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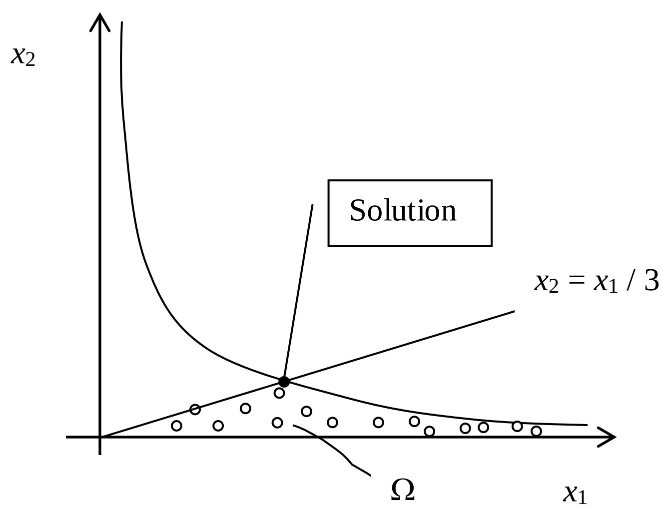
CHAPTER 1
PRELIMINARY CONCEPTS

P1.1 Since the old feasible region is $\subseteq$ new feasible region, the new minimum will be less than or equal to the old minimum, or $f_{\text{new}}^* \leq f_{\text{old}}^*$.

P1.2 The objective corresponding to any feasible point is an upper bound. Thus, we may choose $x_3 = 0.1$, $x_2 = 1$, $x_1 = 1.1$, which satisfies the inequality constraint and the bounds, giving $f = 1.1+1+0.1 = 2.2$. The minimum value f^* has to less than or equal to 2.2.

P1.3 They are sufficient. That is, a problem can have a solution even if Weierstraas conditions are not satisfied. On the other hand, if the conditions are satisfied, then an optimum solution exists.

P1.4. For the cantilever problem in Example 1.14 in text, the feasible region (Fig. E1.14b) with the additional constraint: $x_2 \leq x_1 /3$ is shown below.



Now, the problem does have a solution as shown in the figure. However, Weierstraas conditions are still not satisfied as the feasible region Ω remains unbounded.

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P1.5 Writing the equation of the line as $\mathbf{a}^{\mathrm{T}}\mathbf{x} - b = 0$, with $x_1 = x$ and $x_2 = y$, we have $\mathbf{a} = [1, -1]^{\mathrm{T}}$, $b = -1$, whence the formula in Example 1.2 in text gives $d = \frac{|-2|}{\sqrt{2}} = \sqrt{2}$.

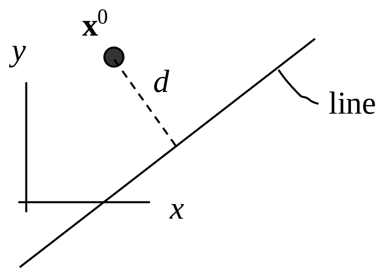

Fig. P1.5

P1.6 At optimum, the deflection at the center must equal the deflection at the end or abs(δ_{center}) = abs(δ_{support}). In case the objective is the bending moment, then abs(M_{center}) = abs(M_{support}).

P1.8 $\alpha_1 \mathbf{V}^1 + \alpha_2 \mathbf{V}^2 + \alpha_3 \mathbf{V}^3 = 0$ yields

$$\begin{bmatrix} 1 & 0 & 3 \\ -5 & 6 & -3 \\ 2 & -1 & 4 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{or} \quad \mathbf{A}\,\boldsymbol{\alpha} = \mathbf{0}$$

Since det $\mathbf{A} = 0$, it does not follow that all α_i have to be zero. Thus, the vectors are linearly <u>dependent</u>.

P1.9. We can express $f = 2\,x_1^2 + 5\,x_2^2 + 3\,x_3^2 - 2\,x_1\,x_2 - 4\,x_2\,x_3$ in matrix form as

$$f = \mathbf{x}^{\mathrm{T}} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 5 & -2 \\ 0 & -2 & 3 \end{bmatrix} \mathbf{x}$$

Using Sylvester's test, we check the determinant of each principal sub-matrix:

$2 > 0$

$10 - (1) = 9 > 0$

$2\,(11) + 1\,(-3) + 0 = 19 > 0$

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Thus, **A** is positive definite and so is the quadratic form. Alternatively, we can conclude the same from the Matlab command "eig(A)" which gives the eigenvalues 1.281, 2.316, 6.403 which are all positive.

