

Chapter 1

Introduction

- 1.1. The mean annual rainfall in Boston is approximately 1050 mm , and the mean annual evapotranspiration is in the range of $380\text{--}630 \text{ mm}$ (USGS). On the basis of rainfall, this indicates a subhumid climate. The mean annual rainfall in Santa Fe is approximately 360 mm and the mean annual evapotranspiration is $< 380 \text{ mm}$. On the basis of rainfall, this indicates an arid climate.

Chapter 2

Fundamentals of Flow in Closed Conduits

2.1. $D_1 = 0.1$ m, $D_2 = 0.15$ m, $V_1 = 2$ m/s, and

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.1)^2 = 0.007854 \text{ m}^2$$
$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (0.15)^2 = 0.01767 \text{ m}^2$$

Volumetric flow rate, Q , is given by

$$Q = A_1 V_1 = (0.007854)(2) = \boxed{0.0157 \text{ m}^3/\text{s}}$$

According to continuity,

$$A_1 V_1 = A_2 V_2 = Q$$

Therefore

$$V_2 = \frac{Q}{A_2} = \frac{0.0157}{0.01767} = \boxed{0.889 \text{ m/s}}$$

At 20°C, the density of water, ρ , is 998 kg/m³, and the mass flow rate, \dot{m} , is given by

$$\dot{m} = \rho Q = (998)(0.0157) = \boxed{15.7 \text{ kg/s}}$$

2.2. From the given data: $D_1 = 200$ mm, $D_2 = 100$ mm, $V_1 = 1$ m/s, and

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4} (0.2)^2 = 0.0314 \text{ m}^2$$
$$A_2 = \frac{\pi}{4} D_2^2 = \frac{\pi}{4} (0.1)^2 = 0.00785 \text{ m}^2$$

The flow rate, Q_1 , in the 200-mm pipe is given by

$$Q_1 = A_1 V_1 = (0.0314)(1) = 0.0314 \text{ m}^3/\text{s}$$

and hence the flow rate, Q_2 , in the 100-mm pipe is

$$Q_2 = \frac{Q_1}{2} = \frac{0.0314}{2} = \boxed{0.0157 \text{ m}^3/\text{s}}$$

The average velocity, V_2 , in the 100-mm pipe is

$$V_2 = \frac{Q_2}{A_2} = \frac{0.0157}{0.00785} = \boxed{2 \text{ m/s}}$$

2.3. The velocity distribution in the pipe is

$$v(r) = V_0 \left[1 - \left(\frac{r}{R} \right)^2 \right] \quad (1)$$

and the average velocity, \bar{V} , is defined as

$$\bar{V} = \frac{1}{A} \int_A V \, dA \quad (2)$$

where

$$A = \pi R^2 \quad \text{and} \quad dA = 2\pi r \, dr \quad (3)$$

Combining Equations 1 to 3 yields

$$\begin{aligned} \bar{V} &= \frac{1}{\pi R^2} \int_0^R V_0 \left[1 - \left(\frac{r}{R} \right)^2 \right] 2\pi r \, dr = \frac{2V_0}{R^2} \left[\int_0^R r \, dr - \int_0^R \frac{r^3}{R^2} \, dr \right] = \frac{2V_0}{R^2} \left[\frac{R^2}{2} - \frac{R^4}{4R^2} \right] \\ &= \frac{2V_0}{R^2} \frac{R^2}{4} = \boxed{\frac{V_0}{2}} \end{aligned}$$

The flow rate, Q , is therefore given by

$$Q = A\bar{V} = \boxed{\frac{\pi R^2 V_0}{2}}$$

2.4.

$$\begin{aligned} \beta &= \frac{1}{A\bar{V}^2} \int_A v^2 \, dA = \frac{4}{\pi R^2 V_0^2} \int_0^R V_0^2 \left[1 - \frac{2r^2}{R^2} + \frac{r^4}{R^4} \right] 2\pi r \, dr \\ &= \frac{8}{R^2} \left[\int_0^R r \, dr - \int_0^R \frac{2r^3}{R^2} \, dr + \int_0^R \frac{r^5}{R^4} \, dr \right] = \frac{8}{R^2} \left[\frac{R^2}{2} - \frac{R^4}{2R^2} + \frac{R^6}{6R^4} \right] \\ &= \boxed{\frac{4}{3}} \end{aligned}$$

2.5. $D = 0.2 \text{ m}$, $Q = 0.06 \text{ m}^3/\text{s}$, $L = 100 \text{ m}$, $p_1 = 500 \text{ kPa}$, $p_2 = 400 \text{ kPa}$, $\gamma = 9.79 \text{ kN/m}^3$.

$$\begin{aligned} R &= \frac{D}{4} = \frac{0.2}{4} = 0.05 \text{ m} \\ \Delta h &= \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{500 - 400}{9.79} = 10.2 \text{ m} \\ \tau_0 &= \frac{\gamma R \Delta h}{L} = \frac{(9.79 \times 10^3)(0.05)(10.2)}{100} = \boxed{49.9 \text{ N/m}^2} \\ A &= \frac{\pi D^2}{4} = \frac{\pi(0.2)^2}{4} = 0.0314 \text{ m}^2 \\ V &= \frac{Q}{A} = \frac{0.06}{0.0314} = 1.91 \text{ m/s} \\ f &= \frac{8\tau_0}{\rho V^2} = \frac{8(49.9)}{(998)(1.91)^2} = \boxed{0.11} \end{aligned}$$

