Chapter 1

Introduction

Problem 1.1 Lepton numbers

The electron is (fortunately) stable. The μ^- and τ^- decay only by weak W^- emission since decay through Z, or γ emission leave the original lepton intact and would violate energy and momentum conservation. The W^- can decay to $\mu^-\bar{\nu}_\mu, e^-\bar{\nu}_e$ or quark pairs $q^{-1/3}\bar{q}^{-2/3}$ conserving lepton number. The only decay mode available to the muon is $\mu^- \to \nu_\mu(e^-\bar{\nu}_e)$ where lepton number is conserved. Decays including quarks can not conserve four momentum since there is no lighter hadron. ($m_\mu < m_\pi$).

Diagrams for μ^- decays appear in Figure 1.1. There are three leading order contributions to $\mu^- \to e^-\bar{\nu}_e\nu_\mu\gamma$ corresponding to radiation of the photon by the μ^-, W^-, or e^-. In each of the three cases, the photon may materialize as an electron positron pair and contribute to $\mu^- \to e^-\bar{\nu}_e\nu_\mu e^+e^-$ as exemplified by Figure 1.1(e). Additional leading order contributions to $\mu^- \to e^-\bar{\nu}_e\nu_\mu e^+e^-$ result from replacing the photon by a Z or H but these are suppressed by the high value of m_Z or m_H.

For each of the allowed decay modes, the initial muon number of the μ^- appears in the final state as a ν_μ. The initial electron number is zero and, in the final state, the e^- and $\bar{\nu}_e$ have opposite electron number. A photon or e^+e^- pair carries no net lepton number. Hence, electron and muon number are both conserved in the allowed decays. In the decays $\mu^- \to e^-\gamma$, $\mu^- \to e^-e^+e^-$, $\mu^- \to e^-\gamma\gamma$, one unit of initial muon number disappears and one unit of net electron number appears in the final state. Hence both muon and electron number are violated. In $\tau^- \to e^-\mu^+\mu^-$ and

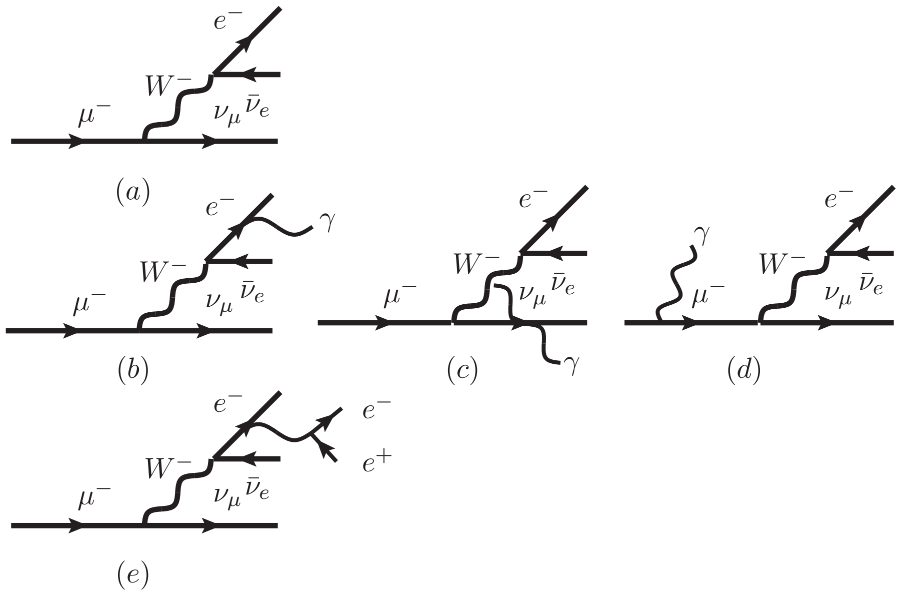

Figure 1.1: Fundamental diagrams for μ^- decays. (a) $\mu^- \to e^-\bar{\nu}_e\nu_\mu$ ($\Gamma_i/\Gamma \simeq 100\%$) , (b)(c)(d) $\mu^- \to e^-\bar{\nu}_e\nu_\mu\gamma$ ($\Gamma_i/\Gamma \simeq 1.4 \pm 0.04\%$) , and (e) $\mu^- \to e^-\bar{\nu}_e\nu_\mu e^+e^-$.

