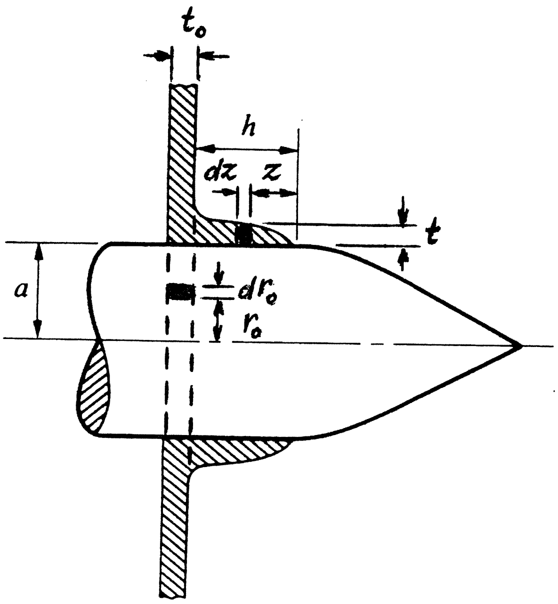
$$= A_1\sigma_1\left(\frac{n_1}{\varepsilon} - 1\right) + A_2\sigma_2\left(\frac{n_2}{\varepsilon} - 1\right)$$

or $$\varepsilon = \frac{A_1\sigma_1 n_1 + A_2\sigma_2 n_2}{A_1\sigma_1 + A_2\sigma_2} = \frac{n_1 + n_2}{2}$$

if $A_1\sigma_1 = A_2\sigma_2$ at instability. Then

$$\frac{A_2}{A_1} = \frac{\sigma_1}{\sigma_2} = \frac{C_1}{C_2}\,\varepsilon^{n_1-n_2} = \frac{C_1}{C_2}\left(\frac{n_1+n_2}{2}\right)^{n_1-n_2} .$$

1.13.



Let r_o be initial radius to an element that is currently at a distance z from the outer edge of the lip. In view of the incompressibility of the material,

$$2\pi r_o t_o dr_o = 2\pi a t dz$$

or $$dz/dr_o = r_o t_o/at .$$

Since the state of stress is uniaxial, the thickness strain is one half in magnitude of the hoop strain. Hence

$$\ln\left(\frac{t}{t_o}\right) = -\frac{1}{2}\ln\left(\frac{a}{r_o}\right), \text{ or } \frac{t}{t_o} = \sqrt{\frac{r_o}{a}}$$

which gives

$$\frac{dz}{dr_o} = \frac{r_o}{a}\sqrt{\frac{a}{r_o}} = \sqrt{\frac{r_o}{a}}$$

or $$\frac{z}{a} = a^{-3/2}\int_0^{r_o}\sqrt{r_o}\,dr_o = \frac{2}{3}\left(\frac{r_o}{a}\right)^{3/2} .$$

Since $z = h$ when $r_o = a$, we get $h = \frac{2}{3}a$. Also, the thickness variation is given by

$$\frac{t}{t_o} = \left(\frac{r_o}{a}\right)^{1/2} = \left(\frac{3z}{2a}\right)^{1/3} .$$

The plastic work per unit volume of a typical element is

$$E = \int\sigma\,d\varepsilon = C\int_0^{\ln(a/r_o)}\varepsilon^n d\varepsilon = \frac{C}{1+n}\left(\ln\frac{a}{r_o}\right)^{1+n} .$$

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The total work done during the perforation is

$$W = 2\pi a \int_0^h E\,t\,dz = \frac{2\pi C t_0}{1+n} \int_0^a \left(\ln \frac{a}{r_0}\right)^{1+n} r_0\, dr_0$$

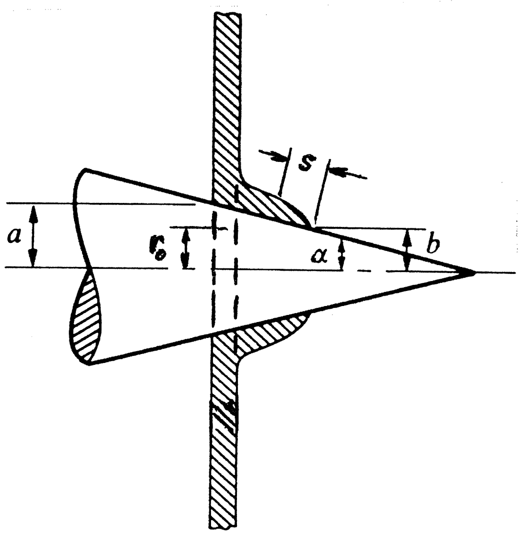
$$= \frac{\pi C t_0 a^2}{(1+n)2^{1+n}} \int_0^\infty e^{-x} x^{1+n}\, dx \quad \left(x = 2 \ln \frac{a}{r_0}\right)$$

or $$W = \frac{\pi C t_0 a^2}{(1+n)2^{1+n}} \Gamma(2+n) = \frac{\pi C t_0 a^2}{2^{1+n}} \Gamma(1+n) \,.$$

When $n = 0.5$, we have

$$W = \frac{\pi\ C\ t_0\ a^2}{2^{5/2}} \Gamma\left(\frac{1}{2}\right) = \frac{\pi \sqrt{\pi}\ C\ t_0\ a^2}{4\sqrt{2}} \cong 0.984\ a^2 t_0 C \,.$$

1.14.



Let s denote the inclined distance of a typical element that was initially at a radial distance r_0 from the axis of symmetry. The incompressibility of the material requires

$$2\pi r_0 t_0\, dr_0 = 2\pi (b + s \sin\alpha)\, t\, ds \,.$$

Since the state of stress is uniaxial,

$$t_0\sqrt{r_0} = t\sqrt{b + s \sin\alpha} \,.$$

Hence $\sqrt{b + s\sin\alpha}\ (ds/dr_0) = r_0\sqrt{r_0}$.

