

## Chapter 2

### Simple Comparative Experiments

### Solutions

**2.1.** Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

Variable	N	Mean	SE Mean	Std. Dev.	Variance	Minimum	Maximum
Y	9	19.96	?	3.12	?	15.94	27.16

$$\text{SE Mean} = 1.04 \quad \text{Variance} = 9.73$$

**2.2.** Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

Variable	N	Mean	SE Mean	Std. Dev.	Sum
Y	16	?	0.159	?	399.851

$$\text{Mean} = 24.991 \quad \text{Std. Dev.} = 0.636$$

**2.3.** Suppose that we are testing  $H_0: \mu = \mu_0$  versus  $H_1: \mu \neq \mu_0$ . Calculate the  $P$ -value for the following observed values of the test statistic:

- (a)  $Z_0 = 2.25$        $P\text{-value} = 0.02445$
- (b)  $Z_0 = 1.55$        $P\text{-value} = 0.12114$
- (c)  $Z_0 = 2.10$        $P\text{-value} = 0.03573$
- (d)  $Z_0 = 1.95$        $P\text{-value} = 0.05118$
- (e)  $Z_0 = -0.10$        $P\text{-value} = 0.92034$

**2.4.** Suppose that we are testing  $H_0: \mu = \mu_0$  versus  $H_1: \mu > \mu_0$ . Calculate the  $P$ -value for the following observed values of the test statistic:

- (a)  $Z_0 = 2.45$        $P\text{-value} = 0.00714$
- (b)  $Z_0 = -1.53$        $P\text{-value} = 0.93699$
- (c)  $Z_0 = 2.15$        $P\text{-value} = 0.01578$
- (d)  $Z_0 = 1.95$        $P\text{-value} = 0.02559$
- (e)  $Z_0 = -0.25$        $P\text{-value} = 0.59871$

2.5. Consider the computer output shown below.

One-Sample Z					
Test of mu = 30 vs not = 30					
The assumed standard deviation = 1.2					
N	Mean	SE Mean	95% CI	Z	P
16	31.2000	0.3000	(30.6120, 31.7880)	?	?

(a) Fill in the missing values in the output. What conclusion would you draw?

$Z = 4$        $P = 0.00006$ ; therefore, the mean is not equal to 30.

(b) Is this a one-sided or two-sided test?

Two-sided.

(c) Use the output and the normal table to find a 99 percent CI on the mean.

CI = 30.42725, 31.97275

(d) What is the  $P$ -value if the alternative hypothesis is  $H_1: \mu > 30$

$P$ -value = 0.00003

2.6. Suppose that we are testing  $H_0: \mu_1 = \mu_2$  versus  $H_1: \mu_1 \neq \mu_2$  with a sample size of  $n_1 = n_2 = 12$ . Both sample variances are unknown but assumed equal. Find bounds on the  $P$ -value for the following observed values of the test statistic:

(a)  $t_0 = 2.30$       Table  $P$ -value = 0.02, 0.05      Computer  $P$ -value = 0.0313

(b)  $t_0 = 3.41$       Table  $P$ -value = 0.002, 0.005      Computer  $P$ -value = 0.0025

(c)  $t_0 = 1.95$       Table  $P$ -value = 0.1, 0.05      Computer  $P$ -value = 0.0640

(d)  $t_0 = -2.45$       Table  $P$ -value = 0.05, 0.02      Computer  $P$ -value = 0.0227

Note that the degrees of freedom is  $(12 + 12) - 2 = 22$ . This is a two-sided test

2.7. Suppose that we are testing  $H_0: \mu_1 = \mu_2$  versus  $H_1: \mu_1 > \mu_2$  with a sample size of  $n_1 = n_2 = 10$ . Both sample variances are unknown but assumed equal. Find bounds on the  $P$ -value for the following observed values of the test statistic:

(a)  $t_0 = 2.31$       Table  $P$ -value = 0.01, 0.025      Computer  $P$ -value = 0.01648

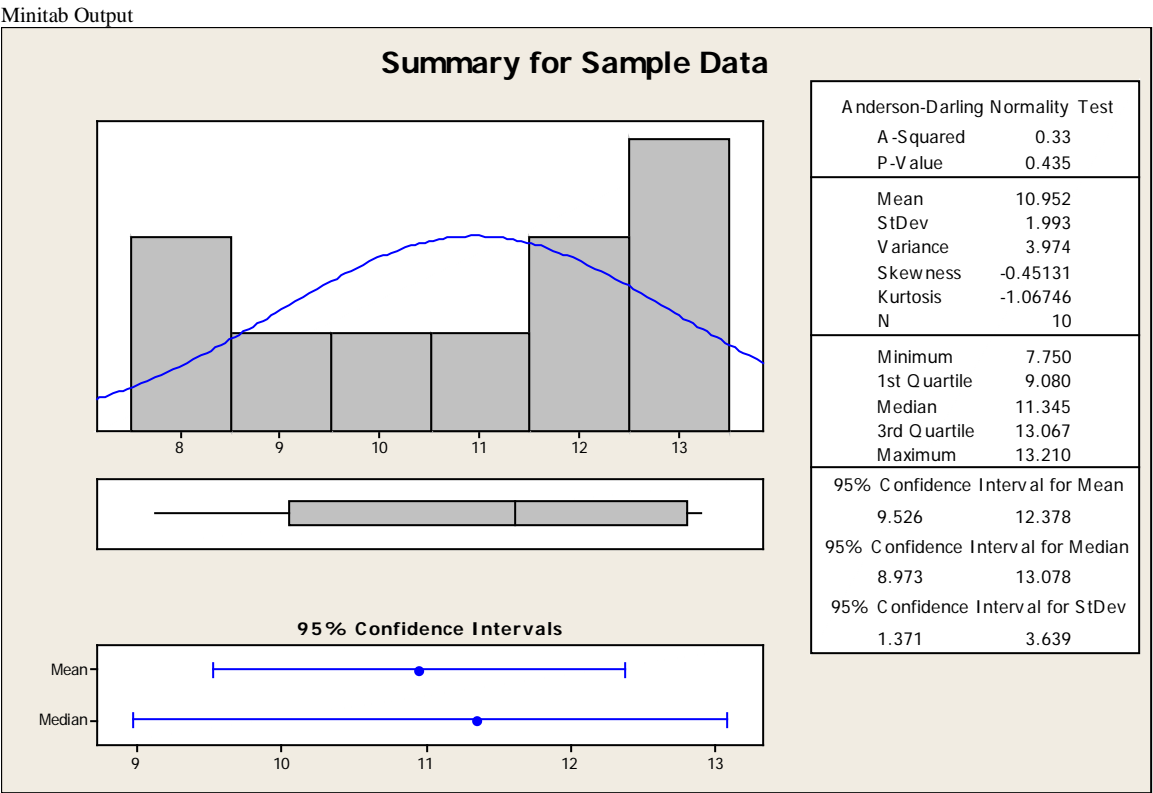
(b)  $t_0 = 3.60$       Table  $P$ -value = 0.001, 0.0005      Computer  $P$ -value = 0.00102

(c)  $t_0 = 1.95$       Table  $P$ -value = 0.05, 0.025      Computer  $P$ -value = 0.03346

(d)  $t_0 = 2.19$       Table  $P$ -value = 0.01, 0.025      Computer  $P$ -value = 0.02097

Note that the degrees of freedom is  $(10 + 10) - 2 = 18$ . This is a one-sided test.

**2.8.** Consider the following sample data: 9.37, 13.04, 11.69, 8.21, 11.18, 10.41, 13.15, 11.51, 13.21, and 7.75. Is it reasonable to assume that this data is from a normal distribution? Is there evidence to support a claim that the mean of the population is 10?



According to the output, the Anderson-Darling Normality Test has a  $P$ -Value of 0.435. The data can be considered normal. The 95% confidence interval on the mean is (9.526,12.378). This confidence interval contains 10, therefore there is evidence that the population mean is 10.

**2.9.** A computer program has produced the following output for the hypothesis testing problem:

Difference in sample means: 2.35
Degrees of freedom: 18
Standard error of the difference in the sample means: ?
Test statistic: $t_0 = 2.01$
$P$ -Value = 0.0298

(a) What is the missing value for the standard error?

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{2.35}{StdError} = 2.01$$

$$StdError = 2.35 / 2.01 = 1.169$$

- (b) Is this a two-sided or one-sided test? One-sided test for a  $t_0 = 2.01$  is a  $P$ -value of 0.0298.  
 (c) If  $\alpha=0.05$ , what are your conclusions? Reject the null hypothesis and conclude that there is a difference in the two samples.  
 (d) Find a 90% two-sided CI on the difference in the means.

$$\bar{y}_1 - \bar{y}_2 - t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{y}_1 - \bar{y}_2 + t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\bar{y}_1 - \bar{y}_2 - t_{0.05, 18} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{y}_1 - \bar{y}_2 + t_{0.05, 18} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$2.35 - 1.734(1.169) \leq \mu_1 - \mu_2 \leq 2.35 + 1.734(1.169)$$

$$0.323 \leq \mu_1 - \mu_2 \leq 4.377$$

**2.10.** A computer program has produced the following output for the hypothesis testing problem:

Difference in sample means: 11.5  
 Degrees of freedom: 24  
 Standard error of the difference in the sample means: ?  
 Test statistic:  $t_0 = -1.88$   
 $P$ -Value = 0.0723

- (a) What is the missing value for the standard error?

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{-11.5}{StdError} = -1.88$$

$$StdError = -11.5 / -1.88 = 6.12$$

- (b) Is this a two-sided or one-sided test? Two-sided test for a  $t_0 = -1.88$  is a  $P$ -value of 0.0723.  
 (c) If  $\alpha=0.05$ , what are your conclusions? Accept the null hypothesis, there is no difference in the means.  
 (d) Find a 90% two-sided CI on the difference in the means.

$$\begin{aligned} \bar{y}_1 - \bar{y}_2 - t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} &\leq \mu_1 - \mu_1 \leq \bar{y}_1 - \bar{y}_2 + t_{\alpha/2, n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \\ \bar{y}_1 - \bar{y}_2 - t_{0.05, 24} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} &\leq \mu_1 - \mu_1 \leq \bar{y}_1 - \bar{y}_2 + t_{0.05, 24} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \\ -11.5 - 1.711(6.12) &\leq \mu_1 - \mu_1 \leq -11.5 + 1.711(6.12) \\ -21.97 &\leq \mu_1 - \mu_1 \leq -1.03 \end{aligned}$$

**2.11.** Suppose that we are testing  $H_0: \mu = \mu_0$  versus  $H_1: \mu > \mu_0$  with a sample size of  $n = 15$ . Calculate bounds on the  $P$ -value for the following observed values of the test statistic:

- |     |              |                                  |                               |
|-----|--------------|----------------------------------|-------------------------------|
| (a) | $t_0 = 2.35$ | Table $P$ -value = 0.01, 0.025   | Computer $P$ -value = 0.01698 |
| (b) | $t_0 = 3.55$ | Table $P$ -value = 0.001, 0.0025 | Computer $P$ -value = 0.00160 |
| (c) | $t_0 = 2.00$ | Table $P$ -value = 0.025, 0.005  | Computer $P$ -value = 0.03264 |
| (d) | $t_0 = 1.55$ | Table $P$ -value = 0.05, 0.10    | Computer $P$ -value = 0.07172 |

The degrees of freedom are  $15 - 1 = 14$ . This is a one-sided test.

**2.12.** Suppose that we are testing  $H_0: \mu = \mu_0$  versus  $H_1: \mu \neq \mu_0$  with a sample size of  $n = 10$ . Calculate bounds on the  $P$ -value for the following observed values of the test statistic:

- |     |               |                                 |                               |
|-----|---------------|---------------------------------|-------------------------------|
| (a) | $t_0 = 2.48$  | Table $P$ -value = 0.02, 0.05   | Computer $P$ -value = 0.03499 |
| (b) | $t_0 = -3.95$ | Table $P$ -value = 0.002, 0.005 | Computer $P$ -value = 0.00335 |
| (c) | $t_0 = 2.69$  | Table $P$ -value = 0.02, 0.05   | Computer $P$ -value = 0.02480 |
| (d) | $t_0 = 1.88$  | Table $P$ -value = 0.05, 0.10   | Computer $P$ -value = 0.09281 |
| (e) | $t_0 = -1.25$ | Table $P$ -value = 0.20, 0.50   | Computer $P$ -value = 0.24282 |

**2.13.** Consider the computer output shown below.

One-Sample T: Y							
Test of mu = 91 vs. not = 91							
Variable	N	Mean	Std. Dev.	SE Mean	95% CI	T	P
Y	25	92.5805	?	0.4675	(91.6160, ? )	3.38	0.002

- (a) Fill in the missing values in the output. Can the null hypothesis be rejected at the 0.05 level? Why?
- Std. Dev. = 2.3365      UCI = 93.5450
- Yes, the null hypothesis can be rejected at the 0.05 level because the  $P$ -value is much lower at 0.002.
- (b) Is this a one-sided or two-sided test?

Two-sided.

- (c) If the hypothesis had been  $H_0: \mu = 90$  versus  $H_1: \mu \neq 90$  would you reject the null hypothesis at the 0.05 level?

Yes.

- (d) Use the output and the  $t$  table to find a 99 percent two-sided CI on the mean.

CI = 91.2735, 93.8875

- (e) What is the  $P$ -value if the alternative hypothesis is  $H_1: \mu > 91$ ?

$P$ -value = 0.001.

**2.14.** Consider the computer output shown below.

One-Sample T: Y							
Test of mu = 25 vs > 25							
Variable	N	Mean	Std. Dev.	SE Mean	95% Lower Bound	T	P
Y	12	25.6818	?	0.3360	?	?	0.034

- (a) How many degrees of freedom are there on the  $t$ -test statistic?

$$(N-1) = (12 - 1) = 11$$

- (b) Fill in the missing information.

$$\text{Std. Dev.} = 1.1639 \qquad 95\% \text{ Lower Bound} = 2.0292$$

**2.15.** Consider the computer output shown below.

Two-Sample T-Test and CI: Y1, Y2				
Two-sample T for Y1 vs Y2				
	N	Mean	Std. Dev.	SE Mean
Y1	20	50.19	1.71	0.38
Y2	20	52.52	2.48	0.55
Difference = mu (X1) - mu (X2)				
Estimate for difference: -2.33341				
95% CI for difference: (-3.69547, -0.97135)				
T-Test of difference = 0 (vs not = ) : T-Value = -3.47				
P-Value = 0.01 DF = 38				
Both use Pooled Std. Dev. = 2.1277				

- (a) Can the null hypothesis be rejected at the 0.05 level? Why?

Yes, the  $P$ -Value of 0.001 is much less than 0.05.

- (b) Is this a one-sided or two-sided test?

Two-sided.