Mode	Fraction
$\mu^-\bar{\nu}_\mu\nu_\tau$	17.36%
$e^-\bar{\nu}_e\nu_\tau$	17.85%
$\pi^-\nu_\tau$	10.9%
$\pi^-\pi^0\nu_\tau$	25.94%
$\pi^-2\pi^0\nu_\tau$	9.3%
$3\pi^0\nu_\tau$	1.18%
$\pi^-3pi^0\nu_\tau$	1.18%
$\pi^-\pi^-\pi^+\nu_\tau$	9.32%
$2\pi^-\pi^0\pi^+\nu_\tau$	4.61%

Table 1.1: Principal decays modes of the τ^-.

$\tau^- \to e^-\pi^+K^-$, one unit of τ^- number disappears and one unit of electron number appears. In $\pi^0 \to \mu^+e^-$, one unit of electron number and negative one unit of muon number appear. The branching fractions from the PDG are $\Gamma_{\mu^-\to e^-\gamma}/\Gamma_\mu < 1.2\times 10^{-11}$, $\Gamma_{\mu^-\to e^-e^+e^-}/\Gamma_\mu < 1.0\times 10^{-12}$, $\Gamma_{\tau^-\to e^-\mu^+\mu^-}/\Gamma_\tau < 3.7\times 10^{-8}$,$\Gamma_{\tau^-\to e^-\pi^+K^-}/\Gamma_\tau < 5.8\times 10^{-8}$,$\Gamma_{\pi^0\to\mu^\pm e^\mp}/\Gamma_\pi < 3.6\times 10^{-10}$.

Problem 1.2 The τ^- lepton

The principal decay modes are listed in Table **??**. The τ^- decays derive from the transition $\tau^- \to W^-\nu_\tau$ with the virtual W^- decaying to $\mu^-\bar{\nu}_\mu, e^-\bar{\nu}_e$ or $q^{-1/3}\bar{q^{-2/3}}$ where the quark pairs must be color neutral. The leptonic decays of the W^- account for 35% of the τ^- decay width. The quark pair combinations consistent with kinematics are $\bar{u}d$ and $\bar{u}s$ and $\bar{u}d$ is favored by the CKM matrix. If $\bar{u}d$ bind into the ground state, a single π^- is produced and accompanies the ν_τ. If $\bar{u}d$ bind into an excited state such as the ρ^- which decays strongly to $\pi^-\pi^0$, an additional π^0 is produced. Production of a resonance such as the ω which decays to three pions and non-resonant color processes must be responsible for decays with more than two pions. The single prong fraction is 85%. The three prong fraction is 15%. The five prong fraction is 0.1 %.

Problem 1.3 Quark and baryon number

When a γ, Z, H, or gluon is radiated or absorbed by a fermion, the fermion flavor is unchanged. When a fermion leg is crossed from initial to final state or from final state to initial state, it is interpreted as an antiparticle and its fermion flavor number is reversed so fermion flavor is conserved. The sum of quark fermion numbers is the quark number so this is conserved. Weak interactions of the $W^\pm$ bosons induce transitions between lepton pairs and between u-like and d-like quarks so conserve total lepton number (as well as individual lepton numbers) and total quark number. So from a fixed number of quarks and antiquarks in the form of mesons and baryons, standard model interactions can only induce transitions to states with the same quark number. Allowed initial states of mesons and baryons have quark number N_q equal to three times the baryon number of the initial state. Since quark number is conserved, the final state following any standard model interaction must contain N_q quarks which must and can combine to form baryons plus an arbitrary number of $q\bar{q}$ pairs which can combine into mesons or a combination of mesons and baryons. If three of the final additional quarks bind into a baryon, three antiquarks must be left unpaired and must bind into an antibaryon. In any event, net zero additional baryon number appears.