P1.10. $n = 3$ (i.e. C^3 continuous)

P1.11 We have

$$\frac{\partial f}{\partial x_r} = \sum_{i=1}^{m} g_i \frac{\partial h_i}{\partial x_r} + \sum_{i=1}^{m} h_i \frac{\partial g_i}{\partial x_r}$$

$$= \left[\frac{\partial h_1}{\partial x_r} \quad \frac{\partial h_2}{\partial x_r} \quad \ldots \quad \frac{\partial h_m}{\partial x_r} \right] \mathbf{g} + \left[\frac{\partial g_1}{\partial x_r} \quad \frac{\partial g_2}{\partial x_r} \quad \ldots \quad \frac{\partial g_m}{\partial x_r} \right] \mathbf{h}$$

Letting $r = 1,2,\ldots,n$, we get

$$\nabla f = \begin{bmatrix} \frac{\partial h_1}{\partial x_1} & \frac{\partial h_2}{\partial x_1} & \ldots & \frac{\partial h_m}{\partial x_1} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial h_1}{\partial x_r} & \frac{\partial h_2}{\partial x_r} & \ldots & \frac{\partial h_m}{\partial x_r} \\ \ldots & \ldots & \ldots & \ldots \\ \frac{\partial h_1}{\partial x_n} & \frac{\partial h_2}{\partial x_n} & \ldots & \frac{\partial h_m}{\partial x_n} \end{bmatrix} \mathbf{g} + \ldots$$

$$= [\nabla \mathbf{h}]\, \mathbf{g} + [\nabla \mathbf{g}]\, \mathbf{h}$$

P1.12 No. Since given an x in the domain, there is no unique vale for f.

P1.13 Since $\mathbf{y}^T \mathbf{A}\, \mathbf{y} > 0$ for *any* nonzero vector $\mathbf{y}$, choose $\mathbf{y} = [1,0,0,\ldots,0]^T$ which gives $A_{11} > 0$. Similarly, choose $\mathbf{y} = [0,1,0,\ldots,0]^T$ and obtain $A_{22} > 0$ etc.

P1.14 (Forward Differences):

(i) central difference for gradient evaluation involves $2n$ evaluation

(ii) every diagonal element of the Hessian involves 2 evaluations and every off–diagonal term involves 3 evaluations thus giving a total of $2n + 3/2\,[n^2 - n]$.

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P1.15 With a suitable choice of ε, Program GRADS.BAS gives:

Analytical	Forw. Diff	Backw. Diff	Central Diff.
-.0342	-.03413577	-.03426522	-.03420003
.0026	.002600874	.002599011	.002599632

P1.16 $\mathbf{x}(\alpha) = [2-\alpha\, , 1]^T$, giving $f(\alpha) = 100\,[1-(2-\alpha)^2]^2 + [1-(2-\alpha)]^2$. At $\mathbf{x}^0$, $\alpha = 0$. The directional derivative df/dα at $\alpha = 0$ equals $[\,f(\alpha = \varepsilon) - f(\alpha = 0)]/\,\varepsilon$. Choosing ε = 0.01, we get $f' = -2380$.

P1.17 *** Correction *** Take c = 1e−8

Experimenting with forward difference parameter ε, the code below shows that forward difference matches well with the analytical formula for $10^{-19} \le \varepsilon \le 10^{-10}$.

```
clear all; close all;
%P1.17
A= [1 -1;-1 1];
c= 1e-8;
B0= c*[1 0;0 1];
[V0,D0] = eig(A,B0);
epsil1 = 1e-26 *[1 .25e2 .5e2 1e2 .25e3 .5e3 1e3 .5e4 1e4 1e5 1e6 1e7
1e8 1e9 1e10 1e11 1e12 1e13 1e14 ...
                 1e15 1e16 .5e17 1e17 .5e18];
%
% analytical derivative below
denom= V0(:,2)'*B0*V0(:,2);
dBdx= [0 1;1 0];
sanaly= -D0(2,2)/denom*V0(:,2)'*dBdx*V0(:,2);
% fwd diff below
for ii=1:length(epsil1)
  epsil=epsil1(ii);
  deltaB= [0 epsil;epsil 0];
  B= B0 + deltaB;
  [V,D] = eig(A,B);
  snumer(ii)= (D(2,2)-D0(2,2))/epsil;
  sanalyt(ii) = sanaly;
end
xscale = log10(epsil1)
plot(xscale,snumer,'k+')
hold
plot(xscale,sanalyt,'rp')
```