- (c) If the hypothesis had been  $H_0: \mu_1 - \mu_2 = 2$  versus  $H_1: \mu_1 - \mu_2 \neq 2$  would you reject the null hypothesis at the 0.05 level?

Yes.

- (d) If the hypothesis had been  $H_0: \mu_1 - \mu_2 = 2$  versus  $H_1: \mu_1 - \mu_2 < 2$  would you reject the null hypothesis at the 0.05 level? Can you answer this question without doing any additional calculations? Why?

Yes, no additional calculations are required because the test is naturally becoming more significant with the change from -2.33341 to -4.33341.

- (e) Use the output and the  $t$  table to find a 95 percent upper confidence bound on the difference in means?

95% upper confidence bound = -1.21.

- (f) What is the  $P$ -value if the alternative hypotheses are  $H_0: \mu_1 - \mu_2 = 2$  versus  $H_1: \mu_1 - \mu_2 \neq 2$ ?

$P$ -value = 1.4E-07.

**2.16.** The breaking strength of a fiber is required to be at least 150 psi. Past experience has indicated that the standard deviation of breaking strength is  $\sigma = 3$  psi. A random sample of four specimens is tested. The results are  $y_1=145$ ,  $y_2=153$ ,  $y_3=150$  and  $y_4=147$ .

- (a) State the hypotheses that you think should be tested in this experiment.

$$H_0: \mu = 150 \quad H_1: \mu > 150$$

- (b) Test these hypotheses using  $\alpha = 0.05$ . What are your conclusions?

$$n = 4, \quad \sigma = 3, \quad \bar{y} = 1/4 (145 + 153 + 150 + 147) = 148.75$$

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{148.75 - 150}{\frac{3}{\sqrt{4}}} = \frac{-1.25}{\frac{3}{2}} = -0.8333$$

Since  $z_{0.05} = 1.645$ , do not reject.

- (c) Find the  $P$ -value for the test in part (b).

$$\text{From the } z\text{-table: } P \cong 1 - [0.7967 + (2/3)(0.7995 - 0.7967)] = 0.2014$$

- (d) Construct a 95 percent confidence interval on the mean breaking strength.

The 95% confidence interval is

$$\begin{aligned}\bar{y} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} &\leq \mu \leq \bar{y} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \\ 148.75 - (1.96)(3/2) &\leq \mu \leq 148.75 + (1.96)(3/2) \\ 145.81 &\leq \mu \leq 151.69\end{aligned}$$

**2.17.** The viscosity of a liquid detergent is supposed to average 800 centistokes at 25°C. A random sample of 16 batches of detergent is collected, and the average viscosity is 812. Suppose we know that the standard deviation of viscosity is  $\sigma = 25$  centistokes.

(a) State the hypotheses that should be tested.

$$H_0: \mu = 800 \quad H_1: \mu \neq 800$$

(b) Test these hypotheses using  $\alpha = 0.05$ . What are your conclusions?

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{812 - 800}{\frac{25}{\sqrt{16}}} = \frac{12}{\frac{25}{4}} = 1.92 \quad \text{Since } z_{\alpha/2} = z_{0.025} = 1.96, \text{ do not reject.}$$

(c) What is the  $P$ -value for the test?

(d) Find a 95 percent confidence interval on the mean.

The 95% confidence interval is

$$\begin{aligned}\bar{y} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} &\leq \mu \leq \bar{y} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \\ 812 - (1.96)(25/4) &\leq \mu \leq 812 + (1.96)(25/4) \\ 812 - 12.25 &\leq \mu \leq 812 + 12.25 \\ 799.75 &\leq \mu \leq 824.25\end{aligned}$$

**2.18.** The diameters of steel shafts produced by a certain manufacturing process should have a mean diameter of 0.255 inches. The diameter is known to have a standard deviation of  $\sigma = 0.0001$  inch. A random sample of 10 shafts has an average diameter of 0.2545 inches.

(a) Set up the appropriate hypotheses on the mean  $\mu$ .

$$H_0: \mu = 0.255 \quad H_1: \mu \neq 0.255$$

(b) Test these hypotheses using  $\alpha = 0.05$ . What are your conclusions?

$$n = 10, \quad \sigma = 0.0001, \quad \bar{y} = 0.2545$$



$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{0.2545 - 0.255}{\frac{0.0001}{\sqrt{10}}} = -15.81$$

Since  $z_{0.025} = 1.96$ , reject  $H_0$ .

- (c) Find the  $P$ -value for this test.  $P = 2.6547 \times 10^{-56}$
- (d) Construct a 95 percent confidence interval on the mean shaft diameter.

The 95% confidence interval is

$$\begin{aligned} \bar{y} - z_{\alpha/2} \frac{\sigma}{\sqrt{n}} &\leq \mu \leq \bar{y} + z_{\alpha/2} \frac{\sigma}{\sqrt{n}} \\ 0.2545 - (1.96) \left( \frac{0.0001}{\sqrt{10}} \right) &\leq \mu \leq 0.2545 + (1.96) \left( \frac{0.0001}{\sqrt{10}} \right) \\ 0.254438 &\leq \mu \leq 0.254562 \end{aligned}$$

**2.19.** A normally distributed random variable has an unknown mean  $\mu$  and a known variance  $\sigma^2 = 9$ . Find the sample size required to construct a 95 percent confidence interval on the mean that has total length of 1.0.

Since  $y \sim N(\mu, 9)$ , a 95% two-sided confidence interval on  $\mu$  is

If the total interval is to have width 1.0, then the half-interval is 0.5. Since  $z_{\alpha/2} = z_{0.025} = 1.96$ ,

$$\begin{aligned} (1.96) \left( \frac{3}{\sqrt{n}} \right) &= 0.5 \\ \sqrt{n} &= (1.96) \left( \frac{3}{0.5} \right) = 11.76 \\ n &= (11.76)^2 = 138.30 \cong 139 \end{aligned}$$

**2.20.** The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

Days	
108	138
124	163
124	159
106	134
115	139

- (a) We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.

$$H_0: \mu = 120 \qquad H_1: \mu > 120$$

- (b) Test these hypotheses using  $\alpha = 0.01$ . What are your conclusions?

$$\begin{aligned}\bar{y} &= 131 \\ S^2 &= 3438 / 9 = 382 \\ S &= \sqrt{382} = 19.54 \\ t_0 &= \frac{\bar{y} - \mu_0}{S/\sqrt{n}} = \frac{131 - 120}{19.54/\sqrt{10}} = 1.78\end{aligned}$$

since  $t_{0.01,9} = 2.821$ ; do not reject  $H_0$

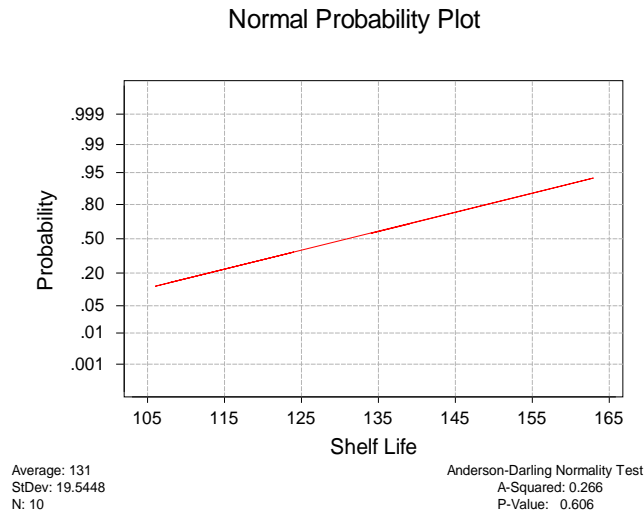
Minitab Output

T-Test of the Mean						
Test of mu = 120.00 vs mu > 120.00						
Variable	N	Mean	StDev	SE Mean	T	P
Shelf Life	10	131.00	19.54	6.18	1.78	0.054
T Confidence Intervals						
Variable	N	Mean	StDev	SE Mean	99.0 % CI	
Shelf Life	10	131.00	19.54	6.18	( 110.91, 151.09 )	

- (c) Find the  $P$ -value for the test in part (b).  $P=0.054$
- (d) Construct a 99 percent confidence interval on the mean shelf life.
- The 99% confidence interval is  $\bar{y} - t_{\alpha/2, n-1} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{y} + t_{\alpha/2, n-1} \frac{S}{\sqrt{n}}$  with  $\alpha = 0.01$ .
- $$131 - (3.250) \left( \frac{19.54}{\sqrt{10}} \right) \leq \mu \leq 131 + (3.250) \left( \frac{19.54}{\sqrt{10}} \right)$$
- $$110.91 \leq \mu \leq 151.08$$

**2.21.** Consider the shelf life data in Problem 2.20. Can shelf life be described or modeled adequately by a normal distribution? What effect would violation of this assumption have on the test procedure you used in solving Problem 2.20?

A normal probability plot, obtained from Minitab, is shown. There is no reason to doubt the adequacy of the normality assumption. If shelf life is not normally distributed, then the impact of this on the  $t$ -test in problem 2.20 is not too serious unless the departure from normality is severe.



**2.22.** The time to repair an electronic instrument is a normally distributed random variable measured in hours. The repair time for 16 such instruments chosen at random are as follows:

Hours			
159	280	101	212
224	379	179	264
222	362	168	250
149	260	485	170

(a) You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.

$$H_0: \mu = 225 \quad H_1: \mu > 225$$

(b) Test the hypotheses you formulated in part (a). What are your conclusions? Use  $\alpha = 0.05$ .

$$\bar{y} = 241.50$$

$$S^2 = 146202 / (16 - 1) = 9746.80$$

$$S = \sqrt{9746.8} = 98.73$$

$$t_o = \frac{\bar{y} - \mu_o}{\frac{S}{\sqrt{n}}} = \frac{241.50 - 225}{\frac{98.73}{\sqrt{16}}} = 0.67$$

since  $t_{0.05,15} = 1.753$ ; do not reject  $H_0$

Minitab Output

**T-Test of the Mean**

Test of mu = 225.0 vs mu > 225.0

Variable	N	Mean	StDev	SE Mean	T	P
Hours	16	241.5	98.7	24.7	0.67	0.26

T Confidence Intervals						
Variable	N	Mean	StDev	SE Mean	95.0 % CI	
Hours	16	241.5	98.7	24.7	(	188.9, 294.1)

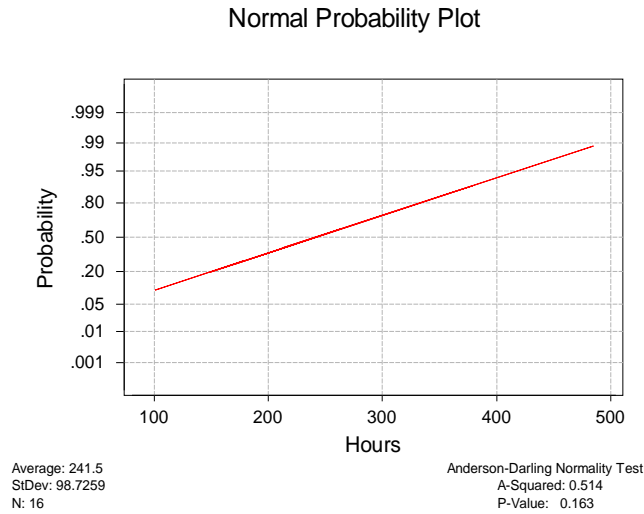
- (c) Find the  $P$ -value for this test.  $P=0.26$
- (d) Construct a 95 percent confidence interval on mean repair time.

The 95% confidence interval is  $\bar{y} - t_{\alpha/2, n-1} \frac{S}{\sqrt{n}} \leq \mu \leq \bar{y} + t_{\alpha/2, n-1} \frac{S}{\sqrt{n}}$

$$241.50 - (2.131) \left( \frac{98.73}{\sqrt{16}} \right) \leq \mu \leq 241.50 + (2.131) \left( \frac{98.73}{\sqrt{16}} \right)$$
$$188.9 \leq \mu \leq 294.1 \quad \checkmark$$

**2.23.** Reconsider the repair time data in Problem 2.22. Can repair time, in your opinion, be adequately modeled by a normal distribution?

The normal probability plot below does not reveal any serious problem with the normality assumption.



**2.24.** Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The filling processes can be assumed to be normal, with standard deviation of  $\sigma_1 = 0.015$  and  $\sigma_2 = 0.018$ . The quality engineering department suspects that both machines fill to the same net volume, whether or not this volume is 16.0 ounces. An experiment is performed by taking a random sample from the output of each machine.

Machine 1		Machine 2	
16.03	16.01	16.02	16.03
16.04	15.96	15.97	16.04
16.05	15.98	15.96	16.02
16.05	16.02	16.01	16.01
16.02	15.99	15.99	16.00

(a) State the hypotheses that should be tested in this experiment.

$$H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 \neq \mu_2$$

(b) Test these hypotheses using  $\alpha=0.05$ . What are your conclusions?

$$\begin{aligned} \bar{y}_1 &= 16.015 & \bar{y}_2 &= 16.005 \\ \sigma_1 &= 0.015 & \sigma_2 &= 0.018 \\ n_1 &= 10 & n_2 &= 10 \end{aligned}$$

$$z_o = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{16.015 - 16.018}{\sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}}} = 1.35$$

$$z_{0.025} = 1.96; \text{ do not reject}$$

(c) What is the  $P$ -value for the test?  $P = 0.1770$

(d) Find a 95 percent confidence interval on the difference in the mean fill volume for the two machines.

The 95% confidence interval is

$$\begin{aligned} \bar{y}_1 - \bar{y}_2 - z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{y}_1 - \bar{y}_2 + z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \\ (16.015 - 16.005) - (1.96) \sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}} \leq \mu_1 - \mu_2 \leq (16.015 - 16.005) + (1.96) \sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}} \\ -0.0045 \leq \mu_1 - \mu_2 \leq 0.0245 \end{aligned}$$

**2.25.** Two types of plastic are suitable for use by an electronic calculator manufacturer. The breaking strength of this plastic is important. It is known that  $\sigma_1 = \sigma_2 = 1.0$  psi. From random samples of  $n_1 = 10$  and  $n_2 = 12$  we obtain  $\bar{y}_1 = 162.5$  and  $\bar{y}_2 = 155.0$ . The company will not adopt plastic 1 unless its breaking strength exceeds that of plastic 2 by at least 10 psi. Based on the sample information, should they use plastic 1? In answering this questions, set up and test appropriate hypotheses using  $\alpha = 0.01$ . Construct a 99 percent confidence interval on the true mean difference in breaking strength.

$$H_0: \mu_1 - \mu_2 = 10 \quad H_1: \mu_1 - \mu_2 > 10$$

$$\begin{aligned} \bar{y}_1 &= 162.5 & \bar{y}_2 &= 155.0 \\ \sigma_1 &= 1 & \sigma_2 &= 1 \\ n_1 &= 10 & n_2 &= 12 \end{aligned}$$

$$z_o = \frac{\bar{y}_1 - \bar{y}_2 - 10}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{162.5 - 155.0 - 10}{\sqrt{\frac{1^2}{10} + \frac{1^2}{12}}} = -5.84$$

$$z_{0.01} = 2.325; \text{ do not reject}$$

The 99 percent confidence interval is

$$\bar{y}_1 - \bar{y}_2 - z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \leq \mu_1 - \mu_2 \leq \bar{y}_1 - \bar{y}_2 + z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(162.5 - 155.0) - (2.575) \sqrt{\frac{1^2}{10} + \frac{1^2}{12}} \leq \mu_1 - \mu_2 \leq (162.5 - 155.0) + (2.575) \sqrt{\frac{1^2}{10} + \frac{1^2}{12}}$$

$$6.40 \leq \mu_1 - \mu_2 \leq 8.60$$

**2.26.** The following are the burning times (in minutes) of chemical flares of two different formulations. The design engineers are interested in both the means and variance of the burning times.