2.6. $T = 20^\circ\text{C}$, $V = 2 \text{ m/s}$, $D = 0.25 \text{ m}$, horizontal pipe, ductile iron. For ductile iron pipe, $k_s = 0.26 \text{ mm}$, and

$$\frac{k_s}{D} = \frac{0.26}{250} = 0.00104$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998.2)(2)(0.25)}{(1.002 \times 10^{-3})} = 4.981 \times 10^5$$

From the Moody diagram:

$$f = 0.0202 \text{ (pipe is smooth)}$$

Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

Substituting for k_s/D and Re gives

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{0.00104}{3.7} + \frac{2.51}{4.981 \times 10^5 \sqrt{f}} \right)$$

By trial and error leads to

$$f = 0.0204$$

Using the Swamee-Jain equation,

$$\begin{aligned} \frac{1}{\sqrt{f}} &= -2 \log \left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right] \\ &= -2 \log \left[\frac{0.00104}{3.7} + \frac{5.74}{(4.981 \times 10^5)^{0.9}} \right] \end{aligned}$$

which leads to

$$f = 0.0205$$

The head loss, h_f , over 100 m of pipeline is given by

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 0.0204 \frac{100}{0.25} \frac{(2)^2}{2(9.81)} = 1.66 \text{ m}$$

Therefore the pressure drop, Δp , is given by

$$\Delta p = \gamma h_f = (9.79)(1.66) = 16.3 \text{ kPa}$$

If the pipe is 1 m lower at the downstream end, f would not change, but the pressure drop, Δp , would then be given by

$$\Delta p = \gamma(h_f - 1.0) = 9.79(1.66 - 1) = 6.46 \text{ kPa}$$

2.7. From the given data: $D = 25$ mm, $k_s = 0.1$ mm, $\theta = 10^\circ$, $p_1 = 550$ kPa, and $L = 100$ m. At 20°C , $\nu = 1.00 \times 10^{-6}$ m²/s, $\gamma = 9.79$ kN/m³, and

$$\begin{aligned}\frac{k_s}{D} &= \frac{0.1}{25} = 0.004 \\ A &= \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.025)^2 = 4.909 \times 10^{-4} \text{ m}^2 \\ h_f &= f \frac{L}{D} \frac{Q^2}{2gA^2} = f \frac{100}{0.025} \frac{Q^2}{2(9.81)(4.909 \times 10^{-4})^2} = 8.46 \times 10^8 f Q^2\end{aligned}$$

The energy equation applied over 100 m of pipe is

$$\frac{p_1}{\gamma} + \frac{V^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V^2}{2g} + z_2 + h_f$$

which simplifies to

$$\begin{aligned}p_2 &= p_1 - \gamma(z_2 - z_1) - \gamma h_f \\ p_2 &= 550 - 9.79(100 \sin 10^\circ) - 9.79(8.46 \times 10^8 f Q^2) \\ p_2 &= 380.0 - 8.28 \times 10^9 f Q^2\end{aligned}$$

(a) For $Q = 2$ L/min $= 3.333 \times 10^{-5}$ m³/s,

$$\begin{aligned}V &= \frac{Q}{A} = \frac{3.333 \times 10^{-5}}{4.909 \times 10^{-4}} = 0.06790 \text{ m/s} \\ \text{Re} &= \frac{VD}{\nu} = \frac{(0.06790)(0.025)}{1 \times 10^{-6}} = 1698\end{aligned}$$

Since $\text{Re} < 2000$, the flow is laminar when $Q = 2$ L/min. Hence,

$$\begin{aligned}f &= \frac{64}{\text{Re}} = \frac{64}{1698} = 0.03770 \\ p_2 &= 380.0 - 8.28 \times 10^9 (0.03770)(3.333 \times 10^{-5})^2 = 380 \text{ kPa}\end{aligned}$$

Therefore, when the flow is 2 L/min, the pressure at the downstream section is 380 kPa.

For $Q = 20$ L/min $= 3.333 \times 10^{-4}$ m³/s,

$$\begin{aligned}V &= \frac{Q}{A} = \frac{3.333 \times 10^{-4}}{4.909 \times 10^{-4}} = 0.6790 \text{ m/s} \\ \text{Re} &= \frac{VD}{\nu} = \frac{(0.6790)(0.025)}{1 \times 10^{-6}} = 16980\end{aligned}$$

Since $\text{Re} > 5000$, the flow is turbulent when $Q = 20$ L/min. Hence,

$$\begin{aligned}f &= \frac{0.25}{\left[\log \left(\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} = \frac{0.25}{\left[\log \left(\frac{0.004}{3.7} + \frac{5.74}{16980^{0.9}} \right) \right]^2} = 0.0342 \\ p_2 &= 380.0 - 8.28 \times 10^9 (0.0342)(3.333 \times 10^{-4})^2 = 349 \text{ kPa}\end{aligned}$$

Therefore, when the flow is 20 L/min, the pressure at the downstream section is 349 kPa.

(b) Using the Colebrook equation with $Q = 20$ L/min,

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right] = -2 \log \left[\frac{0.004}{3.7} + \frac{2.51}{16980\sqrt{f}} \right]$$

which yields $f = 0.0337$. Comparing this with the Swamee-Jain result of $f = 0.0342$ indicates a difference of 1.5% , which is more than the 1% claimed by Swamee-Jain.

2.8. The Colebrook equation is given by

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

Inverting and squaring this equation gives

$$f = \frac{0.25}{\{\log[(k_s/D)/3.7 + 2.51/(\text{Re}\sqrt{f})]\}^2}$$

This equation is “slightly more convenient” than the Colebrook formula since it is quasi-explicit in f , whereas the Colebrook formula gives $1/\sqrt{f}$.