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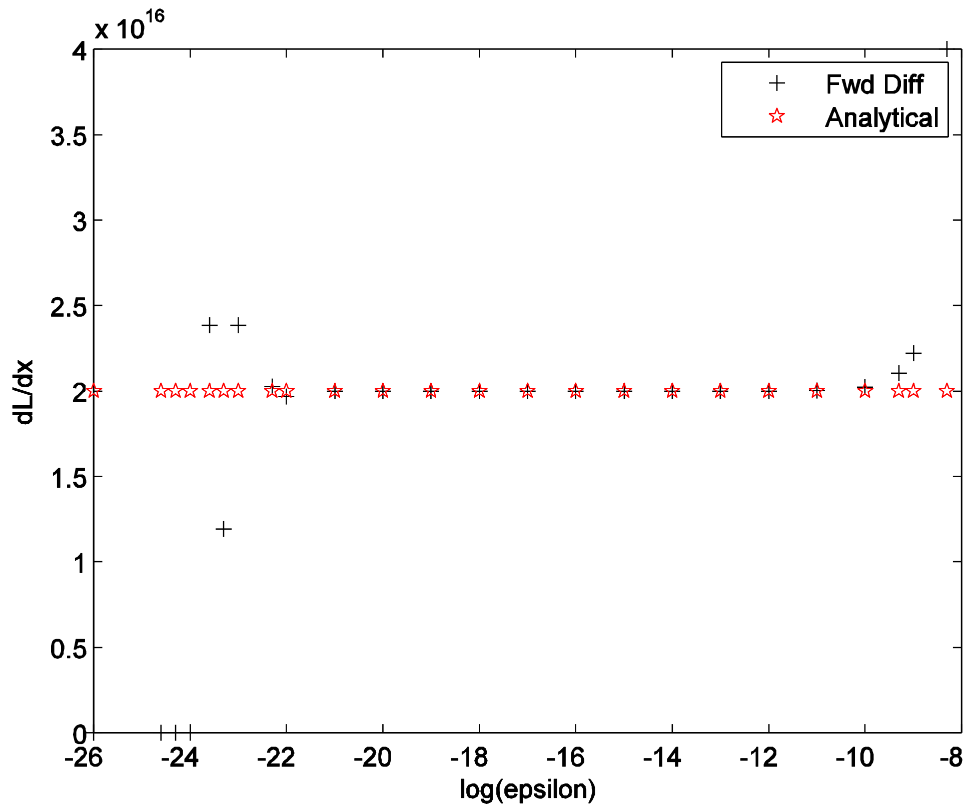

P1.18 Central differences gives a stable range of $10^{-22} \le \varepsilon \le 10^{-10}$, which is almost same as forward difference for this problem.

P1.19 $\ell(\mathbf{x}) = g(\mathbf{x}^0) + \nabla g(\mathbf{x}^0)^T (\mathbf{x} - \mathbf{x}^0)$

$= 0 - 5\,(5, 8)\,.\,(x_1 - 4, x_2 - 5)^T$

$= -5\,(5\,x_1 + 8x_2 - 60)$

and the quadratic approximation

$$q(\mathbf{x}) = \ell(\mathbf{x}) + \tfrac{1}{2}(\mathbf{x} - \mathbf{x}^0)^T [\nabla^2 f(\mathbf{x}^0)] (\mathbf{x} - \mathbf{x}^0)$$

$$= \ell(\mathbf{x}) - (x_1 - 4)(5x_2 - 25) - (x_2 - 5)(5\,x_1 + 4\,x_2 - 40)$$

<u>Plotting:</u> We need to plot, in variable-space or **x**-space, the contours

$$f(\mathbf{x}) = c, \quad \ell(\mathbf{x}) = c, \; q(\mathbf{x}) = c, \quad \text{where } c = g(\mathbf{x}^0) = 0$$

Modifying the code in Example 1.21 in Text, we get the plot below.