In $\pi^- p$ collisions, the initial charge is zero and the initial baryon number is one. If the final state contains one $\bar{n}$ which has baryon number -1, it must contain two baryons such as pp, pn or nn. To conserve charge, the final states must therefore be $\pi^-\pi^- pp\bar{n}$, $\pi^- pn\bar{n}$, or $nn\bar{n}$. The last case could appear via annihilation of a $\bar{u}$ in the π^- with a u in the p producing n plus a gluon, the gluon fragmenting into $u\bar{u}$, two further gluons producing two $d\bar{d}$ pairs, these three quarks and antiquarks combining to form $n\bar{n}$.

Problem 1.4 Strangeness conservation

Diagrams for these processes appear in Figure 1.2.

Problem 1.5 Photon mass

The Earth's field limit corresponds to

$$m_\gamma = \frac{p}{L} = \frac{0.05 \times 0.2 \text{ GeV fm}}{10^{22} \text{ fm}} = 10^{-15} \text{ eV}.$$

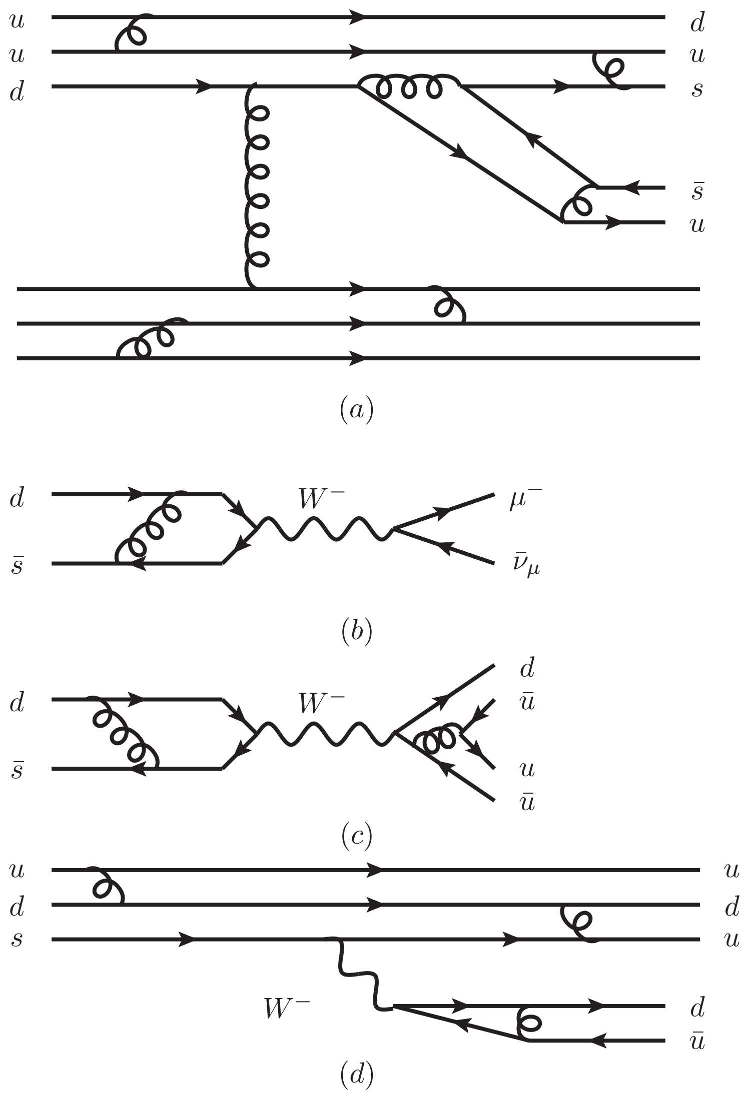

Figure 1.2: Strange quark processes. (a) $\mathrm{p} + \mathrm{Nucleus} \rightarrow \Lambda + \mathrm{K}^+ + \mathrm{Nucleus}$, (b) $K^- \rightarrow \mu^-\bar{\nu}_\mu$, (c) $K^- \rightarrow \pi^-\pi^0$, (d) $\Lambda \rightarrow p\pi^-$.