2.9. The Colebrook equation is preferable since it provides greater accuracy than interpolating from the Moody diagram.

2.10. $D = 0.5$ m, $p_1 = 600$ kPa, $Q = 0.50$ m³/s, $z_1 = 120$ m, $z_2 = 100$ m, $\gamma = 9.79$ kN/m³, $L = 1000$ m, k_s (ductile iron) = 0.26 mm,

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.5)^2 = 0.1963 \text{ m}^2$$

$$V = \frac{Q}{A} = \frac{0.50}{0.1963} = 2.55 \text{ m/s}$$

Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right)$$

where $k_s/D = 0.26/500 = 0.00052$, and at 20°C

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998)(2.55)(0.5)}{1.00 \times 10^{-3}} = 1.27 \times 10^6$$

Substituting k_s/D and Re into the Colebrook equation gives

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{0.00052}{3.7} + \frac{2.51}{1.27 \times 10^6 \sqrt{f}} \right)$$

which leads to

$$f = 0.0172$$

Applying the energy equation

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f$$

Since $V_1 = V_2$, and h_f is given by the Darcy-Weisbach equation, then the energy equation can be written as

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + f \frac{L}{D} \frac{V^2}{2g}$$

Substituting known values leads to

$$\frac{600}{9.79} + 120 = \frac{p_2}{9.79} + 100 + 0.0172 \frac{1000}{0.5} \frac{(2.55)^2}{2(9.81)}$$

which gives

$$p_2 = 684 \text{ kPa}$$

If p is the (static) pressure at the top of a 30 m high building, then

$$p = p_2 - 30\gamma = 684 - 30(9.79) = 390 \text{ kPa}$$

This (static) water pressure is adequate for service.

2.11. The head loss, h_f , in the pipe is estimated by

$$h_f = \left(\frac{p_{\text{main}}}{\gamma} + z_{\text{main}} \right) - \left(\frac{p_{\text{outlet}}}{\gamma} + z_{\text{outlet}} \right)$$

where $p_{\text{main}} = 400 \text{ kPa}$, $z_{\text{main}} = 0 \text{ m}$, $p_{\text{outlet}} = 0 \text{ kPa}$, and $z_{\text{outlet}} = 2.0 \text{ m}$. Therefore,

$$h_f = \left(\frac{400}{9.79} + 0 \right) - (0 + 2.0) = 38.9 \text{ m}$$

Also, since $D = 25 \text{ mm}$, $L = 20 \text{ m}$, $k_s = 0.15 \text{ mm}$ (from Table 2.1), $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$ (at 20°C), the combined Darcy-Weisbach and Colebrook equation (Equation 2.43) yields,

$$\begin{aligned} Q &= -0.965 D^2 \sqrt{\frac{g D h_f}{L}} \ln \left(\frac{k_s/D}{3.7} + \frac{1.774 \nu}{D \sqrt{g D h_f / L}} \right) \\ &= -0.965 (0.025)^2 \sqrt{\frac{(9.81)(0.025)(38.9)}{20}} \ln \left[\frac{0.15/25}{3.7} + \frac{1.774(1.00 \times 10^{-6})}{(0.025) \sqrt{(9.81)(0.025)(38.9)/20}} \right] \\ &= 0.00265 \text{ m}^3/\text{s} = 2.65 \text{ L/s} \end{aligned}$$

The faucet can therefore be expected to deliver 2.65 L/s when fully open.

2.12. From the given data: $Q = 300 \text{ L/s} = 0.300 \text{ m}^3/\text{s}$, $L = 40 \text{ m}$, and $h_f = 45 \text{ m}$. Assume that $\nu = 10^{-6} \text{ m}^2/\text{s}$ (at 20°C) and take $k_s = 0.15 \text{ mm}$ (from Table 2.1). Substituting these data

into Equation 2.43 gives

$$Q = -0.965D^2 \sqrt{\frac{gDh_f}{L}} \ln \left(\frac{k_s/D}{3.7} + \frac{1.784\nu}{D\sqrt{gDh_f/L}} \right)$$

$$0.2 = -0.965D^2 \sqrt{\frac{(9.81)D(45)}{(40)}} \ln \left(\frac{0.00015}{3.7D} + \frac{1.784(10^{-6})}{D\sqrt{(9.81)D(45)/(40)}} \right)$$

This is an implicit equation in D that can be solved numerically to yield $D = 166 \text{ mm}$.

2.13. Since $k_s = 0.15 \text{ mm}$, $L = 40 \text{ m}$, $Q = 0.3 \text{ m}^3/\text{s}$, $h_f = 45 \text{ m}$, $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$, the Swamee-Jain approximation (Equation 2.44) gives

$$D = 0.66 \left[k_s^{1.25} \left(\frac{LQ^2}{gh_f} \right)^{4.75} + \nu Q^{9.4} \left(\frac{L}{gh_f} \right)^{5.2} \right]^{0.04}$$

$$= 0.66 \left\{ (0.00015)^{1.25} \left[\frac{(40)(0.3)^2}{(9.81)(45)} \right]^{4.75} + (1.00 \times 10^{-6})(0.3)^{9.4} \left[\frac{40}{(9.81)(45)} \right]^{5.2} \right\}^{0.04}$$

$$= 0.171 \text{ m} = 171 \text{ mm}$$

The calculated pipe diameter (171 mm) is about 3% higher than calculated by the Colebrook equation (166 mm).