The integration of this equation gives

$$(b + s \sin\alpha)^{3/2} - b^{3/2} = r_0^{3/2} \sin\alpha \,.$$

Since $b + s\sin\alpha = a$ when $r_0 = a$, we have

$$a^{3/2} - b^{3/2} = a^{3/2}\sin\alpha, \quad \text{or} \quad b = a(1 - \sin\alpha)^{2/3} \,.$$

Also, $$\left(1 + \frac{s}{b}\sin\alpha\right)^{3/2} = 1 + \left(\frac{r_0}{b}\right)^{3/2} \sin\alpha \,.$$

The thickness variation of the perforated plate is therefore given by

$$\frac{t}{t_0} = \sqrt{\frac{r_0}{b}} \left(1 + \frac{s}{b}\sin\alpha\right)^{-1/2} = \sqrt{\frac{r_0}{b}} \left\{1 + \left(\frac{r_0}{b}\right)^{3/2} \sin\alpha\right\}^{-1/3} \,.$$

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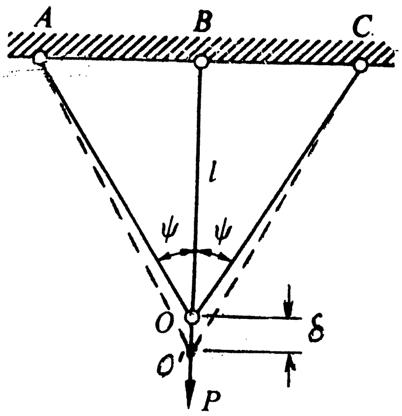
1.15.



In the purely elastic range, the stresses are

$$\sigma_1 = E\,\varepsilon_1 = \frac{E\,\delta}{\ell}$$

$$\sigma_2 = E\,\varepsilon_2 = \frac{E\,\delta}{\ell}\cos^2\psi = \sigma_1\cos^2\psi$$

and the applied load is

$$P = A(\sigma_1 + 2\sigma_2\cos\psi) = A\,\sigma_1\,(1 + 2\cos^3\psi).$$

Hence the elastic stresses become

$$\sigma_1 = \frac{P/A}{1+2\cos^3\psi}\,, \quad \sigma_2 + \frac{(P/A)\cos^2\psi}{1+2\cos^3\psi}\,.$$

Subtracting these stresses from the fully plastic stresses of Prob. 1.8, we obtain the residual stresses. Thus

$$\frac{\sigma_1'}{Y} = \left(\frac{E\,\delta}{Y\ell}\right)^n - \frac{P/AY}{1+2\cos^3\psi} = \left(\frac{E\,\delta}{Y\ell}\right)^n \left\{1 - \frac{1+2\cos^{2n+1}\psi}{1+2\cos^3\psi}\right\}$$

or $$\frac{\sigma_1'}{2Y}\sec\psi = -\left(\frac{E\,\delta}{Y\ell}\right)^n \left(\frac{\cos^{2n}\psi - \cos^2\psi}{1+2\cos^3\psi}\right)\,.$$

Since $\sigma_1 + 2\sigma_2'\cos\psi = 0$ for the unloaded structure,

$$\sigma_2'/Y = -(\sigma_1/2Y)\sec\psi\ .$$

The residual deflection δ' of point O is obtained by subtracting $(\sigma_1/E)\ell$ from the plastic deflection δ. Thus

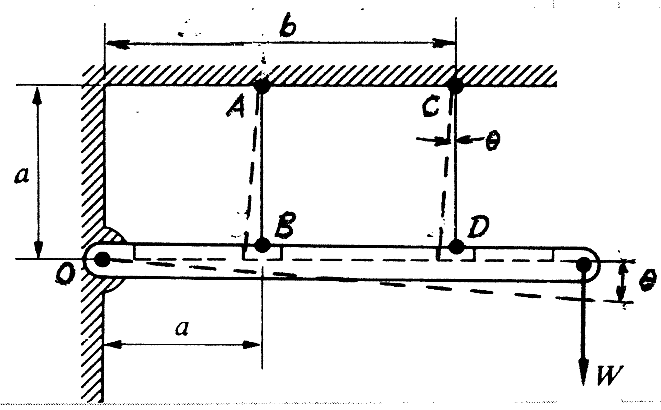
$$\frac{E\,\delta'}{Y\ell} = \frac{E\,\delta}{Y\ell} - \left(\frac{E\,\delta}{Y\ell}\right)^n \left(\frac{1+2\cos^{2n+1}\psi}{1+2\cos^3\psi}\right)\,.$$

When $n = 0.25$, $\psi = \pi/4$ and $E\delta/Y\ell = 3$, we get

$$\frac{\sigma_1'}{Y} = -\sqrt{2}\,(3)^{0.25}\left\{\frac{(0.5)^{0.25} - 0.5}{1+0.707}\right\} = -0.372\ .$$

$$\frac{E\,\delta'}{Y\ell} = 3 - (3)^{0.25}\left\{\frac{1+\sqrt{2}\,(0.5)^{0.25}}{1\,+\,0.707}\right\} = 1.312\ .$$

1.16



At the onset of instability, the horizontal rigid bar is inclined at an angle θ. Since the groove is smooth, the stretched wires AB' and CD' must remain perpendicular to the center line of the bar. From geometry, we have

$$AB' = a(\cos\theta + \sin\theta)$$

$$CD' = a\cos\theta + b\sin\theta\ .$$

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