Type 1		Type 2	
65	82	64	56
81	67	71	69
57	59	83	74
66	75	59	82
82	70	65	79

(a) Test the hypotheses that the two variances are equal. Use  $\alpha = 0.05$ .

$$H_0 : \sigma_1^2 = \sigma_2^2$$

$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

Do not reject.

(b) Using the results of (a), test the hypotheses that the mean burning times are equal. Use  $\alpha = 0.05$ . What is the  $P$ -value for this test?

Do not reject.

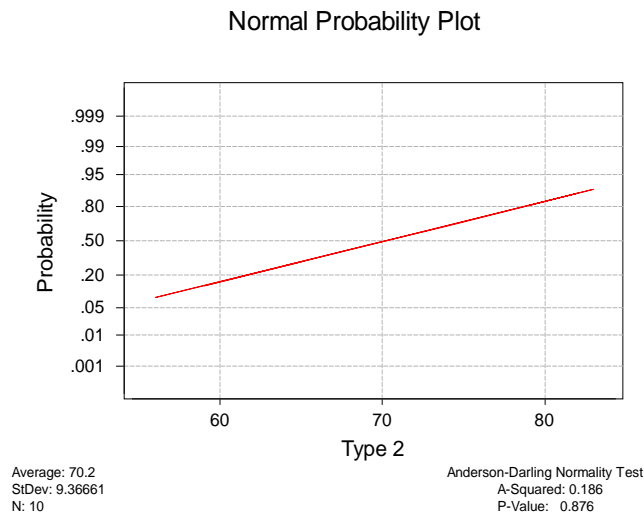
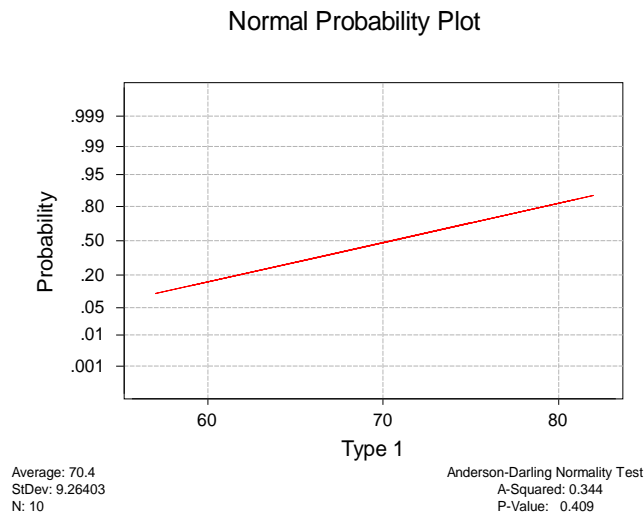
From the computer output,  $t=0.05$ ; do not reject. Also from the computer output  $P=0.96$

Minitab Output

Two Sample T-Test and Confidence Interval				
Two sample T for Type 1 vs Type 2				
	N	Mean	StDev	SE Mean
Type 1	10	70.40	9.26	2.9
Type 2	10	70.20	9.37	3.0
95% CI for mu Type 1 - mu Type 2: ( -8.6, 9.0)				
T-Test mu Type 1 = mu Type 2 (vs not =): T = 0.05 P = 0.96 DF = 18				
Both use Pooled StDev = 9.32				

(c) Discuss the role of the normality assumption in this problem. Check the assumption of normality for both types of flares.

The assumption of normality is required in the theoretical development of the *t*-test. However, moderate departure from normality has little impact on the performance of the *t*-test. The normality assumption is more important for the test on the equality of the two variances. An indication of nonnormality would be of concern here. The normal probability plots shown below indicate that burning time for both formulations follow the normal distribution.



2.27. An article in *Solid State Technology*, "Orthogonal Design of Process Optimization and Its Application to Plasma Etching" by G.Z. Yin and D.W. Jillie (May, 1987) describes an experiment to determine the effect of C<sub>2</sub>F<sub>6</sub> flow rate on the uniformity of the etch on a silicon wafer used in integrated circuit manufacturing. Data for two flow rates are as follows:

C <sub>2</sub> F <sub>6</sub> (SCCM)	Uniformity Observation					
	1	2	3	4	5	6
125	2.7	4.6	2.6	3.0	3.2	3.8
200	4.6	3.4	2.9	3.5	4.1	5.1

(a) Does the  $C_2F_6$  flow rate affect average etch uniformity? Use  $\alpha = 0.05$ .

No,  $C_2F_6$  flow rate does not affect average etch uniformity.

Minitab Output

Two Sample T-Test and Confidence Interval				
Two sample T for Uniformity				
Flow Rat	N	Mean	StDev	SE Mean
125	6	3.317	0.760	0.31
200	6	3.933	0.821	0.34
95% CI for mu (125) - mu (200): ( -1.63, 0.40)				
T-Test mu (125) = mu (200) (vs not =): T = -1.35 P = 0.21 DF = 10				
Both use Pooled StDev = 0.791				

(b) What is the  $P$ -value for the test in part (a)? From the *Minitab* output,  $P=0.21$

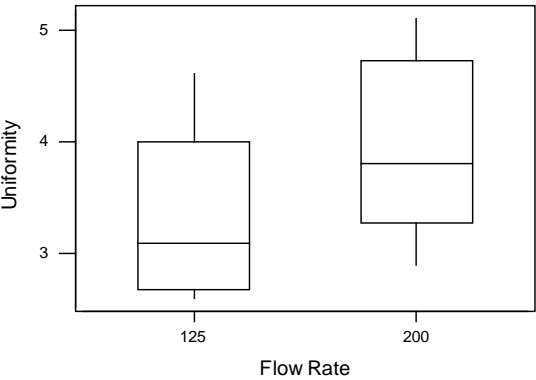
(c) Does the  $C_2F_6$  flow rate affect the wafer-to-wafer variability in etch uniformity? Use  $\alpha = 0.05$ .

$$\begin{aligned} H_0 : \sigma_1^2 &= \sigma_2^2 \\ H_1 : \sigma_1^2 &\neq \sigma_2^2 \\ F_{0.025,5,5} &= 7.15 \\ F_{0.975,5,5} &= 0.14 \\ F_0 &= \frac{0.5776}{0.6724} = 0.86 \end{aligned}$$

Do not reject;  $C_2F_6$  flow rate does not affect wafer-to-wafer variability.

(d) Draw box plots to assist in the interpretation of the data from this experiment.

The box plots shown below indicate that there is little difference in uniformity at the two gas flow rates. Any observed difference is not statistically significant. See the  $t$ -test in part (a).





**2.28.** A new filtering device is installed in a chemical unit. Before its installation, a random sample yielded the following information about the percentage of impurity:  $\bar{y}_1 = 12.5$ ,  $S_1^2 = 101.17$ , and  $n_1 = 8$ . After installation, a random sample yielded  $\bar{y}_2 = 10.2$ ,  $S_2^2 = 94.73$ ,  $n_2 = 9$ .

(a) Can you conclude that the two variances are equal? Use  $\alpha = 0.05$ .

$$\begin{aligned}H_0 : \sigma_1^2 &= \sigma_2^2 \\H_1 : \sigma_1^2 &\neq \sigma_2^2 \\F_{0.025, 7, 8} &= 4.53 \\F_0 &= \frac{S_1^2}{S_2^2} = \frac{101.17}{94.73} = 1.07\end{aligned}$$

Do not reject. Assume that the variances are equal.

(b) Has the filtering device reduced the percentage of impurity significantly? Use  $\alpha = 0.05$ .

$$\begin{aligned}H_0 : \mu_1 &= \mu_2 \\H_1 : \mu_1 &> \mu_2 \\S_p^2 &= \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{(8 - 1)(101.17) + (9 - 1)(94.73)}{8 + 9 - 2} = 97.74 \\S_p &= 9.89 \\t_0 &= \frac{\bar{y}_1 - \bar{y}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{12.5 - 10.2}{9.89 \sqrt{\frac{1}{8} + \frac{1}{9}}} = 0.479 \\t_{0.05, 15} &= 1.753\end{aligned}$$

Do not reject. There is no evidence to indicate that the new filtering device has affected the mean.

**2.29.** Photoresist is a light-sensitive material applied to semiconductor wafers so that the circuit pattern can be imaged on to the wafer. After application, the coated wafers are baked to remove the solvent in the photoresist mixture and to harden the resist. Here are measurements of photoresist thickness (in  $\text{\AA}$ ) for eight wafers baked at two different temperatures. Assume that all of the runs were made in random order.

95 °C	100 °C
11.176	5.623
7.089	6.748
8.097	7.461
11.739	7.015
11.291	8.133
10.759	7.418
6.467	3.772
8.315	8.963

- (a) Is there evidence to support the claim that the higher baking temperature results in wafers with a lower mean photoresist thickness? Use  $\alpha = 0.05$ .

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 > \mu_2$$

$$S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2} = \frac{(8 - 1)(4.41) + (8 - 1)(2.54)}{8 + 8 - 2} = 3.48$$

$$S_p = 1.86$$

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{9.37 - 6.89}{1.86 \sqrt{\frac{1}{8} + \frac{1}{8}}} = 2.65$$

$$t_{0.05,14} = 1.761$$

Since  $t_{0.05,14} = 1.761$ , reject  $H_0$ . There appears to be a lower mean thickness at the higher temperature. This is also seen in the computer output.

Minitab Output

**Two-Sample T-Test and CI: Thickness, Temp**

Two-sample T for Thick@95 vs Thick@100

	N	Mean	StDev	SE Mean
Thick@95	8	9.37	2.10	0.74
Thick@10	8	6.89	1.60	0.56

Difference = mu Thick@95 - mu Thick@100

Estimate for difference: 2.475

95% lower bound for difference: 0.833

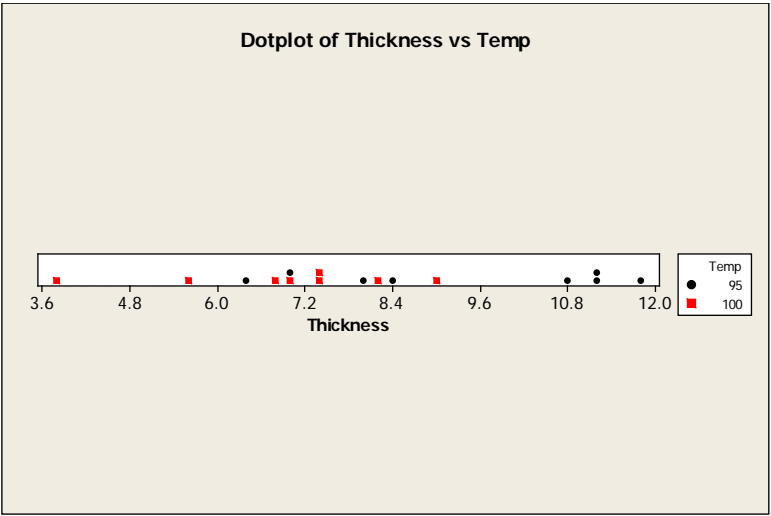
T-Test of difference = 0 (vs >): T-Value = 2.65 P-Value = 0.009 DF = 14

Both use Pooled StDev = 1.86

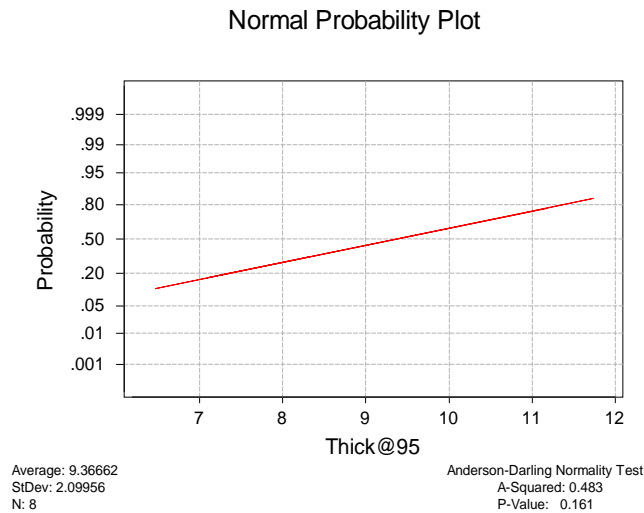
- (b) What is the  $P$ -value for the test conducted in part (a)?  $P = 0.009$
- (c) Find a 95% confidence interval on the difference in means. Provide a practical interpretation of this interval.

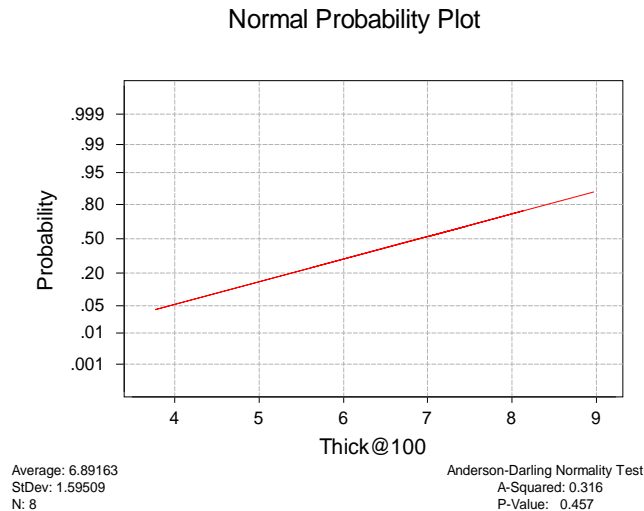
From the computer output the 95% lower confidence bound is  $0.833 \leq \mu_1 - \mu_2$ . This lower confidence bound is greater than 0; therefore, there is a difference in the two temperatures on the thickness of the photoresist.

(d) Draw dot diagrams to assist in interpreting the results from this experiment.



(e) Check the assumption of normality of the photoresist thickness.