2.14. The kinetic energy correction factor, α , is defined by

$$\int_A \rho \frac{v^3}{2} dA = \alpha \rho \frac{V^3}{2} A$$

or

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \quad (1)$$

Using the velocity distribution in Problem 2.3 gives

$$\begin{aligned} \int_A v^3 dA &= \int_0^R V_0^3 \left[1 - \left(\frac{r}{R} \right)^2 \right]^2 2\pi r dr \\ &= 2\pi V_0^3 \int_0^R \left[1 - 3 \left(\frac{r}{R} \right)^2 + 3 \left(\frac{r}{R} \right)^4 - \left(\frac{r}{R} \right)^6 \right] r dr \\ &= 2\pi V_0^3 \int_0^R \left[r - \frac{3r^3}{R^2} + \frac{3r^5}{R^4} - \frac{r^7}{R^6} \right] dr \\ &= 2\pi V_0^3 \left[\frac{r^2}{2} - \frac{3r^4}{4R^2} + \frac{r^6}{2R^4} - \frac{r^8}{8R^6} \right]_0^R \\ &= 2\pi R^2 V_0^3 \left[\frac{1}{2} - \frac{3}{4} + \frac{1}{2} - \frac{1}{8} \right] \\ &= \frac{\pi R^2 V_0^3}{4} \end{aligned} \quad (2)$$

The average velocity, V , was calculated in Problem 2.3 as

$$V = \frac{V_0}{2}$$

hence

$$V^3 A = \left(\frac{V_0}{2}\right)^3 \pi R^2 = \frac{\pi R^2 V_0^3}{8} \quad (3)$$

Combining Equations 1 to 3 gives

$$\alpha = \frac{\pi R^2 V_0^3 / 4}{\pi R^2 V_0^3 / 8} = \boxed{2}$$

2.15. The kinetic energy correction factor, α , is defined by

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \quad (1)$$

Using the given velocity distribution gives

$$\begin{aligned} \int_A v^3 dA &= \int_0^R V_0^3 \left(1 - \frac{r}{R}\right)^{\frac{3}{7}} 2\pi r \, dr \\ &= 2\pi V_0^3 \int_0^R \left(1 - \frac{r}{R}\right)^{\frac{3}{7}} r \, dr \end{aligned} \quad (2)$$

To facilitate integration, let

$$x = 1 - \frac{r}{R} \quad (3)$$

which gives

$$r = R(1 - x) \quad (4)$$

$$dr = -R \, dx \quad (5)$$

Combining Equations 2 to 5 gives

$$\begin{aligned} \int_A v^3 dA &= 2\pi V_0^3 \int_0^1 x^{\frac{3}{7}} R(1 - x)(-R) dx \\ &= 2\pi R^2 V_0^3 \int_0^1 x^{\frac{3}{7}} (1 - x) dx = 2\pi R^2 V_0^3 \int_0^1 (x^{\frac{3}{7}} - x^{\frac{10}{7}}) dx \\ &= 2\pi R^2 V_0^3 \left[\frac{7}{10} x^{\frac{10}{7}} - \frac{7}{17} x^{\frac{17}{7}} \right]_0^1 \\ &= 0.576\pi R^2 V_0^3 \end{aligned} \quad (6)$$

The average velocity, V , is given by (using the same substitution as above)

$$\begin{aligned} V &= \frac{1}{A} \int_A v \, dA \\ &= \frac{1}{\pi R^2} \int_0^R V_0 \left(1 - \frac{r}{R}\right)^{\frac{1}{7}} 2\pi r \, dr = \frac{2V_0}{R^2} \int_0^1 x^{\frac{1}{7}} R(1 - x)(-R) dx \\ &= 2V_0 \int_0^1 (x^{\frac{1}{7}} - x^{\frac{8}{7}}) dx = 2V_0 \left[\frac{7}{8} x^{\frac{8}{7}} - \frac{7}{15} x^{\frac{15}{7}} \right]_0^1 \\ &= 0.817V_0 \end{aligned} \quad (7)$$

Using this result,

$$V^3 A = (0.817V_0)^3 \pi R^2 = 0.545\pi R^2 V_0^3 \quad (8)$$

Combining Equations 1, 6, and 8 gives

$$\alpha = \frac{0.576\pi R^2 V_0^3}{0.545\pi R^2 V_0^3} = \boxed{1.06}$$

The momentum correction factor, β , is defined by

$$\beta = \frac{\int_A v^2 dA}{AV^2} \quad (9)$$

In this case,

$$AV^2 = \pi R^2 (0.817V_0)^2 = 0.667\pi R^2 V_0^2 \quad (10)$$

and

$$\begin{aligned} \int_A v^2 dA &= \int_0^R V_0^2 \left(1 - \frac{r}{R}\right)^{\frac{2}{7}} 2\pi r \, dr \\ &= 2\pi V_0^2 \int_1^0 x^{\frac{2}{7}} R(1-x)(-R) dx = 2\pi R^2 V_0^2 \int_0^1 (x^{\frac{2}{7}} - x^{\frac{9}{7}}) dx \\ &= 2\pi R^2 V_0^2 \left[\frac{7}{9} x^{\frac{9}{7}} - \frac{7}{16} x^{\frac{16}{7}} \right]_0^1 = 0.681\pi R^2 V_0^2 \end{aligned} \quad (11)$$

Combining Equations 9 to 11 gives

$$\beta = \frac{0.681\pi R^2 V_0^2}{0.667\pi R^2 V_0^2} = \boxed{1.02}$$

2.16. The kinetic energy correction factor, α , is defined by

$$\alpha = \frac{\int_A v^3 dA}{V^3 A} \quad (1)$$

Using the velocity distribution given by Equation 2.73 gives

$$\begin{aligned} \int_A v^3 dA &= \int_0^R V_0^3 \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} 2\pi r \, dr \\ &= 2\pi V_0^3 \int_0^R \left(1 - \frac{r}{R}\right)^{\frac{3}{n}} r \, dr \end{aligned} \quad (2)$$