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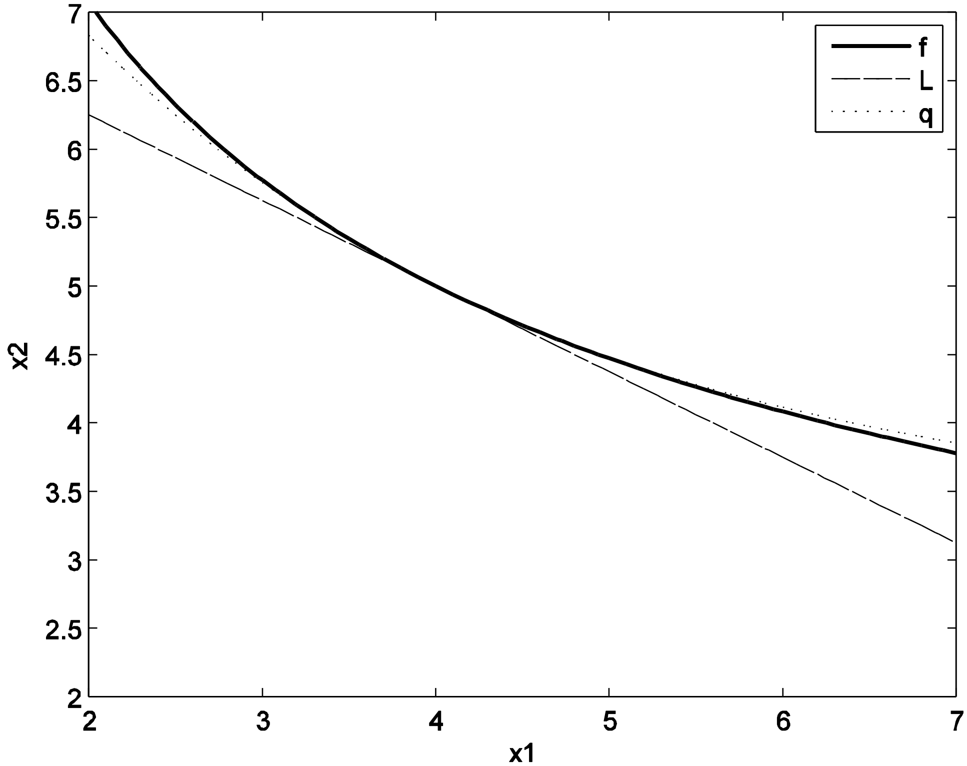

P1.20 Similar to the above problem. **P1.19** $\ell(\mathbf{x}) = g(\mathbf{x}^0) + \nabla g(\mathbf{x}^0)^T (\mathbf{x} - \mathbf{x}^0)$

$= 0 - (25, 4) \cdot (x_1 - 4, x_2 - 25)^T$ at (4,25), a point on the boundary

and

$= -25 - (25, 5) \cdot (x_1 - 5, x_2 - 25)^T$ at (5,25), a point inside the feasible region (red line)

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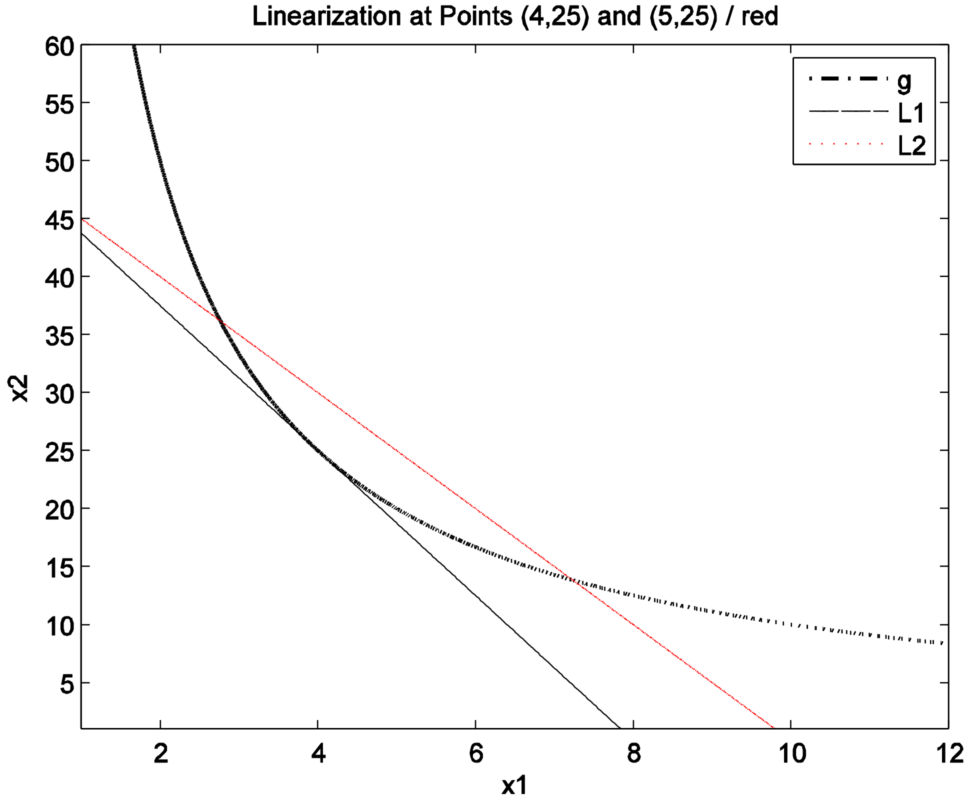