There are no significant deviations from the normality assumptions.

(f) Find the power of this test for detecting an actual difference in means of 2.5 kÅ.

Minitab Output

Power and Sample Size

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)  
Calculating power for mean 1 = mean 2 + difference  
Alpha = 0.05 Sigma = 1.86

Difference	Sample Size	Power
2.5	8	0.7056

(g) What sample size would be necessary to detect an actual difference in means of 1.5 kÅ with a power of at least 0.9?.

Minitab Output

Power and Sample Size

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)  
Calculating power for mean 1 = mean 2 + difference  
Alpha = 0.05 Sigma = 1.86

Difference	Sample Size	Target Power	Actual Power
1.5	34	0.9000	0.9060

This result makes intuitive sense. More samples are needed to detect a smaller difference.

**2.30.** Front housings for cell phones are manufactured in an injection molding process. The time the part is allowed to cool in the mold before removal is thought to influence the occurrence of a particularly troublesome cosmetic defect, flow lines, in the finished housing. After manufacturing, the housings are inspected visually and assigned a score between 1 and 10 based on their appearance, with 10 corresponding to a perfect part and 1 corresponding to a completely defective part. An experiment was conducted using

two cool-down times, 10 seconds and 20 seconds, and 20 housings were evaluated at each level of cool-down time. All 40 observations in this experiment were run in random order. The data are shown below.

10 Seconds		20 Seconds	
1	3	7	6
2	6	8	9
1	5	5	5
3	3	9	7
5	2	5	4
1	1	8	6
5	6	6	8
2	8	4	5
3	2	6	8
5	3	7	7

- (a) Is there evidence to support the claim that the longer cool-down time results in fewer appearance defects? Use  $\alpha = 0.05$ .

From the analysis shown below, there is evidence that the longer cool-down time results in fewer appearance defects.

Minitab Output

**Two-Sample T-Test and CI: 10 seconds, 20 seconds**

Two-sample T for 10 seconds vs 20 seconds

	N	Mean	StDev	SE Mean
10 secon	20	3.35	2.01	0.45
20 secon	20	6.50	1.54	0.34

Difference =  $\mu$  10 seconds -  $\mu$  20 seconds

Estimate for difference: -3.150

95% upper bound for difference: -2.196

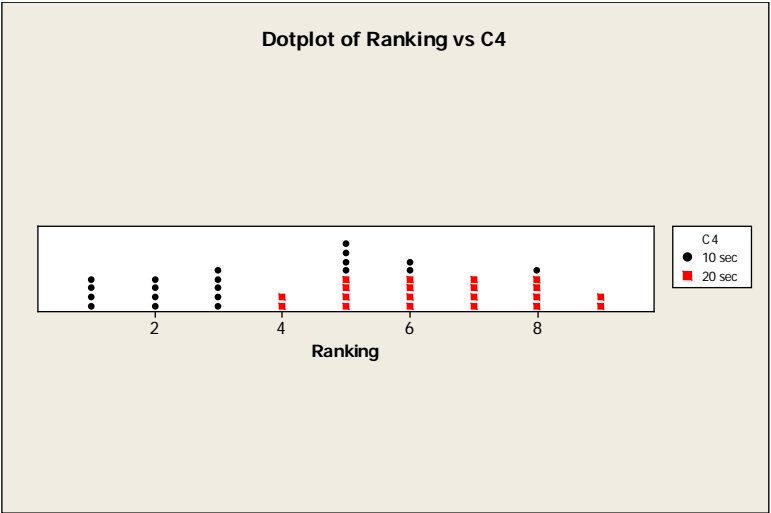
T-Test of difference = 0 (vs <): T-Value = -5.57 P-Value = 0.000 DF = 38

Both use Pooled StDev = 1.79

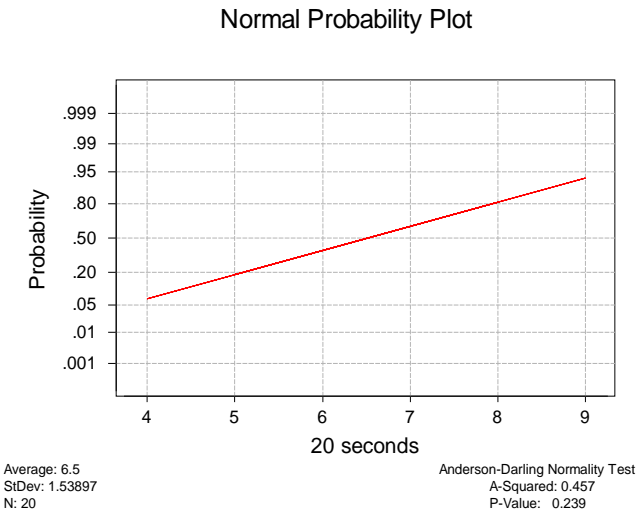
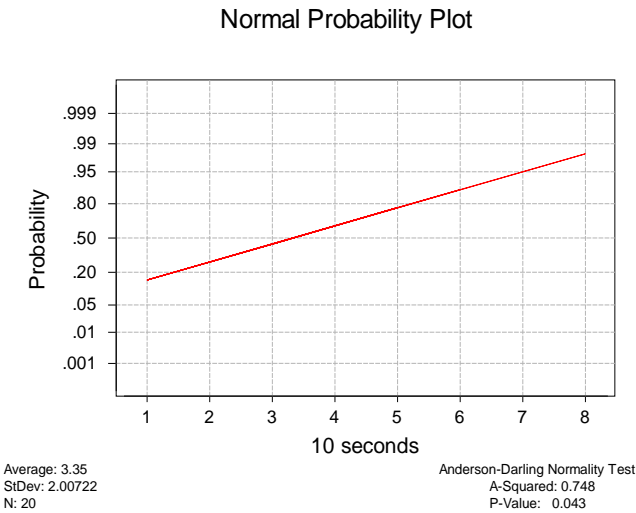
- (b) What is the  $P$ -value for the test conducted in part (a)? From the *Minitab* output,  $P = 0.000$
- (c) Find a 95% confidence interval on the difference in means. Provide a practical interpretation of this interval.

From the *Minitab* output,  $\mu_1 - \mu_2 \leq -2.196$ . This lower confidence bound is less than 0. The two samples are different. The 20 second cooling time gives a cosmetically better housing.

- (d) Draw dot diagrams to assist in interpreting the results from this experiment.



(e) Check the assumption of normality for the data from this experiment.



There are no significant departures from normality.

**2.31.** Twenty observations on etch uniformity on silicon wafers are taken during a qualification experiment for a plasma etcher. The data are as follows:

Etch Uniformity				
5.34	6.65	4.76	5.98	7.25
6.00	7.55	5.54	5.62	6.21
5.97	7.35	5.44	4.39	4.98
5.25	6.35	4.61	6.00	5.32

(a) Construct a 95 percent confidence interval estimate of  $\sigma^2$ .

$$\frac{(n-1)S^2}{\chi^2_{\alpha/2, n-1}} \leq \sigma^2 \leq \frac{(n-1)S^2}{\chi^2_{(1-\alpha/2), n-1}}$$

$$\frac{(20-1)(0.88907)^2}{32.852} \leq \sigma^2 \leq \frac{(20-1)(0.88907)^2}{8.907}$$

$$0.457 \leq \sigma^2 \leq 1.686$$

(b) Test the hypothesis that  $\sigma^2 = 1.0$ . Use  $\alpha = 0.05$ . What are your conclusions?

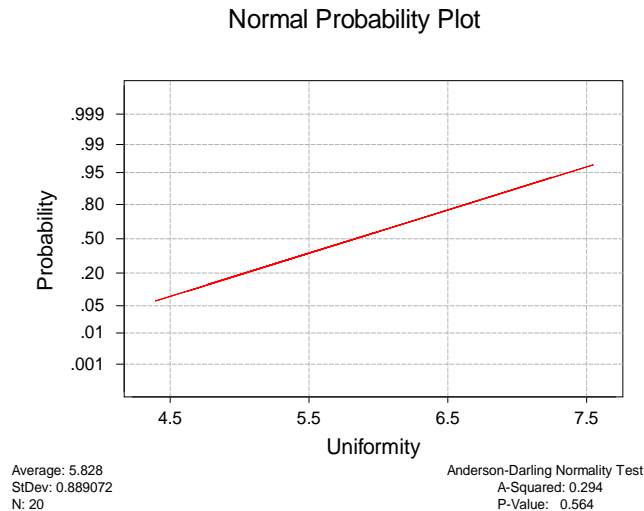
Do not reject. There is no evidence to indicate that  $\sigma^2 \neq 1$

(c) Discuss the normality assumption and its role in this problem.

The normality assumption is much more important when analyzing variances than when analyzing means. A moderate departure from normality could cause problems with both statistical tests and confidence intervals. Specifically, it will cause the reported significance levels to be incorrect.

(d) Check normality by constructing a normal probability plot. What are your conclusions?

The normal probability plot indicates that there is not a serious problem with the normality assumption.



**2.32.** The diameter of a ball bearing was measured by 12 inspectors, each using two different kinds of calipers. The results were:

Inspector	Caliper 1	Caliper 2	Difference	Difference^2
1	0.265	0.264	0.001	0.000001
2	0.265	0.265	0.000	0
3	0.266	0.264	0.002	0.000004
4	0.267	0.266	0.001	0.000001
5	0.267	0.267	0.000	0
6	0.265	0.268	-0.003	0.000009
7	0.267	0.264	0.003	0.000009
8	0.267	0.265	0.002	0.000004
9	0.265	0.265	0.000	0
10	0.268	0.267	0.001	0.000001
11	0.268	0.268	0.000	0
12	0.265	0.269	-0.004	0.000016
			$\sum = 0.003$	$\sum = 0.000045$

(a) Is there a significant difference between the means of the population of measurements represented by the two samples? Use  $\alpha = 0.05$ .

$$\begin{aligned}
 H_0 : \mu_1 &= \mu_2 && \text{or equivalently} && H_0 : \mu_d &= 0 \\
 H_1 : \mu_1 &\neq \mu_2 && && H_1 : \mu_d &\neq 0
 \end{aligned}$$

Minitab Output

Paired T-Test and Confidence Interval				
Paired T for Caliper 1 - Caliper 2				
	N	Mean	StDev	SE Mean
Caliper	12	0.266250	0.001215	0.000351
Caliper	12	0.266000	0.001758	0.000508
Difference	12	0.000250	0.002006	0.000579
95% CI for mean difference: (-0.001024, 0.001524)				
T-Test of mean difference = 0 (vs not = 0): T-Value = 0.43 P-Value = 0.674				



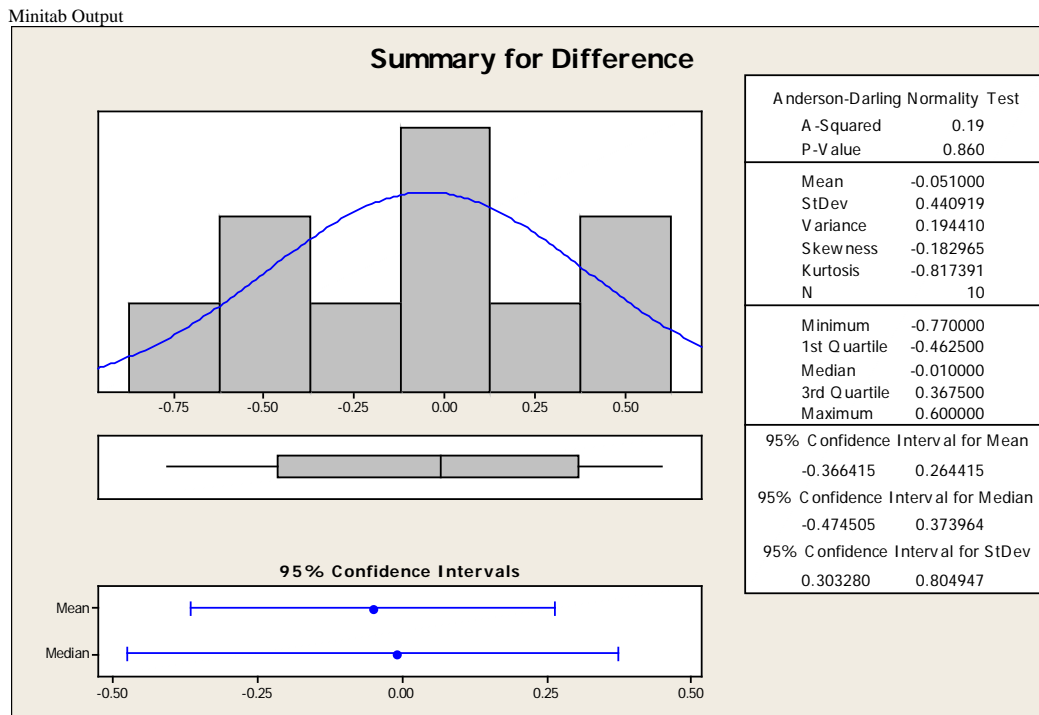
- (b) Find the  $P$ -value for the test in part (a).  $P=0.674$
- (c) Construct a 95 percent confidence interval on the difference in the mean diameter measurements for the two types of calipers.

$$\begin{aligned}\bar{d} - t_{\alpha/2, n-1} \frac{S_d}{\sqrt{n}} &\leq \mu_D (= \mu_1 - \mu_2) \leq \bar{d} + t_{\alpha/2, n-1} \frac{S_d}{\sqrt{n}} \\ 0.00025 - 2.201 \frac{0.002}{\sqrt{12}} &\leq \mu_d \leq 0.00025 + 2.201 \frac{0.002}{\sqrt{12}} \\ -0.00102 &\leq \mu_d \leq 0.00152\end{aligned}$$

**2.33.** An article in the journal of *Neurology* (1998, Vol. 50, pp.1246-1252) observed that the monozygotic twins share numerous physical, psychological and pathological traits. The investigators measured an intelligence score of 10 pairs of twins. The data are obtained as follows:

Pair	Birth Order: 1	Birth Order: 2
1	6.08	5.73
2	6.22	5.80
3	7.99	8.42
4	7.44	6.84
5	6.48	6.43
6	7.99	8.76
7	6.32	6.32
8	7.60	7.62
9	6.03	6.59
10	7.52	7.67

(a) Is the assumption that the difference in score is normally distributed reasonable?



By plotting the differences, the output shows that the Anderson-Darling Normality Test shows a P-Value of 0.860. The data is assumed to be normal.

(b) Find a 95% confidence interval on the difference in the mean score. Is there any evidence that mean score depends on birth order?

The 95% confidence interval on the difference in mean score is (-0.366415, 0.264415) contains the value of zero. There is no difference in birth order.