The Compton wavelength for photon mass $m_\gamma = 10^{-17}$ eV is

$$\lambda_C = \frac{0.2 \text{ GeV fm}}{10^{-26} \text{ GeV}} = 0.2 \times 10^{11} \text{ m} \simeq 0.1 \text{ a.u.}$$

Problem 1.6 Static weak interaction energy

a.) A particle of mass μ confined to a range r has momentum $p \sim r^{-1} = m_Z$ and, if nonrelativistic, has kinetic energy $K = p^2/(2\mu) = (2\mu r^2)^{-1} = m_Z^2/(2\mu)$. The potential energy is of order $U = -g^2/r = -g^2 m_Z$ and $K+U < 0$ gives the result. b.) $\mu = \alpha^{-1} m_Z \simeq$12 TeV. $m_Z = \alpha m_e = 3.7$ keV. c.) Since the weak interaction is short range,

$$\Delta E \simeq |\psi(0)|^2 \int d\mathbf{r}(-g^2 \frac{e^{-m_Z r}}{r}) = \frac{\alpha m_e^3}{\pi}(4\pi) \int_0^\infty dr \quad re^{-m_Z r}.$$

The integral is of the form

$$\int_0^\infty dr \quad re^{ar} = \frac{d}{da} \int_0^\infty e^a = a^{-2}.$$

Since the Rydberg is $E_0 = m_e \alpha^2/2$, we have

$$\Delta E \simeq 4\alpha m_e^3 m_Z^{-2} = 8\alpha (m_e/m_Z)^2 E_0.$$

Problem 1.7 Light hadron decays

Feynman diagrams for pion decay appear in Figure 1.3. The decay of the π^0 results from a second order electromagnetic process. The decay of the π^- results from a second order weak process. The decay of the ρ^+ results from a second order color process. In each case, additional color interactions bind the quarks into hadrons. The lifetime of the π^0 is $\tau_{\pi^0} = 8.4 \times 10^{-17}$ s. The similar process $\eta \to \gamma\gamma$ has a branching fraction of 0.71 and total width of 1.3 keV so partial width of 0.92 keV. The lifetime corresponding to the $\gamma\gamma$ decay is

$$\tau_\eta = \frac{197 \text{ MeV fm}}{(3 \times 10^{23} \text{ fm s}^{-1})(.92 \times 10^{-3} \text{ MeV})} = 0.71 \times 10^{-18} \text{ s.}$$