Let

$$x = 1 - \frac{r}{R} \quad (3)$$

which gives

$$r = R(1 - x) \quad (4)$$

$$dr = -R \, dx \quad (5)$$

Combining Equations 2 to 5 gives

$$\begin{aligned}
 \int_A v^3 dA &= 2\pi V_0^3 \int_0^1 x^{\frac{3}{n}} R(1-x)(-R) dx \\
 &= 2\pi R^2 V_0^3 \int_0^1 x^{\frac{3}{n}} (1-x) dx = 2\pi R^2 V_0^3 \int_0^1 (x^{\frac{3}{n}} - x^{\frac{3+n}{n}}) dx \\
 &= 2\pi R^2 V_0^3 \left[\frac{n}{3+n} x^{\frac{3+n}{n}} - \frac{n}{3+2n} x^{\frac{3+2n}{n}} \right]_0^1 \\
 &= \frac{2n^2}{(3+n)(3+2n)} \pi R^2 V_0^3
 \end{aligned} \tag{6}$$

The average velocity, V , is given by

$$\begin{aligned}
 V &= \frac{1}{A} \int_A v dA \\
 &= \frac{1}{\pi R^2} \int_0^R V_0 \left(1 - \frac{r}{R}\right)^{\frac{1}{n}} 2\pi r dr = \frac{2V_0}{R^2} \int_1^0 x^{\frac{1}{n}} R(1-x)(-R) dx \\
 &= 2V_0 \int_0^1 (x^{\frac{1}{n}} - x^{\frac{1+n}{n}}) dx = 2V_0 \left[\frac{n}{1+n} x^{\frac{1+n}{n}} - \frac{n}{1+2n} x^{\frac{1+2n}{n}} \right]_0^1 \\
 &= \left[\frac{2n^2}{(1+n)(1+2n)} \right] V_0
 \end{aligned} \tag{7}$$

Using this result,

$$V^3 A = \left[\frac{2n^2}{(1+n)(1+2n)} \right]^3 V_0^3 \pi R^2 = \frac{8n^6}{(1+n)^3(1+2n)^3} \pi R^2 V_0^3 \tag{8}$$

Combining Equations 1, 6, and 8 gives

$$\begin{aligned}
 \alpha &= \frac{\frac{2n^2}{(3+n)(3+2n)} \pi R^2 V_0^3}{\frac{8n^6}{(1+n)^3(1+2n)^3} \pi R^2 V_0^3} \\
 &= \frac{(1+n)^3(1+2n)^3}{4n^4(3+n)(3+2n)}
 \end{aligned}$$

Putting $n = 7$ gives $\boxed{\alpha = 1.06}$, the same result obtained in Problem 2.15.

2.17. $p_1 = 30$ kPa, $p_2 = 500$ kPa, therefore head, h_p , added by pump is given by

$$h_p = \frac{p_2 - p_1}{\gamma} = \frac{500 - 30}{9.79} = \boxed{48.0 \text{ m}}$$

Power, P , added by pump is given by

$$P = \gamma Q h_p = (9.79)(Q)(48.0) = \boxed{470 \text{ kW per m}^3/\text{s}}$$

- 2.18.** $Q = 0.06 \text{ m}^3/\text{s}$, $D = 0.2 \text{ m}$, $k_s = 0.9 \text{ mm}$ (riveted steel), $k_s/D = 0.9/200 = 0.00450$, for 90° bend $K = 0.3$, for the entrance $K = 1.0$, at 20°C $\rho = 998 \text{ kg/m}^3$, and $\mu = 1.00 \times 10^{-3} \text{ Pa}\cdot\text{s}$, therefore

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.2)^2 = 0.0314 \text{ m}^2$$

$$V = \frac{Q}{A} = \frac{0.06}{0.0314} = 1.91 \text{ m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998)(1.91)(0.2)}{1.00 \times 10^{-3}} = 3.81 \times 10^5$$

Substituting k_s/D and Re into the Colebrook equation gives

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{0.00450}{3.7} + \frac{2.51}{3.81 \times 10^5 \sqrt{f}} \right)$$

which leads to

$$f = 0.0297$$

Minor head loss, h_m , is given by

$$h_m = \sum K \frac{V^2}{2g} = (1.0 + 0.3) \frac{(1.91)^2}{2(9.81)} = 0.242 \text{ m}$$

If friction losses, h_f , account for 90% of the total losses, then

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = 9h_m$$

which means that

$$0.0297 \frac{L}{0.2} \frac{(1.91)^2}{2(9.81)} = 9(0.242)$$

Solving for L gives

$$\boxed{L = 78.9 \text{ m}}$$

For pipe lengths shorter than the length calculated in this problem, the word “minor” should not be used.

- 2.19.** From the given data: $p_0 = 480 \text{ kPa}$, $v_0 = 5 \text{ m/s}$, $z_0 = 2.44 \text{ m}$, $D = 19 \text{ mm} = 0.019 \text{ m}$, $L = 40 \text{ m}$, $z_1 = 7.62 \text{ m}$, and $\sum K_m = 3.5$. For copper tubing it can be assumed that $k_s = 0.0023 \text{ mm}$. Applying the energy and Darcy-Weisbach equations between the water main and the faucet gives

$$\frac{p_0}{\gamma} + z_0 - h_f - h_m = \frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1$$

$$\frac{480}{9.79} + 2.44 - \frac{f(40)}{0.019} \frac{v^2}{2(9.81)} - 3.5 \frac{v^2}{2(9.81)} = \frac{0}{\gamma} + \frac{v^2}{2(9.81)} + 7.62$$

which simplifies to

$$v = \frac{6.622}{\sqrt{107.3f - 0.2141}} \quad (1)$$

The Colebrook equation, with $\nu = 1 \times 10^{-6} \text{ m}^2/\text{s}$ gives

$$\begin{aligned}\frac{1}{\sqrt{f}} &= -2 \log \left[\frac{k_s}{3.7D} + \frac{2.51}{\text{Re}\sqrt{f}} \right] \\ \frac{1}{\sqrt{f}} &= -2 \log \left[\frac{0.0025}{3.7(19)} + \frac{2.51}{\frac{v(0.019)}{1 \times 10^{-6}} \sqrt{f}} \right] \\ \frac{1}{\sqrt{f}} &= -2 \log \left[3.556 \times 10^{-5} + \frac{1.321 \times 10^{-4}}{v\sqrt{f}} \right]\end{aligned}\quad (2)$$