P1.21 $f(x+h) = f(x_0) + f'(x_0)\,h + \dfrac{f''(\hat{x})}{2}\,h^2 \le f(x_0) + f'(x_0)\,h + \dfrac{M}{2}\,h^2$

where $x_0 \le \hat{x} \le x_0 + h$.

Thus, $\dfrac{f(x_0+h) - f(x_0)}{h} - f'(x_0) \;\le\; \dfrac{M}{2}\,h$

The error is proportional to *h*.

P1.22 $f(x+\dfrac{h}{2}) = f(x_0) + f'(x_0)\,\dfrac{h}{2} + \dfrac{f''(x_0)}{2}\left(\dfrac{h}{2}\right)^2 + \dfrac{f'''(\hat{x})}{6}\left(\dfrac{h}{2}\right)^3$

$f(x-\dfrac{h}{2}) = f(x_0) - f'(x_0)\,\dfrac{h}{2} + \dfrac{f''(x_0)}{2}\left(\dfrac{h}{2}\right)^2 - \dfrac{f'''(\hat{x})}{6}\left(\dfrac{h}{2}\right)^3$

Thus, $\dfrac{f(x_0+\frac{h}{2}) - f(x_0-\frac{h}{2})}{h} - f'(x_0) \;\le\; \dfrac{M}{12}\,h^2$

where *M* represents an upper bound on the third derivative. The error is proportional to h^2, and is consequently more accurate than forward or backward difference where error is proportional to *h*.

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P1.23 Taylor series expansion is shown using a triangular pattern as (binomial expansion):

$$
\begin{array}{ccccccc}
 & & & f_0 & & & \\
 & & f_x & & f_y & & \\
 & f_{xx} & & f_{xy} & & f_{yy} & \\
f_{xxx} & & f_{xxy} & & f_{xyy} & & f_{yyy}
\end{array}
$$

Entry in the 1st row is a constant $f_0 = f(x_0,y_0)$. Then, $f_x = \frac{df}{dx}$, $f_{xxy} = \frac{d^3 f}{dx^2\, dy}$ etc. are all evaluated at (x_0,y_0). Denoting the linear, quadratic, and cubic expansions by *L*, *Q*, *C*, respectively, we have