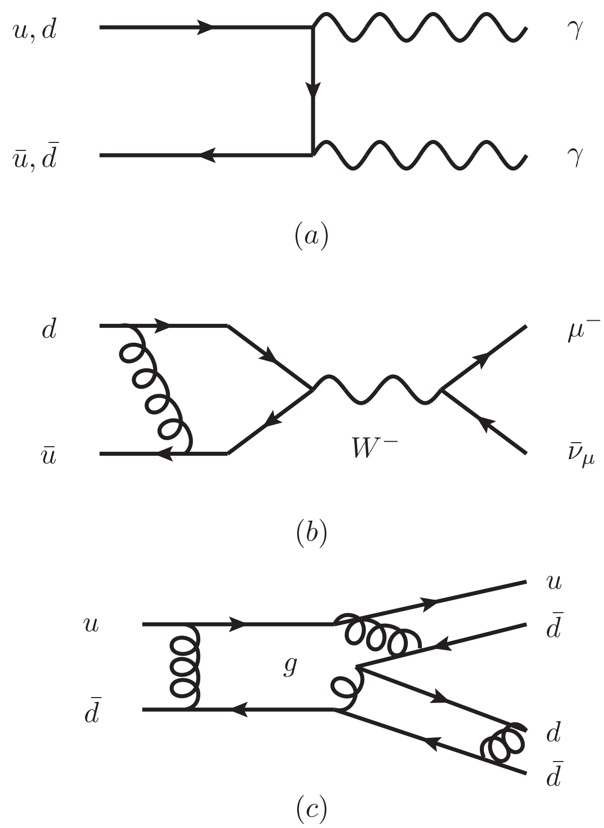

Figure 1.3: Feynman diagrams for (a) $\pi^0 \to \gamma\gamma$, (b) $\pi^- \to \mu^-\bar{\nu}_\mu$, and (c) $\rho^+ \to \pi^+\pi^0$.

The rest masses $m_\pi = 135$ MeV and $m_\eta = 547$ MeV are converted to photon energy and a shorter lifetime is associated with higher energy. The lifetime of the π^- is 2.6×10^{-8} s. The lifetime of the K^- is 1.2×10^{-8} s and the lifetime associated with the decay to $\mu^- \bar{\nu}_\mu$ is 1.9×10^{-8} s. The masses are $m_\pi = 139$ MeV and $m_K = 493$ MeV. The CKM factor in the amplitude for $\pi^- \to \mu^- \bar{\nu}_\mu$ is $V_{ud} \simeq 1$. The CKM factor in the amplitude for $K^- \to \mu^- \bar{\nu}_\mu$ is $V_{su} \simeq 0.22$. These factors appear squared in the decay rate so the K^- lifetime is longer by a factor of 20 than might otherwise be expected. The decay width of the $\rho(770)$ is $\Gamma_\rho = 139$ MeV. The energy release is $m_\rho - 2m_\pi$=496 MeV. The decay width of the $K^+(890)$ is 46 MeV and the energy release is $m_{K^*} - m_K - m_\pi$ = 262 MeV. The smaller energy release implies a lifetime somewhat longer than that associated with the $\rho^- \to \pi^- \pi^0$ decay.

Problem 1.8 CKM matrix and heavy quark decay

The CKM matrix is

$$|V_{\rm CKM}| \equiv \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} 0.97 & 0.22 & \sim 0.003 \\ 0.22 & 0.97 & \sim 0.04 \\ \sim 0.01 & 0.04 & 0.999 \end{pmatrix}$$

We have $m_t > m_b > m_c > m_s > m_u \simeq m_d$. If the CKM matrix were diagonal, t-quarks would decay to b-quarks which would then be stable and c-quarks would decay to s-quarks which would also be stable. Since m_u and m_d are both much smaller than Λ_{QCD}, their masses do not govern their dynamics and both would occur as presently observed. Starting with the t, all other things being equal, the ratio of decay rates

$$\frac{\Gamma_{t\to s}}{\Gamma_{t\to b}} = \frac{|V_{ts}|^2}{|V_{tb}|^2} \simeq 1.6 \times 10^{-3}$$

and $\Gamma_{t\to d}/\Gamma_{t\to b} \simeq 10^{-4}$. Hence the t-quark decays predominantly to a b-quark. In fact, since $m_t >> m_b$ and $m_t >> m_s$, the energy release and phase space factors cancel. The off-diagonal element $V_{cb} \simeq 0.04$ permits the the decay $b \to c$. The ratio

$$\frac{\Gamma_{b\to u}}{\Gamma_{b\to c}} = \frac{|V_{bu}|^2}{|V_{bc}|^2} \simeq 5 \times 10^{-3}$$

characterizes the fraction of b-quark decays proceeding directly to the first generation. Similarly, the c-quark decays predominantly to the s-quark. The s-quark can only decay to a u-quark.