Combining Equations 1 and 2 gives

$$\frac{1}{\sqrt{f}} = -2 \log \left[3.556 \times 10^{-5} + \frac{1.995 \times 10^{-5} \sqrt{107.3f - 0.2141}}{\sqrt{f}} \right]$$

which yields

$$f = 0.0189$$

Substituting into Equation 1 yields

$$\begin{aligned}v &= \frac{6.622}{\sqrt{107.3(0.0189) - 0.2141}} = 4.92 \text{ m/s} \\ Q &= Av = \left(\frac{\pi}{4} 0.019^2 \right) (4.92) = 0.00139 \text{ m}^3/\text{s} = \boxed{1.39 \text{ L/s} (= 22 \text{ gpm})}\end{aligned}$$

This flow is very high for a faucet. The flow would be reduced if other faucets are open, this is due to increased pipe flow and frictional resistance between the water main and the faucet.

- 2.20.** From the given data: $z_1 = -1.5 \text{ m}$, $z_2 = 40 \text{ m}$, $p_1 = 450 \text{ kPa}$, $\sum k = 10.0$, $Q = 20 \text{ L/s} = 0.02 \text{ m}^3/\text{s}$, $D = 150 \text{ mm}$ (PVC), $L = 60 \text{ m}$, $T = 20^\circ\text{C}$, and $p_2 = 150 \text{ kPa}$. The combined energy and Darcy-Weisbach equations give

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_p = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \left[\frac{fL}{D} + \sum k \right] \frac{V^2}{2g} \quad (1)$$

where

$$V_1 = V_2 = V = \frac{Q}{A} = \frac{0.02}{\frac{\pi(0.15)^2}{4}} = 1.13 \text{ m/s} \quad (2)$$

At 20°C , $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$, and

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.13)(0.15)}{1.00 \times 10^{-6}} = 169500$$

Since PVC pipe is smooth ($k_s = 0$), the friction factor, f , is given by

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{2.51}{\text{Re}\sqrt{f}} \right) = -2 \log \left(\frac{2.51}{169500\sqrt{f}} \right)$$

which yields

$$f = 0.0162 \quad (3)$$

Taking $\gamma = 9.79 \text{ kN/m}^3$ and combining Equations 1 to 3 yields

$$\frac{450}{9.79} + \frac{1.13^2}{2(9.81)} + (-1.5) + h_p = \frac{150}{9.79} + \frac{1.13^2}{2(9.81)} + 40 + \left[\frac{(0.0162)(60)}{0.15} + 10 \right] \frac{1.13^2}{2(9.81)}$$

which gives

$$h_p = 11.9 \text{ m}$$

Since $h_p > 0$, a booster pump is required. The power, P , to be supplied by the pump is given by

$$P = \gamma Q h_p = (9.79)(0.02)(11.9) = \boxed{2.3 \text{ kW}}$$

2.21. (a) Diameter of pipe, $D = 0.75 \text{ m}$, area, A given by

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.75)^2 = 0.442 \text{ m}^2$$

and velocity, V , in pipe

$$V = \frac{Q}{A} = \frac{1}{0.442} = 2.26 \text{ m/s}$$

Energy equation between reservoir and A:

$$7 + h_p - h_f = \frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A \quad (1)$$

where $p_A = 350 \text{ kPa}$, $\gamma = 9.79 \text{ kN/m}^3$, $V_A = 2.26 \text{ m/s}$, $z_A = 10 \text{ m}$, and

$$h_f = \frac{fL}{D} \frac{V^2}{2g}$$

where f depends on Re and k_s/D . At 20°C , $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$ and

$$\text{Re} = \frac{VD}{\nu} = \frac{(2.26)(0.75)}{1.00 \times 10^{-6}} = 1.70 \times 10^6$$

$$\frac{k_s}{D} = \frac{0.26}{750} = 3.47 \times 10^{-4}$$

Using the Swamee-Jain equation,

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right] = -2 \log \left[\frac{3.47 \times 10^{-4}}{3.7} + \frac{5.74}{(1.70 \times 10^6)^{0.9}} \right] = 7.93$$

which leads to

$$f = 0.0159$$

The head loss, h_f , between the reservoir and A is therefore given by

$$h_f = \frac{fL}{D} \frac{V^2}{2g} = \frac{(0.0159)(1000)}{0.75} \frac{(2.26)^2}{2(9.81)} = 5.52 \text{ m}$$

Substituting into Equation 1 yields

$$7 + h_p - 5.52 = \frac{350}{9.81} + \frac{2.26^2}{2(9.81)} + 10$$

which leads to

$$\boxed{h_p = 44.5 \text{ m}}$$

(b) Power, P , supplied by the pump is given by

$$P = \gamma Q h_p = (9.79)(1)(44.5) = \boxed{436 \text{ kW}}$$

(c) Energy equation between A and B is given by

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A - h_f = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B$$

and since $V_A = V_B$,

$$\begin{aligned} p_B &= p_A + \gamma(z_A - z_B - h_f) = 350 + 9.79(10 - 4 - 5.52) \\ &= \boxed{355 \text{ kPa}} \end{aligned}$$