$L(x,y) = f_0 + f_x\,(x-x_0) + f_y\,(y-y_0)$

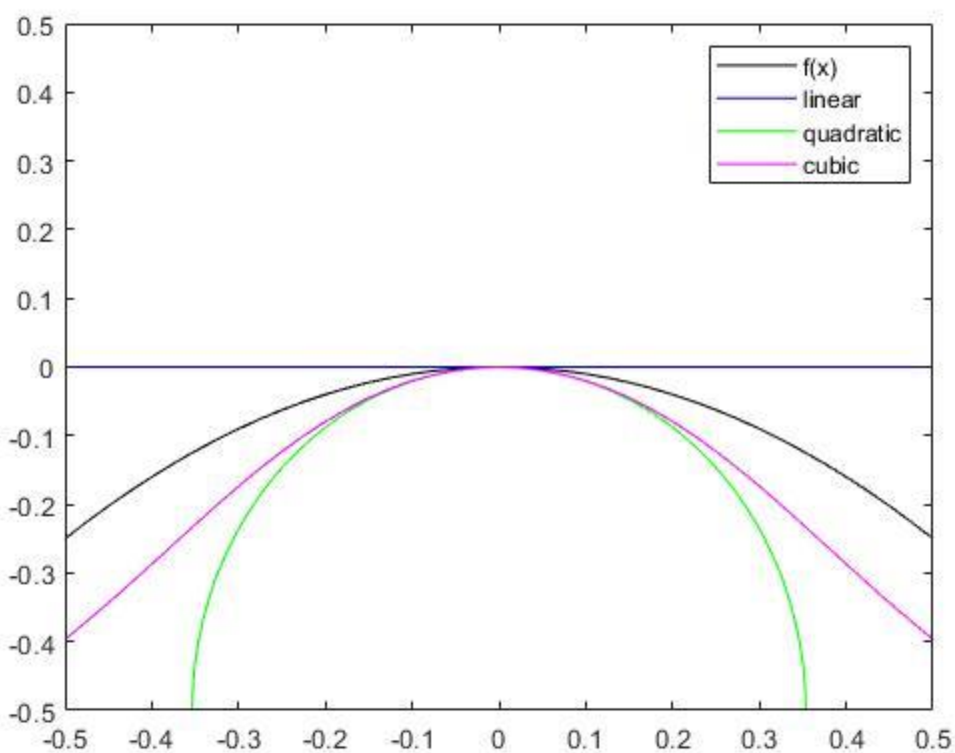
$Q = L(x,y) + f_{xx}\,(x-x_0)^2 + f_{xy}\,(x-x_0)(y-y_0) + f_{yy}\,(y-y_0)^2$

$C = Q(x,y) + f_{xxx}\,(x-x_0)^3 + f_{xxy}\,(x-x_0)^2(y-y_0) + f_{xyy}\,(x-x_0)(y-y_0)^2 + f_{yyy}\,(y-y_0)^3$

Here, we have $f = \exp(x^2 + y)$, $(x_0, y_0) = (0,0)$. Thus,

$L = 1 + y$, $Q = L + 2x^2 + y^2$, $C = Q + 2x^2y + y^3$

A contour plot of the original function with the approximations is given below, to validate the above calculations.



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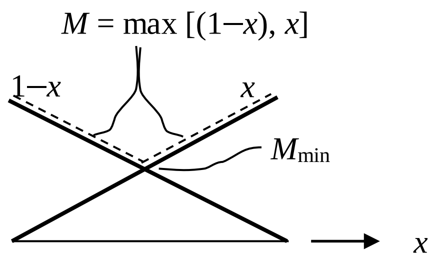
P1.24 Note: part (ii) is easier than part (i) and should have been asked first.

(ii) From a free body diagram of the platform, we have the cable tensions equal to $P/2$ which should be $\leq F$ (given). Thus, $P_{max} = 2F$ is the maximum load we can apply.
(i) A free body diagram gives the forces in each cable to be $P\,(1-x)$ and $P\,x$, respectively.
Engineering Approach: Thus, if the load P is towards the left, the right cable is less stressed and vice versa. The worst design is when the load is at either end, when $P_{max} = F$. The best value will be to choose $x = 0.5$ giving $P_{max} = 2F$.
Mathematical Approach: We want $P\,(1-x) \leq F$ and $P\,x \leq F$. Equivalently, we want $P \max\,[(1-x), x] \leq F$. Let $M = \max\,[(1-x), x]$. Then, $P \leq F / M$. Thus, $P_{max} = F / M_{min}$. Now, M_{min} occurs at $x = 0.5$ as seen from the plot below.