Problem 1.9 Rare decays of t and Z

The diagrams for $t \to bW^+Z$ appear in Figure 1.4a. The process can be thought of as the leading order $t \to W^+b$ decay with an additional Z radiated from any one of the three charged particles. A diagram for $Z \to W^+\pi^-$ appears in Figure 1.4b. The masses are $m_t = 172.0 \pm 0.9 \pm 1.3$ GeV, $m_Z = 91.1876 \pm 0.0021$ GeV, $m_W = 80.4 \pm 0.02$ GeV, $m_b = 4.2 \pm 0.2$ GeV, $m_\pi = 0.139$ GeV. We have $m_Z + m_W$=171.58 GeV which is within one standard deviation of the measured value of m_t. We have $m_Z + m_W + m_b$=175 GeV so if the final state particles are real, the t-quark must be ever so slightly virtual. We have $m_W + m_\pi = 80.53$, some 10 GeV below m_Z so $Z \to W^+\pi^-$ can proceed with all particles real.

Problem 1.10 Fourth generation fermions

Assuming the fourth generation lepton number is conserved, the ω^- would decay exclusively to $\nu_\omega W^-$ with the W^- real and ν_ω would be stable. The decay $t \to bW^+ \to b\nu_\omega\omega^+$ would be allowed with the W^+ virtual. If the 4-dimensional quark CKM matrix were diagonal, then the x would decay exclusively by real W^+ emission to y which would be stable. The energy release would be $m_x - m_y - m_W = 170$ GeV. The decay rate scaled from t-decay would be naively

$$\Gamma_{x \to yW^+} \simeq \frac{\rho((m_x - m_y - m_W)}{\rho(m_t - m_W - m_b)} \Gamma_{t \to bW^+}$$

where ρ is the phase space factor. Assuming small off-diagonal matrix elements to the nearest neighbor generation, the x could decay to b by real W^+ emission with energy release $m_x - m_b - m_W \simeq 215$ GeV. The decay rate would be naively

$$\Gamma_{x \to bW^-} \simeq |V_{xb}|^2 \frac{\rho(m_x - m_b - m_W)}{\rho(m_t - m_W - m_b)} \Gamma_{t \to bW^+}.$$

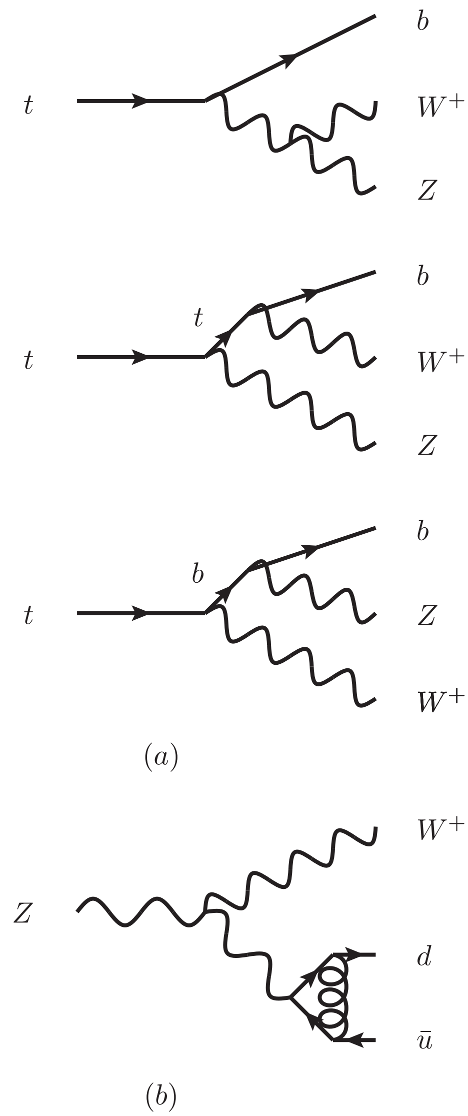

Figure 1.4: Feynman diagrams for a) $t \to W^+bZ$ and b) $Z \to W^+\pi^-$.