2.22. From the given data: $L = 3 \text{ km} = 3000 \text{ m}$, $Q_{\text{ave}} = 0.0175 \text{ m}^3/\text{s}$, and $Q_{\text{peak}} = 0.578 \text{ m}^3/\text{s}$. If the velocity, V_{peak} , during peak flow conditions is 2.5 m/s , then

$$2.5 = \frac{Q_{\text{peak}}}{\pi D^2/4} = \frac{0.578}{\pi D^2/4}$$

which gives

$$D = \sqrt{\frac{0.578}{\pi(2.5)/4}} = 0.543 \text{ m}$$

Rounding to the nearest 25 mm gives

$$\boxed{D = 550 \text{ mm}}$$

with a cross-sectional area, A , given by

$$A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.550)^2 = 0.238 \text{ m}^2$$

During average demand conditions, the head, h_{ave} , at the suburban development is given by

$$h_{\text{ave}} = \frac{p_{\text{ave}}}{\gamma} + \frac{V_{\text{ave}}^2}{2g} + z_0 \quad (1)$$

where $p_{\text{ave}} = 340 \text{ kPa}$, $\gamma = 9.79 \text{ kN/m}^3$, $V_{\text{ave}} = Q_{\text{ave}}/A = 0.0175/0.238 = 0.0735 \text{ m/s}$, and $z_0 = 8.80 \text{ m}$. Substituting into Equation 1 gives

$$h_{\text{ave}} = \frac{340}{9.79} + \frac{0.0735^2}{2(9.81)} + 8.80 = 43.5 \text{ m}$$

For ductile-iron pipe, $k_s = 0.26$ mm, $k_s/D = 0.26/550 = 4.73 \times 10^{-4}$, at 20°C $\nu = 1.00 \times 10^{-6}$ m²/s, and therefore

$$\text{Re} = \frac{V_{\text{ave}} D}{\nu} = \frac{(0.0735)(0.550)}{1.00 \times 10^{-6}} = 4.04 \times 10^4$$

and the Swamee-Jain equation gives

$$\frac{1}{\sqrt{f_{\text{ave}}}} = -2 \log \left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right] = -2 \log \left[\frac{4.73 \times 10^{-4}}{3.7} + \frac{5.74}{(4.04 \times 10^4)^{0.9}} \right]$$

and yields

$$f_{\text{ave}} = 0.0234$$

The head loss between the water treatment plant and the suburban development is therefore given by

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = (0.0234) \frac{3000}{0.550} \frac{0.0735^2}{2(9.81)} = 0.035 \text{ m}$$

Since the head at the water treatment plant is 10.00 m, the pump head, h_p , that must be added is

$$h_p = (43.5 + 0.035) - 10.00 = 33.5 \text{ m}$$

and the power requirement, P , is given by

$$P = \gamma Q h_p = (9.79)(0.0175)(33.5) = \boxed{5.74 \text{ kW}}$$

During peak demand conditions, the head, h_{peak} , at the suburban development is given by

$$h_{\text{peak}} = \frac{p_{\text{peak}}}{\gamma} + \frac{V_{\text{peak}}^2}{2g} + z_0 \quad (2)$$

where $p_{\text{peak}} = 140$ kPa, $\gamma = 9.79$ kN/m³, $V_{\text{peak}} = Q_{\text{peak}}/A = 0.578/0.238 = 2.43$ m/s, and $z_0 = 8.80$ m. Substituting into Equation 2 gives

$$h_{\text{peak}} = \frac{140}{9.79} + \frac{2.43^2}{2(9.81)} + 8.80 = 23.4 \text{ m}$$

For pipe, $k_s/D = 4.73 \times 10^{-4}$, and

$$\text{Re} = \frac{V_{\text{peak}} D}{\nu} = \frac{(2.43)(0.550)}{1.00 \times 10^{-6}} = 1.34 \times 10^6$$

and the Swamee-Jain equation gives

$$\frac{1}{\sqrt{f_{\text{peak}}}} = -2 \log \left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right] = -2 \log \left[\frac{4.73 \times 10^{-4}}{3.7} + \frac{5.74}{(1.34 \times 10^6)^{0.9}} \right]$$

and yields

$$f_{\text{peak}} = 0.0170$$

The head loss between the water treatment plant and the suburban development is therefore given by

$$h_f = f \frac{L}{D} \frac{V^2}{2g} = (0.0170) \frac{3000}{0.550} \frac{2.43^2}{2(9.81)} = 27.9 \text{ m}$$

Since the head at the water treatment plant is 10.00 m, the pump head, h_p , that must be added is

$$h_p = (23.4 + 27.9) - 10.00 = 41.3 \text{ m}$$

and the power requirement, P , is given by

$$P = \gamma Q h_p = (9.79)(0.578)(41.3) = \boxed{234 \text{ kW}}$$

2.23. The energy equation applied between the reservoir and the outlet is given by

$$40 - \left[K_e + \frac{fL}{D} + K_v \right] \frac{V^2}{2g} - h_t = \frac{V^2}{2g}$$

which can be put in the form

$$h_t = 40 - \left[K_e + \frac{fL}{D} + K_v + 1 \right] \frac{V^2}{2g} \quad (1)$$