- (c) Test an appropriate set of hypothesis indicating that the mean score does not depend on birth order.

$$\begin{aligned} H_0 : \mu_1 &= \mu_2 & \text{or equivalently} & H_0 : \mu_d = 0 \\ H_1 : \mu_1 &\neq \mu_2 & & H_1 : \mu_d \neq 0 \end{aligned}$$

Minitab Output

Paired T for Birth Order: 1 - Birth Order: 2

	N	Mean	StDev	SE Mean
Birth Order: 1	10	6.967	0.810	0.256
Birth Order: 2	10	7.018	1.053	0.333
Difference	10	-0.051	0.441	0.139

95% CI for mean difference: (-0.366, 0.264)

T-Test of mean difference = 0 (vs not = 0): T-Value = -0.37 P-Value = 0.723

Do not reject. The  $P$ -value is 0.723.

**2.34.** An article in the *Journal of Strain Analysis* (vol.18, no. 2, 1983) compares several procedures for predicting the shear strength for steel plate girders. Data for nine girders in the form of the ratio of predicted to observed load for two of these procedures, the Karlsruhe and Lehigh methods, are as follows:

Girder	Karlsruhe Method	Lehigh Method	Difference	Difference^2
S1/1	1.186	1.061	0.125	0.015625
S2/1	1.151	0.992	0.159	0.025281
S3/1	1.322	1.063	0.259	0.067081
S4/1	1.339	1.062	0.277	0.076729
S5/1	1.200	1.065	0.135	0.018225
S2/1	1.402	1.178	0.224	0.050176
S2/2	1.365	1.037	0.328	0.107584
S2/3	1.537	1.086	0.451	0.203401
S2/4	1.559	1.052	0.507	0.257049
Sum =			2.465	0.821151
Average =			0.274	

- (a) Is there any evidence to support a claim that there is a difference in mean performance between the two methods? Use  $\alpha = 0.05$ .

$$\begin{aligned} H_0 : \mu_1 &= \mu_2 & \text{or equivalently} & H_0 : \mu_d = 0 \\ H_1 : \mu_1 &\neq \mu_2 & & H_1 : \mu_d \neq 0 \end{aligned}$$

$$\bar{d} = \frac{1}{n} \sum_{i=1}^n d_i = \frac{1}{9}(2.465) = 0.274$$

$$s_d = \left[ \frac{\sum_{i=1}^n d_i^2 - \frac{1}{n} \left( \sum_{i=1}^n d_i \right)^2}{n-1} \right]^{1/2} = \left[ \frac{0.821151 - \frac{1}{9}(2.465)^2}{9-1} \right]^{1/2} = 0.135$$

$$t_0 = \frac{\bar{d}}{\frac{S_d}{\sqrt{n}}} = \frac{0.274}{\frac{0.135}{\sqrt{9}}} = 6.08$$

$$t_{\alpha/2, n-1} = t_{0.025, 8} = 2.306, \text{ reject the null hypothesis.}$$

Minitab Output

Paired T-Test and Confidence Interval				
Paired T for Karlsruhe - Lehigh				
	N	Mean	StDev	SE Mean
Karlsruhe	9	1.3401	0.1460	0.0487
Lehigh	9	1.0662	0.0494	0.0165
Difference	9	0.2739	0.1351	0.0450
95% CI for mean difference: (0.1700, 0.3777)				
T-Test of mean difference = 0 (vs not = 0): T-Value = 6.08 P-Value = 0.000				

(b) What is the  $P$ -value for the test in part (a)?

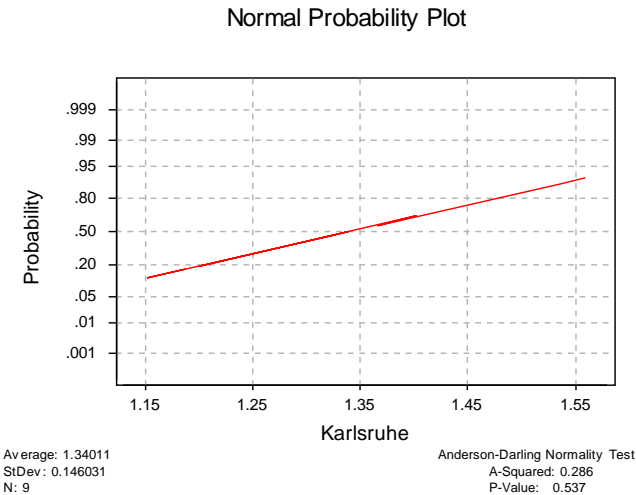
$$P=0.0002$$

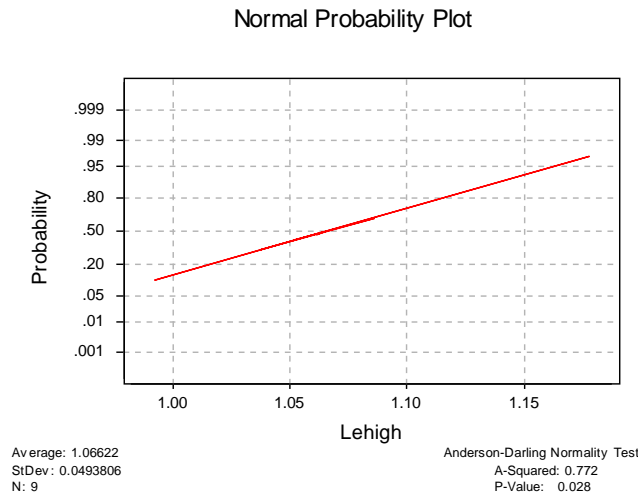
(c) Construct a 95 percent confidence interval for the difference in mean predicted to observed load.

$$\begin{aligned} \bar{d} - t_{\alpha/2, n-1} \frac{S_d}{\sqrt{n}} &\leq \mu_d \leq \bar{d} + t_{\alpha/2, n-1} \frac{S_d}{\sqrt{n}} \\ 0.274 - 2.306 \frac{0.135}{\sqrt{9}} &\leq \mu_d \leq 0.274 + 2.306 \frac{0.135}{\sqrt{9}} \\ 0.17023 &\leq \mu_d \leq 0.37777 \end{aligned}$$

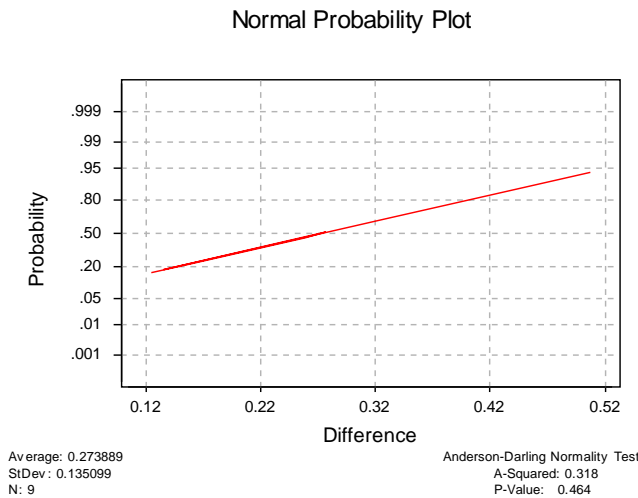
(d) Investigate the normality assumption for both samples.

The normal probability plots of the observations for each method follow. There are no serious concerns with the normality assumption, but there is an indication of a possible outlier (1.178) in the Lehigh method data.





(e) Investigate the normality assumption for the difference in ratios for the two methods.



There is no issue with normality in the difference of ratios of the two methods.

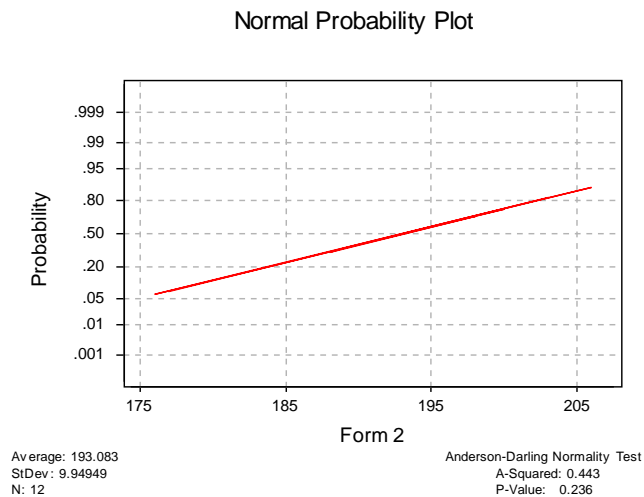
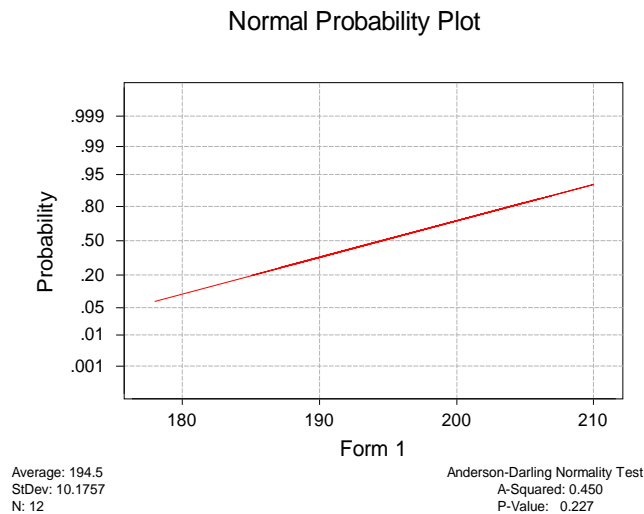
(f) Discuss the role of the normality assumption in the paired  $t$ -test.

As in any  $t$ -test, the assumption of normality is of only moderate importance. In the paired  $t$ -test, the assumption of normality applies to the distribution of the differences. That is, the individual sample measurements do not have to be normally distributed, only their difference.

**2.35.** The deflection temperature under load for two different formulations of ABS plastic pipe is being studied. Two samples of 12 observations each are prepared using each formulation, and the deflection temperatures (in °F) are reported below:

Formulation 1			Formulation 2		
206	193	192	177	176	198
188	207	210	197	185	188
205	185	194	206	200	189
187	189	178	201	197	203

- (a) Construct normal probability plots for both samples. Do these plots support assumptions of normality and equal variance for both samples?



- (b) Do the data support the claim that the mean deflection temperature under load for formulation 1 exceeds that of formulation 2? Use  $\alpha = 0.05$ .

No, formulation 1 does not exceed formulation 2 per the *Minitab* output below.

Minitab Output

Two Sample T-Test and Confidence Interval				
	N	Mean	StDev	SE Mean
Form 1	12	194.5	10.2	2.9
Form 2	12	193.08	9.95	2.9
Difference = mu Form 1 - mu Form 2				
Estimate for difference: 1.42				
95% lower bound for difference: -5.64				
T-Test of difference = 0 (vs >): T-Value = 0.34 P-Value = 0.367 DF = 22				
Both use Pooled StDev = 10.1				

(c) What is the  $P$ -value for the test in part (a)?

$$P = 0.367$$

**2.36.** Refer to the data in problem 2.35. Do the data support a claim that the mean deflection temperature under load for formulation 1 exceeds that of formulation 2 by at least 3 °F?

No, formulation 1 does not exceed formulation 2 by at least 3 °F.

Minitab Output

Two-Sample T-Test and CI: Form1, Form2				
Two-sample T for Form 1 vs Form 2				
	N	Mean	StDev	SE Mean
Form 1	12	194.5	10.2	2.9
Form 2	12	193.08	9.95	2.9
Difference = mu Form 1 - mu Form 2				
Estimate for difference: 1.42				
95% lower bound for difference: -5.64				
T-Test of difference = 3 (vs >): T-Value = -0.39 P-Value = 0.648 DF = 22				
Both use Pooled StDev = 10.1				

**2.37.** In semiconductor manufacturing, wet chemical etching is often used to remove silicon from the backs of wafers prior to metalization. The etch rate is an important characteristic of this process. Two different etching solutions are being evaluated. Eight randomly selected wafers have been etched in each solution and the observed etch rates (in mils/min) are shown below:

Solution 1		Solution 2	
9.9	10.6	10.2	10.6
9.4	10.3	10.0	10.2
10.0	9.3	10.7	10.4
10.3	9.8	10.5	10.3

(a) Do the data indicate that the claim that both solutions have the same mean etch rate is valid? Use  $\alpha = 0.05$  and assume equal variances.

No, the solutions do not have the same mean etch rate. See the Minitab output below.

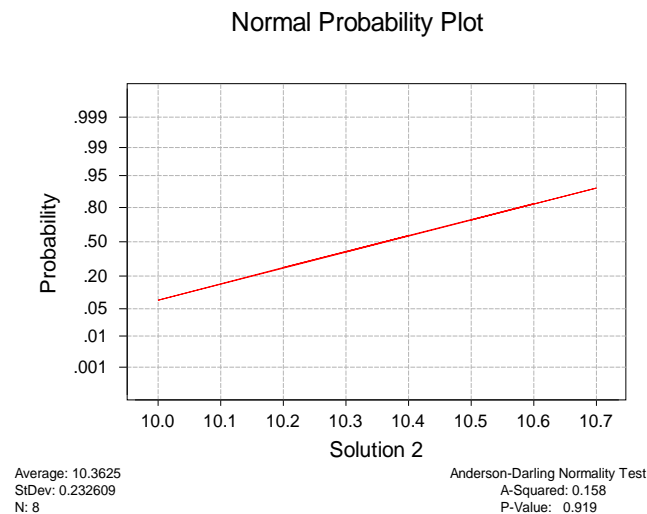
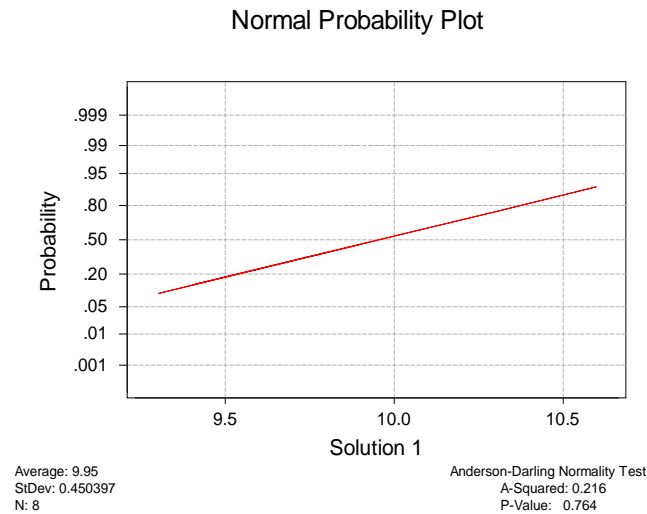
Minitab Output

Two Sample T-Test and Confidence Interval				
Two-sample T for Solution 1 vs Solution 2				
	N	Mean	StDev	SE Mean
Solution 1	8	9.950	0.450	0.16
Solution 2	8	10.363	0.233	0.082
Difference = mu Solution 1 - mu Solution 2				
Estimate for difference: -0.413				
95% CI for difference: (-0.797, -0.028)				
T-Test of difference = 0 (vs not =): T-Value = -2.30 P-Value = 0.037 DF = 14				
Both use Pooled StDev = 0.358				

(b) Find a 95% confidence interval on the difference in mean etch rate.