P1.25
(a) First, we will show that ERO operations produce a zero row, then rows of **A** are linearly dependent. Here, the right-hand-side vector is not relevant. ERO can be denoted by premultiplication by **T** matrices as shown in the Text. Let **A** be of dimension (m,n), $m \leq n$. For simplicity of explanation, let the basic variables in the ERO be in the order x_1, x_2,...Thus, the 1st column of $\mathbf{T}_1\,\mathbf{A} = [1\ 0\ ...\ 0]'$ where $'$ = transpose, 2nd column of $\mathbf{T}_2\,\mathbf{T}_1\,\mathbf{A} = [0\ 1\ 0\ ...\ 0]'$ etc. After r steps, we observe a zero $(r+1)$th row. Denoting $\mathbf{T} = \mathbf{T}_r\,\mathbf{T}_{r-1}\,...\,\mathbf{T}_1$, this means $(r+1)$th row of $\mathbf{T\,A} = 0$, or

$$\sum_{j=1}^{m} T_{r+1,j}\;A(j,:) = [0,0,...,0]$$

where $A(j,:)$ = jth row of **A**. Since not all entries in the r+1 th row of **T** are zero, we divide by the nonzero entry, say the entry $T_{r+1,s}$ to write

$$T(s,:) = -\sum_{j=1,\,j\neq s}^{m} \frac{T_{r+1,j}}{T_{r+1,s}}\,A(j,:)$$

Since the sth row is written in terms of the others, the rows of **A** are linearly dependent.

Now for the only if part: we have to show that if rows of A are linearly dependent, then ERO leads to a zero row. Let the rank of A be m-1. Then, after $(m$-1) ERO

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operations, the structure of the T A matrix will be (shown below for m = 5, n = 7, rank(A) = 4)

$$\mathbf{T\,A} = \begin{bmatrix} 1 & 0 & 0 & X & X & X & X \\ 0 & 1 & 0 & X & X & X & X \\ 0 & 0 & 1 & X & X & X & X \\ 0 & 0 & 0 & X & X & X & X \\ 0 & 0 & 0 & X & X & X & X \end{bmatrix}$$

Since rank(A) = 4, the last two rows in the matrix above are dependent on each other as

TA(5,:) = α TA(4,:)

The pivot row is p = 4 here. From Eq. (1.17), Text, we have

$$j = 1, \ldots, n$$
$$A_{5,j} = \alpha\, A_{4,j}$$
$$A_{5,j}{}' = A_{5,j} - \frac{A_{5,q}}{A_{4,q}} A_{4,j}$$
$$= A_{5,j} - \frac{\alpha\, A_{4,q}}{A_{4,q}} A_{4,j}$$
$$= A_{5,j} - \alpha\, A_{4,j}$$
$$= 0$$

as is to be proved.

If rank(A) = m-2, then the Gauss-Jordan row reduction ultimately yield two zero rows.

(b) A zero reduced row with nonzero reduced right hand side implies inconsistent equations (no solution exists).

P1.26 The basic solution is $x_1 = 5$, $x_2 = 3$, $x_3 = -1$, with $x_4 = x_5 = x_6 = 0$.

To switch x_2 with x_5, the pivot element is the coefficient of x_5 in row 2 = -3. Divide row 2 by -3, to get

$$-\frac{1}{3}x_2 - \frac{2}{3}x_4 + x_5 - \frac{1}{3}x_6 = -1$$

Thus, rows 1 and 2 will now be

$$x_1 + 0x_2 + 0x_3 + x_4 + x_5 - x_6 = 5$$
$$0x_1 - \frac{1}{3}x_2 + 0x_3 - \frac{2}{3}x_4 + x_5 - \frac{1}{3}x_6 = -1$$

Subtracting, we obtain

$$x_1 + \frac{1}{3}x_2 + 0x_3 + \frac{5}{3}x_4 + 0x_5 - \frac{2}{3}x_6 = 6$$

Similar row operations involving rows 2 and 3 yields

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$$0x_1 + \frac{2}{3}x_2 + x_3 + \frac{1}{3}x_4 + 0x_5 - \frac{1}{3}x_6 = 1$$

The final set of equations can also be obtained by premultiplying the original matrix with a [T] matrix given in Sec. 1.11, Text :

$$\begin{bmatrix} 1 & 1/3 & 0 \\ 0 & -1/3 & 0 \\ 0 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & -1 & 5 \\ 0 & 1 & 0 & 2 & -3 & 1 & 3 \\ 0 & 0 & 1 & -1 & 2 & -1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1/3 & 0 & 5/3 & 0 & -2/3 & 6 \\ 0 & -1/3 & 0 & -2/3 & 1 & -1/3 & -1 \\ 0 & 2/3 & 1 & 1/3 & 0 & -1/3 & 1 \end{bmatrix}$$

The basic solution is now $x_1 = 6$, $x_3 = 1$, $x_5 = -1$, with $x_2 = x_4 = x_6 = 0$. ■