From the Minitab output, -0.797 to -0.028.

- (c) Use normal probability plots to investigate the adequacy of the assumptions of normality and equal variances.



Both the normality and equality of variance assumptions are valid.

**2.38.** Two popular pain medications are being compared on the basis of the speed of absorption by the body. Specifically, tablet 1 is claimed to be absorbed twice as fast as tablet 2. Assume that  $\sigma_1^2$  and  $\sigma_2^2$  are known. Develop a test statistic for

$$H_0: 2\mu_1 = \mu_2$$

$$H_1: 2\mu_1 \neq \mu_2$$

$$2\bar{y}_1 - \bar{y}_2 \sim N\left(2\mu_1 - \mu_2, \frac{4\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right), \text{ assuming that the data is normally distributed.}$$



The test statistic is: 
$$z_o = \frac{2\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{4\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}, \text{ reject if } |z_o| > z_{\alpha/2}$$

**2.39. Continuation of Problem 2.38.** An article in *Nature* (1972, pp.225-226) reported on the levels of monoamine oxidase in blood platelets for a sample of 43 schizophrenic patients resulting in  $\bar{y}_1 = 2.69$  and  $s_1 = 2.30$  while for a sample of 45 normal patients the results were  $\bar{y}_2 = 6.35$  and  $s_2 = 4.03$ . The units are nm/mg protein/h. Use the results of the previous problem to test the claim that the mean monoamine oxidase level for normal patients is at least twice the mean level for schizophrenic patients. Assume that the sample sizes are large enough to use the sample standard deviations as the true parameter values.

$$z_o = \frac{2\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{4\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{2(2.69) - 6.35}{\sqrt{\frac{4(2.30)^2}{43} + \frac{4.03^2}{45}}} = \frac{-0.97}{.92357} = -1.05$$

$z_0 = -1.05$ ; using  $\alpha=0.05$ ,  $z_{\alpha/2} = 1.96$ , do not reject.

**2.40.** Suppose we are testing

$$\begin{aligned} H_0: & \mu_1 = \mu_2 \\ H_1: & \mu_1 \neq \mu_2 \end{aligned}$$

where  $\sigma_1^2$  and  $\sigma_2^2$  are known. Our sampling resources are constrained such that  $n_1 + n_2 = N$ . How should we allocate the  $n_1, n_2$  to the two samples that lead to the most powerful test?

The most powerful test is attained by the  $n_1$  and  $n_2$  that maximize  $z_o$  for given  $\bar{y}_1 - \bar{y}_2$ .

Thus, we chose  $n_1$  and  $n_2$  to 
$$\max z_o = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}, \text{ subject to } n_1 + n_2 = N.$$

This is equivalent to 
$$\min L = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2} = \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{N - n_1}, \text{ subject to } n_1 + n_2 = N.$$

Now 
$$\frac{dL}{dn_1} = \frac{-\sigma_1^2}{n_1^2} + \frac{\sigma_2^2}{(N - n_1)^2} = 0, \text{ implies that } n_1 / n_2 = \sigma_1 / \sigma_2.$$

Thus  $n_1$  and  $n_2$  are assigned proportionally to the ratio of the standard deviations. This has intuitive appeal, as it allocates more observations to the population with the greatest variability.

**2.41 Continuation of Problem 2.40.** Suppose that we want to construct a 95% two-sided confidence interval on the difference in two means where the two sample standard deviations are known to be  $\sigma_1 = 4$  and  $\sigma_2 = 8$ . The total sample size is restricted to  $N = 30$ . What is the length of the 95% CI if the sample sizes used by the experimenter are  $n_1 = n_2 = 15$ ? How much shorter would the 95% CI have been if the experiment had used the optimal sample size calculation?

The 95% confidence interval for  $n_1 = n_2 = 15$  is

$$\begin{aligned}
 (\bar{y}_1 - \bar{y}_2) - z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} &\leq \mu_1 - \mu_2 \leq (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \\
 (\bar{y}_1 - \bar{y}_2) - z_{\alpha/2} \sqrt{\frac{4^2}{15} + \frac{8^2}{15}} &\leq \mu_1 - \mu_2 \leq (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2} \sqrt{\frac{4^2}{15} + \frac{8^2}{15}} \\
 (\bar{y}_1 - \bar{y}_2) - z_{\alpha/2} (2.31) &\leq \mu_1 - \mu_2 \leq (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2} (2.31)
 \end{aligned}$$

The 95% confidence interval for the proportions is,

$$n_1 = 30 - n_2$$

$$\frac{n_1}{n_2} = \frac{\sigma_1}{\sigma_2} = \frac{30 - n_2}{n_2} = \frac{4}{8}$$

Therefore  $n_2 = 20$  and  $n_1 = 10$

$$\begin{aligned}
 (\bar{y}_1 - \bar{y}_2) - z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} &\leq \mu_1 - \mu_2 \leq (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \\
 (\bar{y}_1 - \bar{y}_2) - z_{\alpha/2} \sqrt{\frac{4^2}{10} + \frac{8^2}{20}} &\leq \mu_1 - \mu_2 \leq (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2} \sqrt{\frac{4^2}{10} + \frac{8^2}{20}} \\
 (\bar{y}_1 - \bar{y}_2) - z_{\alpha/2} (2.19) &\leq \mu_1 - \mu_2 \leq (\bar{y}_1 - \bar{y}_2) + z_{\alpha/2} (2.19)
 \end{aligned}$$

The confidence interval decreases from a multiple of 2.31 to a multiple of 2.19.

**2.42.** Develop Equation 2.46 for a  $100(1 - \alpha)$  percent confidence interval for the variance of a normal distribution.

$$\begin{aligned}
 \frac{SS}{\sigma^2} &\sim \chi_{n-1}^2 \quad \text{Thus, } P\left\{\chi_{1-\frac{\alpha}{2}, n-1}^2 \leq \frac{SS}{\sigma^2} \leq \chi_{\frac{\alpha}{2}, n-1}^2\right\} = 1 - \alpha \quad \text{Therefore,} \\
 P\left\{\frac{SS}{\chi_{\frac{\alpha}{2}, n-1}^2} \leq \sigma^2 \leq \frac{SS}{\chi_{1-\frac{\alpha}{2}, n-1}^2}\right\} &= 1 - \alpha, \\
 \text{so } \left[\frac{SS}{\chi_{\frac{\alpha}{2}, n-1}^2}, \frac{SS}{\chi_{1-\frac{\alpha}{2}, n-1}^2}\right] &\text{ is the } 100(1 - \alpha)\% \text{ confidence interval on } \sigma^2.
 \end{aligned}$$

**2.43.** Develop Equation 2.50 for a  $100(1 - \alpha)$  percent confidence interval for the ratio  $\sigma_1^2 / \sigma_2^2$ , where  $\sigma_1^2$  and  $\sigma_2^2$  are the variances of two normal distributions.

$$\begin{aligned}
 \frac{S_2^2 / \sigma_2^2}{S_1^2 / \sigma_1^2} &\sim F_{n_2-1, n_1-1} \\
 P\left\{F_{1-\alpha/2, n_2-1, n_1-1} \leq \frac{S_2^2 / \sigma_2^2}{S_1^2 / \sigma_1^2} \leq F_{\alpha/2, n_2-1, n_1-1}\right\} &= 1 - \alpha \quad \text{or} \\
 P\left\{\frac{S_1^2}{S_2^2} F_{1-\alpha/2, n_2-1, n_1-1} \leq \frac{\sigma_1^2}{\sigma_2^2} \leq \frac{S_1^2}{S_2^2} F_{\alpha/2, n_2-1, n_1-1}\right\} &= 1 - \alpha
 \end{aligned}$$

**2.44.** Develop an equation for finding a  $100(1 - \alpha)$  percent confidence interval on the difference in the means of two normal distributions where  $\sigma_1^2 \neq \sigma_2^2$ . Apply your equation to the portland cement experiment data, and find a 95% confidence interval.

$$\frac{(\bar{y}_1 - \bar{y}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim t_{\alpha/2, \nu}$$

$$t_{\alpha/2, \nu} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \leq (\bar{y}_1 - \bar{y}_2) - (\mu_1 - \mu_2) \leq t_{\alpha/2, \nu} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$(\bar{y}_1 - \bar{y}_2) - t_{\alpha/2, \nu} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}} \leq (\mu_1 - \mu_2) \leq (\bar{y}_1 - \bar{y}_2) + t_{\alpha/2, \nu} \sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}$$

$$\text{where } \nu = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{\left(\frac{S_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{S_2^2}{n_2}\right)^2}{n_2 - 1}}$$

Using the data from Table 2.1

$$(16.764 - 17.343) - 2.110 \sqrt{\frac{0.100138}{10} + \frac{0.0614622}{10}} \leq (\mu_1 - \mu_2) \leq$$

$$(16.764 - 17.343) + 2.110 \sqrt{\frac{0.100138}{10} + \frac{0.0614622}{10}}$$

$$\text{where } \nu = \frac{\left(\frac{0.100138}{10} + \frac{0.0614622}{10}\right)^2}{\frac{\left(\frac{0.100138}{10}\right)^2}{10 - 1} + \frac{\left(\frac{0.0614622}{10}\right)^2}{10 - 1}} = 17.024 \cong 17$$

$$-1.426 \leq (\mu_1 - \mu_2) \leq -0.889$$

2.45. Construct a data set for which the paired  $t$ -test statistic is very large, but for which the usual two-sample or pooled  $t$ -test statistic is small. In general, describe how you created the data. Does this give you any insight regarding how the paired  $t$ -test works?

A	B	delta
7.1662	8.2416	-1.0754
2.3590	2.4555	-0.0965
19.9977	21.1018	-1.1041
0.9077	2.3401	-1.4324
-15.9034	-15.0013	-0.9021
-6.0722	-5.5941	-0.4781
9.9501	10.6910	-0.7409
-1.0944	-0.1358	-0.9586
-4.6907	-3.3446	-1.3461
-6.6929	-5.9303	-0.7626

Minitab Output

**Paired T-Test and Confidence Interval**

Paired T for A - B

	N	Mean	StDev	SE Mean
A	10	0.59	10.06	3.18
B	10	1.48	10.11	3.20
Difference	10	-0.890	0.398	0.126

95% CI for mean difference: (-1.174, -0.605)

T-Test of mean difference = 0 (vs not = 0): T-Value = -7.07 P-Value = 0.000

**Two Sample T-Test and Confidence Interval**

Two-sample T for A vs B

	N	Mean	StDev	SE Mean
A	10	0.6	10.1	3.2
B	10	1.5	10.1	3.2

Difference = mu A - mu B

Estimate for difference: -0.89

95% CI for difference: (-10.37, 8.59)

T-Test of difference = 0 (vs not =): T-Value = -0.20 P-Value = 0.846 DF = 18

Both use Pooled StDev = 10.1

These two sets of data were created by making the observation for  $A$  and  $B$  moderately different within each pair (or block), but making the observations between pairs very different. The fact that the difference between pairs is large makes the pooled estimate of the standard deviation large and the two-sample  $t$ -test statistic small. Therefore the fairly small difference between the means of the two treatments that is present when they are applied to the same experimental unit cannot be detected. Generally, if the blocks are very different, then this will occur. Blocking eliminates the variability associated with the nuisance variable that they represent.

**2.46.** Consider the experiment described in problem 2.26. If the mean burning times of the two flames differ by as much as 2 minutes, find the power of the test. What sample size would be required to detect an actual difference in mean burning time of 1 minute with a power of at least 0.90?

From the *Minitab* output below, the power is 0.0740. This answer was obtained by using the pooled estimate of  $\sigma$  from Problem 2-11,  $S_p = 9.32$ . Because the difference in means is very small relative to the standard deviation, the power is very low.

Minitab Output

Power and Sample Size			
2-Sample t Test			
Testing mean 1 = mean 2 (versus not =)			
Calculating power for mean 1 = mean 2 + difference			
Alpha = 0.05    Sigma = 9.32			
	Sample		
Difference	Size	Power	
2	10	0.0740	

From the *Minitab* output below, the required sample size is 1827. The sample size is huge because the difference in means is very small relative to the standard deviation.

Minitab Output

Power and Sample Size			
2-Sample t Test			
Testing mean 1 = mean 2 (versus not =)			
Calculating power for mean 1 = mean 2 + difference			
Alpha = 0.05    Sigma = 9.32			
	Sample	Target	Actual
Difference	Size	Power	Power
1	1827	0.9000	0.9001

**2.47.** Reconsider the bottle filling experiment described in Problem 2.24. Rework this problem assuming that the two population variances are unknown but equal.

Minitab Output

Two-Sample T-Test and CI: Machine 1, Machine 2				
Two-sample T for Machine 1 vs Machine 2				
	N	Mean	StDev	SE Mean
Machine	10	16.0150	0.0303	0.0096
Machine	10	16.0050	0.0255	0.0081
Difference = mu Machine 1 - mu Machine 2				
Estimate for difference: 0.0100				
95% CI for difference: (-0.0163, 0.0363)				
T-Test of difference = 0 (vs not =): T-Value = 0.80    P-Value = 0.435    DF = 18				
Both use Pooled StDev = 0.0280				

The hypothesis test is the same:  $H_0: \mu_1 = \mu_2$        $H_1: \mu_1 \neq \mu_2$

The conclusions are the same as Problem 2.19, do not reject  $H_0$ . There is no difference in the machines. The  $P$ -value for this analysis is 0.435.

The confidence interval is (-0.0163, 0.0363). This interval contains 0. There is no difference in machines.

**2.48.** Consider the data from problem 2.24. If the mean fill volume of the two machines differ by as much as 0.25 ounces, what is the power of the test used in problem 2.19? What sample size could result in a power of at least 0.9 if the actual difference in mean fill volume is 0.25 ounces?

The power is 1.0000 as shown in the analysis below.

Minitab Output

Power and Sample Size			
2-Sample t Test			
Testing mean 1 = mean 2 (versus not =)			
Calculating power for mean 1 = mean 2 + difference			
Alpha = 0.05   Sigma = 0.028			
	Sample		
Difference	Size	Power	
0.25	10	1.0000	

The required sample size is 2 as shown below.

Minitab Output

Power and Sample Size			
2-Sample t Test			
Testing mean 1 = mean 2 (versus not =)			
Calculating power for mean 1 = mean 2 + difference			
Alpha = 0.05   Sigma = 0.028			
	Sample	Target	Actual
Difference	Size	Power	Power
0.25	2	0.9000	0.9805

## Chapter 3

### Experiments with a Single Factor: The Analysis of Variance

### Solutions

**3.1.** An experimenter has conducted a single-factor experiment with four levels of the factor, and each factor level has been replicated six times. The computed value of the  $F$ -statistic is  $F_0 = 3.26$ . Find bounds on the  $P$ -value.

Table  $P$ -value = 0.025, 0.050

Computer  $P$ -value = 0.043

**3.2.** An experimenter has conducted a single-factor experiment with six levels of the factor, and each factor level has been replicated three times. The computed value of the  $F$ -statistic is  $F_0 = 5.81$ . Find bounds on the  $P$ -value.

Table  $P$ -value < 0.010

Computer  $P$ -value = 0.006

**3.3.** A computer ANOVA output is shown below. Fill in the blanks. You may give bounds on the  $P$ -value.

One-way ANOVA					
Source	DF	SS	MS	F	P
Factor	3	36.15	?	?	?
Error	?	?	?		
Total	19	196.04			

Completed table is:

One-way ANOVA					
Source	DF	SS	MS	F	P
Factor	3	36.15	12.05	1.21	0.3395
Error	16	159.89	9.99		
Total	19	196.04			

**3.4.** A computer ANOVA output is shown below. Fill in the blanks. You may give bounds on the  $P$ -value.

One-way ANOVA					
Source	DF	SS	MS	F	P
Factor	?	?	246.93	?	?
Error	25	186.53	?		
Total	29	1174.24			

Completed table is:

One-way ANOVA					
Source	DF	SS	MS	F	P
Factor	4	987.71	246.93	33.09	< 0.0001
Error	25	186.53	7.46		
Total	29	1174.24			

**3.5.** An article appeared in *The Wall Street Journal* on Tuesday, April 27, 2010, with the title “Eating Chocolate Is Linked to Depression.” The article reported on a study funded by the National Heart, Lung and Blood Institute (part of the National Institutes of Health) and conducted by the faculty at the University of California, San Diego, and the University of California, Davis. The research was also published in the *Archives of Internal Medicine* (2010, pp. 699-703). The study examined 931 adults who were not taking antidepressants and did not have known cardiovascular disease or diabetes. The group was about 70% men and the average age of the group was reported to be about 58. The participants were asked about chocolate consumption and then screened for depression using a questionnaire. People who scored less than 16 on the questionnaire are not considered depressed, while those with scores above 16 and less than or equal to 22 are considered possibly depressed, while those with scores above 22 are considered likely to be depressed. The survey found that people who were not depressed ate an average of 8.4 servings of chocolate per month, while those individuals who scored above 22 were likely to be depressed ate the most chocolate, an average of 11.8 servings per month. No differentiation was made between dark and milk chocolate. Other foods were also examined, but no patterned emerged between other foods and depression. Is this study really a designed experiment? Does it establish a cause-and-effect link between chocolate consumption and depression? How would the study have to be conducted to establish such a link?

This is not a designed experiment, and it does not establish a cause-and-effect link between chocolate consumption and depression. An experiment could be run by giving a group of people a defined amount of chocolate servings per month for several months, while not giving another group any chocolate. Ideally it would be good to have the participants not eat any chocolate for a period of time before the experiment, and measure depression for each participant before and after the experiment.

**3.6.** An article in *Bioelectromagnetics* (“Electromagnetic Effects on Forearm Disuse Osteopenia: A Randomized, Double-Blind, Sham-Controlled Study,” Vol. 32, 2011, pp. 273 – 282) describes a randomized, double-blind, sham-controlled, feasibility and dosing study to determine if a common pulsing electromagnetic field (PEMF) treatment could moderate the substantial osteopenia that occurs after forearm disuse. Subjects were randomized into four groups after a distal radius fracture, or carpal surgery requiring immobilization in a cast. Active of identical sham PEMF transducers were worn on a distal forearm for 1, 2, or 4h/day for 8 weeks starting after cast removal (“baseline”) when bone density continues to decline. Bone mineral density (BMD) and bone geometry were measured in the distal forearm by dual energy X-ray absorptiometry (DXA) and peripheral quantitative computed tomography (pQCT). The data below are the percent losses in BMD measurements on the radius after 16weeks for patients wearing the active or sham PEMF transducers for 1, 2, or 4h/day (data were constructed to match the means and standard deviations read from a graph in the paper).

	PEMF 1h/day	PEMF 2h/day	PEMF 4h/day
Sham			
4.51	5.32	4.73	7.03
7.95	6.00	5.81	4.65
4.97	5.12	5.69	6.65



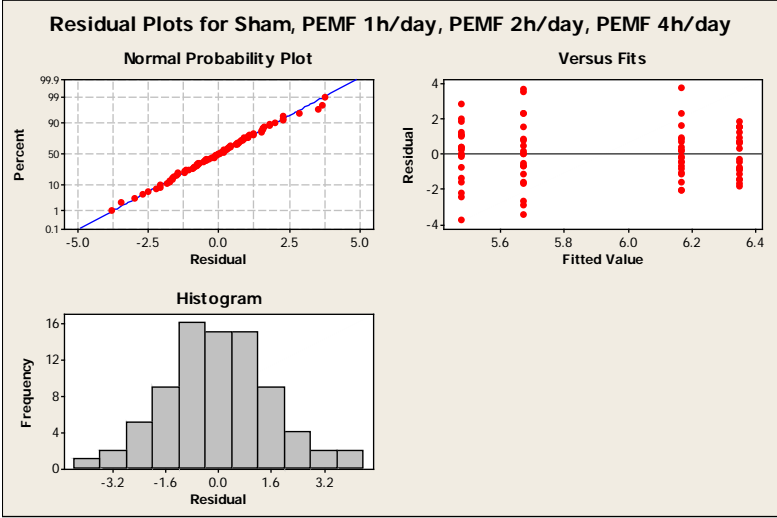
3.00	7.08	3.86	5.49
7.97	5.48	4.06	6.98
2.23	6.52	6.56	4.85
3.95	4.09	8.34	7.26
5.64	6.28	3.01	5.92
9.35	7.77	6.71	5.58
6.52	5.68	6.51	7.91
4.96	8.47	1.70	4.90
6.10	4.58	5.89	4.54
7.19	4.11	6.55	8.18
4.03	5.72	5.34	5.42
2.72	5.91	5.88	6.03
9.19	6.89	7.50	7.04
5.17	6.99	3.28	5.17
5.70	4.98	5.38	7.60
5.85	9.94	7.30	7.90
6.45	6.38	5.46	7.91

- (a) Is there evidence to support a claim that PEMF usage affects BMD loss? If so, analyze the data to determine which specific treatments produce the differences. The ANOVA from the Minitab output shows that there is no difference between the treatments;  $P=0.281$ .

Minitab Output

One-way ANOVA: Sham, PEMF 1h/day, PEMF 2h/day, PEMF 4h/day					
Source	DF	SS	MS	F	P
Factor	3	10.04	3.35	1.30	0.281
Error	76	196.03	2.58		
Total	79	206.07			
S = 1.606    R-Sq = 4.87%    R-Sq(adj) = 1.12%					
Individual 95% CIs For Mean Based on Pooled StDev					
Level	N	Mean	StDev		
Sham	20	5.673	2.002	-----+-----	
PEMF 1h/day	20	6.165	1.444	(-----*-----)	
PEMF 2h/day	20	5.478	1.645	(-----*-----)	
PEMF 4h/day	20	6.351	1.232	(-----*-----)	
				-----+-----	
				4.80    5.40    6.00    6.60	

- (b) Analyze the residuals from this experiment and comment on the underlying assumptions and model adequacy. The residuals show the model is good.



3.7. The tensile strength of Portland cement is being studied. Four different mixing techniques can be used economically. A completely randomized experiment was conducted and the following data were collected.

Mixing Technique	Tensile Strength (lb/in <sup>2</sup> )			
1	3129	3000	2865	2890
2	3200	3300	2975	3150
3	2800	2900	2985	3050
4	2600	2700	2600	2765

(a) Test the hypothesis that mixing techniques affect the strength of the cement. Use  $\alpha = 0.05$ .

Design Expert Output

Response: Tensile Strength in lb/in<sup>2</sup>

ANOVA for Selected Factorial Model

Analysis of variance table [Partial sum of squares]

Source	Sum of Squares	DF	Mean Square	F Value	Prob > F	
Model	4.897E+005	3	1.632E+005	12.73	0.0005	significant
A	4.897E+005	3	1.632E+005	12.73	0.0005	
Residual	1.539E+005	12	12825.69			
Lack of Fit	0.000	0				
Pure Error	1.539E+005	12	12825.69			
Cor Total	6.436E+005	15				

The Model F-value of 12.73 implies the model is significant. There is only a 0.05% chance that a "Model F-Value" this large could occur due to noise.

Treatment Means (Adjusted, If Necessary)

	Estimated Mean	Standard Error
1-1	2971.00	56.63
2-2	3156.25	56.63
3-3	2933.75	56.63
4-4	2666.25	56.63

Treatment	Mean Difference	DF	Standard Error	t for H0 Coeff=0	Prob >  t
1 vs 2	-185.25	1	80.08	-2.31	0.0392
1 vs 3	37.25	1	80.08	0.47	0.6501
1 vs 4	304.75	1	80.08	3.81	0.0025

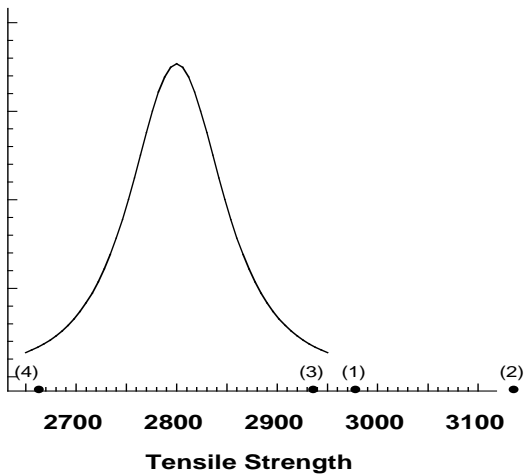
2 vs 3	222.50	1	80.08	2.78	0.0167
2 vs 4	490.00	1	80.08	6.12	< 0.0001
3 vs 4	267.50	1	80.08	3.34	0.0059

The  $F$ -value is 12.73 with a corresponding  $P$ -value of .0005. Mixing technique has an effect.

(b) Construct a graphical display as described in Section 3.5.3 to compare the mean tensile strengths for the four mixing techniques. What are your conclusions?

$$S_{\bar{y}_i} = \sqrt{\frac{MS_E}{n}} = \sqrt{\frac{12825.7}{4}} = 56.625$$

Scaled t Distribution



Based on examination of the plot, we would conclude that  $\mu_1$  and  $\mu_3$  are the same; that  $\mu_4$  differs from  $\mu_1$  and  $\mu_3$ , that  $\mu_2$  differs from  $\mu_1$  and  $\mu_3$ , and that  $\mu_2$  and  $\mu_4$  are different.

(c) Use the Fisher LSD method with  $\alpha=0.05$  to make comparisons between pairs of means.

$$LSD = t_{\alpha/2, N-a} \sqrt{\frac{2MS_E}{n}}$$

$$LSD = t_{0.025, 16-4} \sqrt{\frac{2(12825.7)}{4}}$$

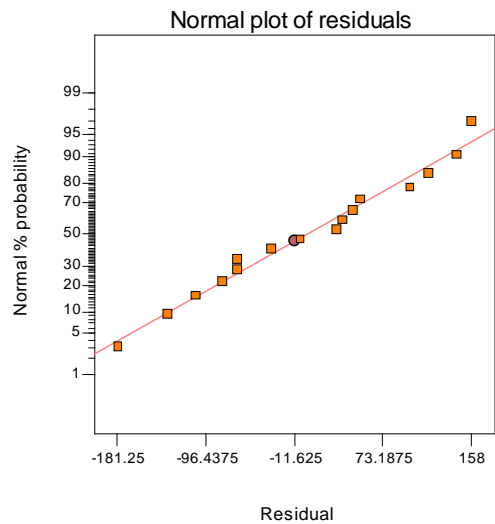
$$LSD = 2.179\sqrt{6412.85} = 174.495$$

Treatment 2 vs. Treatment 4 = 3156.250 - 2666.250 = 490.000 > 174.495  
 Treatment 2 vs. Treatment 3 = 3156.250 - 2933.750 = 222.500 > 174.495  
 Treatment 2 vs. Treatment 1 = 3156.250 - 2971.000 = 185.250 > 174.495  
 Treatment 1 vs. Treatment 4 = 2971.000 - 2666.250 = 304.750 > 174.495  
 Treatment 1 vs. Treatment 3 = 2971.000 - 2933.750 = 37.250 < 174.495  
 Treatment 3 vs. Treatment 4 = 2933.750 - 2666.250 = 267.500 > 174.495

The Fisher LSD method is also presented in the Design-Expert computer output above. The results agree with the graphical method for this experiment.

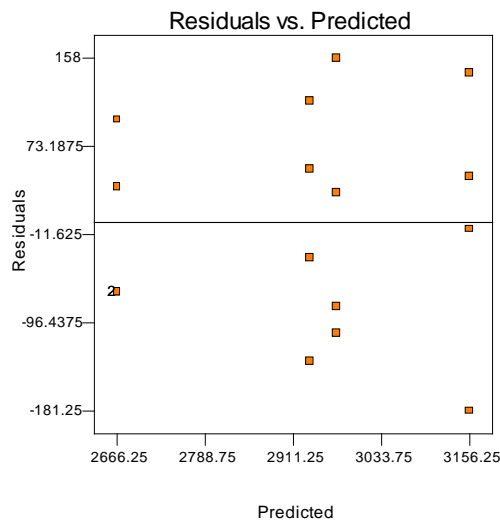
(d) Construct a normal probability plot of the residuals. What conclusion would you draw about the validity of the normality assumption?

There is nothing unusual about the normal probability plot of residuals.



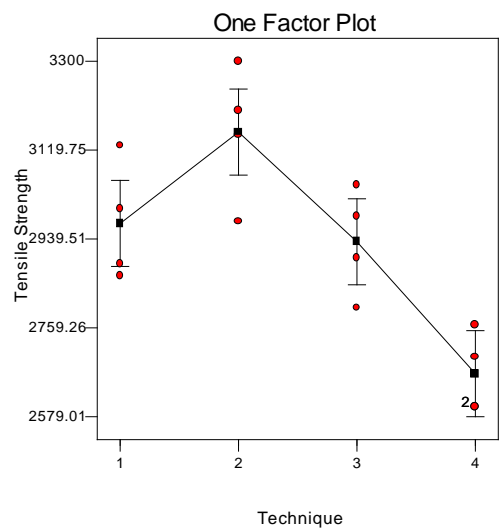
(e) Plot the residuals versus the predicted tensile strength. Comment on the plot.

There is nothing unusual about this plot.



(f) Prepare a scatter plot of the results to aid the interpretation of the results of this experiment.

Design-Expert automatically generates the scatter plot. The plot below also shows the sample average for each treatment and the 95 percent confidence interval on the treatment mean.



**3.8.** (a) Rework part (c) of Problem 3.7 using Tukey’s test with  $\alpha = 0.05$ . Do you get the same conclusions from Tukey’s test that you did from the graphical procedure and/or the Fisher LSD method?

Minitab Output

Tukey's pairwise comparisons

Family error rate = 0.0500

Individual error rate = 0.0117

Critical value = 4.20

Intervals for (column level mean) - (row level mean)

	1	2	3
2	-423 53		
3	-201 275	-15 460	
4	67 543	252 728	30 505

No, the conclusions are not the same. The mean of Treatment 4 is different than the means of Treatments 1, 2, and 3. However, the mean of Treatment 2 is not different from the means of Treatments 1 and 3 according to Tukey’s method, they were found to be different using the graphical method and the Fisher LSD method.

(b) Explain the difference between the Tukey and Fisher procedures.

Both Tukey and Fisher utilize a single critical value; however, Tukey’s is based on the studentized range statistic while Fisher’s is based on  $t$  distribution.

**3.9.** Reconsider the experiment in Problem 3.7. Find a 95 percent confidence interval on the mean tensile strength of the portland cement produced by each of the four mixing techniques. Also find a 95 percent confidence interval on the difference in means for techniques 1 and 3. Does this aid in interpreting the results of the experiment?

$$\bar{y}_i - t_{\alpha/2, N-a} \sqrt{\frac{MS_E}{n}} \leq \mu_i \leq \bar{y}_i + t_{\alpha/2, N-a} \sqrt{\frac{MS_E}{n}}$$

Treatment 1:  $2971 \pm 2.179 \sqrt{\frac{12825.69}{4}}$

$2971 \pm 123.387$

$2847.613 \leq \mu_1 \leq 3094.387$

Treatment 2:  $3156.25 \pm 123.387$

$3032.863 \leq \mu_2 \leq 3279.637$

Treatment 3:  $2933.75 \pm 123.387$

$2810.363 \leq \mu_3 \leq 3057.137$

Treatment 4:  $2666.25 \pm 123.387$

$2542.863 \leq \mu_4 \leq 2789.637$

Treatment 1 - Treatment 3:  $\bar{y}_i - \bar{y}_j - t_{\alpha/2, N-a} \sqrt{\frac{2MS_E}{n}} \leq \mu_i - \mu_j \leq \bar{y}_i - \bar{y}_j + t_{\alpha/2, N-a} \sqrt{\frac{2MS_E}{n}}$

$2971.00 - 2933.75 \pm 2.179 \sqrt{\frac{2(12825.7)}{4}}$

$-137.245 \leq \mu_1 - \mu_3 \leq 211.745$

Because the confidence interval for the difference between means 1 and 3 spans zero, we agree with the statement in Problem 3.5 (b); there is not a statistical difference between these two means.

**3.10.** A product developer is investigating the tensile strength of a new synthetic fiber that will be used to make cloth for men’s shirts. Strength is usually affected by the percentage of cotton used in the blend of materials for the fiber. The engineer conducts a completely randomized experiment with five levels of cotton content and replicated the experiment five times. The data are shown in the following table.

Cotton Weight Percentage		Observations			
15	7	7	15	11	9
20	12	17	12	18	18
25	14	19	19	18	18
30	19	25	22	19	23
35	7	10	11	15	11

(a) Is there evidence to support the claim that cotton content affects the mean tensile strength? Use  $\alpha = 0.05$ .

Minitab Output

One-way ANOVA: Tensile Strength versus Cotton Percentage					
Analysis of Variance for Tensile					
Source	DF	SS	MS	F	P
Cotton P	4	475.76	118.94	14.76	0.000
Error	20	161.20	8.06		
Total	24	636.96			

Yes, the  $F$ -value is 14.76 with a corresponding  $P$ -value of 0.000. The percentage of cotton in the fiber appears to have an affect on the tensile strength.

(b) Use the Fisher LSD method to make comparisons between the pairs of means. What conclusions can you draw?

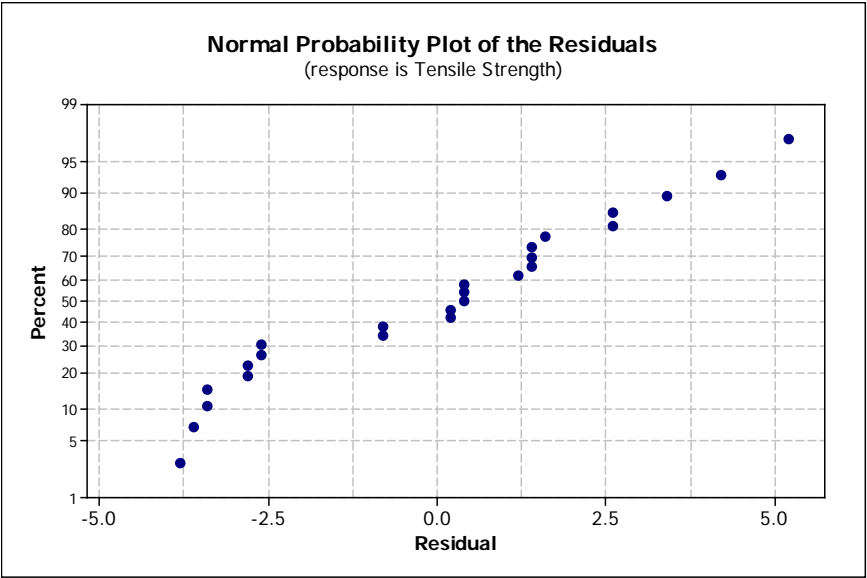
Minitab Output

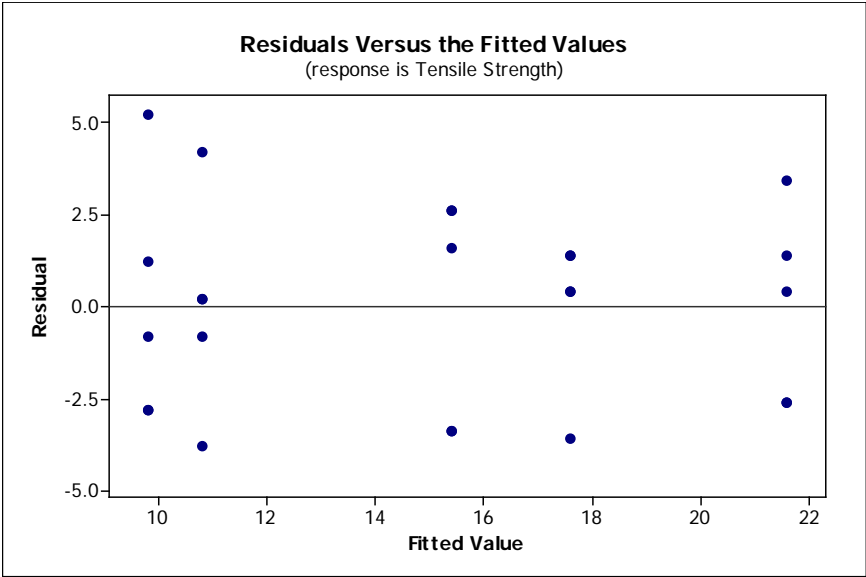
Fisher's pairwise comparisons				
Family error rate = 0.264				
Individual error rate = 0.0500				
Critical value = 2.086				
Intervals for (column level mean) - (row level mean)				
	15	20	25	30
20	-9.346 -1.854			
25	-11.546 -4.054	-5.946 1.546		
30	-15.546 -8.054	-9.946 -2.454	-7.746 -0.254	
35	-4.746 2.746	0.854 8.346	3.054 10.546	7.054 14.546

In the Minitab output the pairs of treatments that do not contain zero in the pair of numbers indicates that there is a difference in the pairs of the treatments. 15% cotton is different than 20%, 25% and 30%. 20% cotton is different than 30% and 35% cotton. 25% cotton is different than 30% and 35% cotton. 30% cotton is different than 35%.

(c) Analyze the residuals from this experiment and comment on model adequacy.

The residual plots below show nothing unusual.





**3.11.** Reconsider the experiment described in Problem 3.10. Suppose that 30 percent cotton content is a control. Use Dunnett’s test with  $\alpha = 0.05$  to compare all of the other means with the control.

For this problem:  $a = 5, a-1 = 4, f=20, n=5$  and  $\alpha = 0.05$

$$d_{0.05}(4, 20) \sqrt{\frac{2MS_E}{n}} = 2.65 \sqrt{\frac{2(8.06)}{5}} = 4.76$$

$$\begin{aligned} 1 \text{ vs. } 4: \bar{y}_1 - \bar{y}_4 &= 9.8 - 21.6 = -11.8^* \\ 2 \text{ vs. } 4: \bar{y}_2 - \bar{y}_4 &= 15.4 - 21.6 = -6.2^* \\ 3 \text{ vs. } 4: \bar{y}_3 - \bar{y}_4 &= 17.6 - 21.6 = -4.0 \\ 5 \text{ vs. } 4: \bar{y}_5 - \bar{y}_4 &= 10.8 - 21.6 = -10.8^* \end{aligned}$$

The control treatment, treatment 4, differs from treatments 1, 2 and 5.

**3.12.** A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single-factor experiment was conducted with three dosage levels, and the following results were obtained.

Dosage		Observations		
20g	24	28	37	30
30g	37	44	31	35
40g	42	47	52	38

(a) Is there evidence to indicate that dosage level affects bioactivity? Use  $\alpha = 0.05$ .



Minitab Output

One-way ANOVA: Activity versus Dosage

Analysis of Variance for Activity					
Source	DF	SS	MS	F	P
Dosage	2	450.7	225.3	7.04	0.014
Error	9	288.3	32.0		
Total	11	738.9			

There appears to be a different in the dosages.

(b) If it is appropriate to do so, make comparisons between the pairs of means. What conclusions can you draw?

Because there appears to be a difference in the dosages, the comparison of means is appropriate.

Minitab Output

Tukey's pairwise comparisons

Family error rate = 0.0500  
Individual error rate = 0.0209

Critical value = 3.95

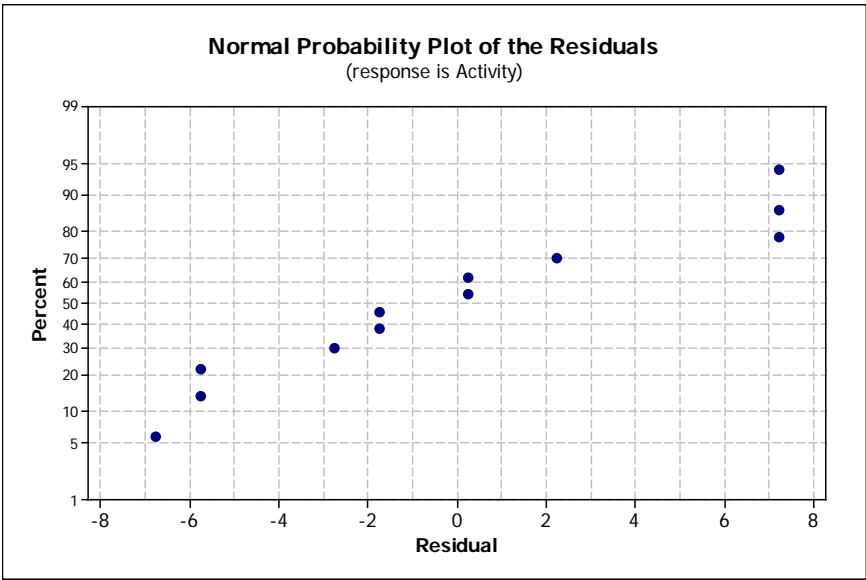
Intervals for (column level mean) - (row level mean)

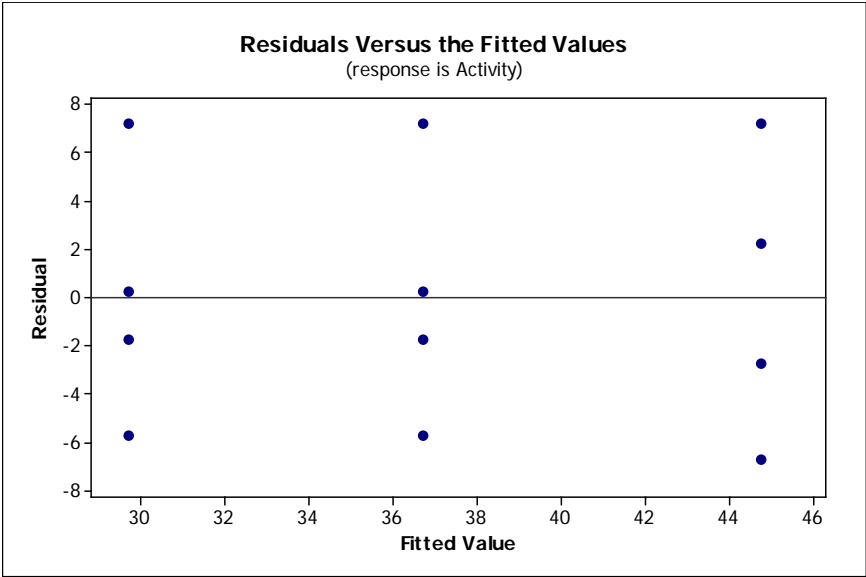
	20g	30g
30g	-18.177 4.177	
40g	-26.177 -3.823	-19.177 3.177

The Tukey comparison shows a difference in the means between the 20g and the 40g dosages.

(c) Analyze the residuals from this experiment and comment on the model adequacy.

There is nothing too unusual about the residual plots shown below.





**3.13.** A rental car company wants to investigate whether the type of car rented affects the length of the rental period. An experiment is run for one week at a particular location, and 10 rental contracts are selected at random for each car type. The results are shown in the following table.

Type of Car	Observations									
Sub-compact	3	5	3	7	6	5	3	2	1	6
Compact	1	3	4	7	5	6	3	2	1	7
Midsize	4	1	3	5	7	1	2	4	2	7
Full Size	3	5	7	5	10	3	4	7	2	7

- (a) Is there evidence to support a claim that the type of car rented affects the length of the rental contract? Use  $\alpha = 0.05$ . If so, which types of cars are responsible for the difference?

Minitab Output

One-way ANOVA: Days versus Car Type					
Analysis of Variance for Days					
Source	DF	SS	MS	F	P
Car Type	3	16.68	5.56	1.11	0.358
Error	36	180.30	5.01		
Total	39	196.98			

There is no difference.

- (b) Analyze the residuals from this experiment and comment on the model adequacy.