For a sharp-edged entrance, $K_e = 0.5$, for an open globe valve, $K_v = 10.0$, and from the given data: $D = 0.05 \text{ m}$, $A = \pi D^2/4 = 0.001963 \text{ m}^2$, $Q = 4 \text{ L/s} = 0.004 \text{ m}^3/\text{s}$, $V = Q/A = 2.038 \text{ m/s}$, $L = 125 \text{ m}$, $\nu = 1.00 \times 10^{-6} \text{ m}^2/\text{s}$, $k_s = 0.23 \text{ mm}$, $\text{Re} = VD/\nu = 1.02 \times 10^5$, and using the Swamee-Jain equation,

$$\begin{aligned} f &= \frac{0.25}{\left[\log \left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}} \right) \right]^2} \\ &= \frac{0.25}{\left[\log \left(\frac{0.23}{3.7(50)} + \frac{5.74}{(1.02 \times 10^5)^{0.9}} \right) \right]^2} = 0.0308 \end{aligned}$$

Substituting into Equation 1 gives

$$h_t = 40 - \left[0.5 + \frac{(0.0308)(125)}{0.05} + 10.0 + 1 \right] \frac{(2.038)^2}{2(9.81)} = 21.27 \text{ m}$$

Therefore, taking $\gamma = 9.79 \text{ kN/m}^3$, the power extracted by the turbine is given by

$$P = \gamma Q h_t = (9.79)(0.004)(21.27) = \boxed{0.833 \text{ kW}}$$

A similar problem would be encountered in calculating the power output at a hydroelectric facility.

2.24. The head loss is calculated using Equation 2.78. The hydraulic radius, R , is given by

$$R = \frac{A}{P} = \frac{(2)(1)}{2(2+1)} = 0.333 \text{ m}$$

and the mean velocity, V , is given by

$$V = \frac{Q}{A} = \frac{5}{(2)(1)} = 2.5 \text{ m/s}$$

At 20°C, $\rho = 998.2 \text{ kg/m}^3$, $\mu = 1.002 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$, and therefore the Reynolds number, Re , is given by

$$\text{Re} = \frac{\rho V(4R)}{\mu} = \frac{(998.2)(2.5)(4 \times 0.333)}{1.002 \times 10^{-3}} = 3.32 \times 10^6$$

A median equivalent sand roughness for concrete can be taken as $k_s = 1.6 \text{ mm}$ (Table 2.1), and therefore the relative roughness, $k_s/4R$, is given by

$$\frac{k_s}{4R} = \frac{1.6 \times 10^{-3}}{4(0.333)} = 0.00120$$

Substituting Re and $k_s/4R$ into the Swamee-Jain equation (Equation 2.38) for the friction factor yields

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{k_s/4R}{3.7} + \frac{5.74}{\text{Re}^{0.9}} \right] = -2 \log \left[\frac{0.00120}{3.7} + \frac{5.74}{(3.32 \times 10^6)^{0.9}} \right] = 6.96$$

which yields

$$f = 0.0206$$

The frictional head loss in the culvert, h_f , is therefore given by the Darcy-Weisbach equation as

$$h_f = \frac{fL}{4R} \frac{V^2}{2g} = \frac{(0.0206)(100)}{(4 \times 0.333)} \frac{2.5^2}{2(9.81)} = \boxed{0.493 \text{ m}}$$

2.25. The frictional head loss is calculated using Equation 2.78. The hydraulic radius, R , is given by

$$R = \frac{A}{P} = \frac{(2)(2)}{2(2+2)} = 0.500 \text{ m}$$

and the mean velocity, V , is given by

$$V = \frac{Q}{A} = \frac{10}{(2)(2)} = 2.5 \text{ m/s}$$

At 20°C, $\rho = 998 \text{ kg/m}^3$, $\mu = 1.00 \times 10^{-3} \text{ N}\cdot\text{s/m}^2$, and therefore the Reynolds number, Re , is given by

$$\text{Re} = \frac{\rho V(4R)}{\mu} = \frac{(998)(2.5)(4 \times 0.500)}{1.00 \times 10^{-3}} = 4.99 \times 10^6$$

A median equivalent sand roughness for concrete can be taken as $k_s = 1.6 \text{ mm}$ (Table 2.1), and therefore the relative roughness, $k_s/4R$, is given by

$$\frac{k_s}{4R} = \frac{1.6 \times 10^{-3}}{4(0.500)} = 0.0008$$

Substituting Re and $k_s/4R$ into the Swamee-Jain equation (Equation 2.39) for the friction factor yields

$$\frac{1}{\sqrt{f}} = -2 \log \left[\frac{k_s/4R}{3.7} + \frac{5.74}{Re^{0.9}} \right] = -2 \log \left[\frac{0.0008}{3.7} + \frac{5.74}{(4.99 \times 10^6)^{0.9}} \right] = 7.31$$

which yields

$$f = 0.0187$$

The frictional head loss in the culvert, h_f , is therefore given by the Darcy-Weisbach equation as

$$h_f = \frac{fL}{4R} \frac{V^2}{2g} = \frac{(0.0187)(500)}{(4 \times 0.500)} \frac{2.5^2}{2(9.81)} = 1.49 \text{ m}$$

Applying the energy equation between the upstream and downstream sections (Sections 1 and 2 respectively),

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f$$

which gives

$$\frac{p_1}{9.79} + \frac{2.5^2}{2(9.81)} + (0.002)(500) = \frac{p_2}{9.79} + \frac{2.5^2}{2(9.81)} + 0 + 1.49$$

Re-arranging this equation gives

$$p_1 - p_2 = 4.80 \text{ kPa}$$

2.26. The Hazen-Williams formula is given by

$$V = 0.849 C_H R^{0.63} S_f^{0.54} \quad (1)$$

where

$$S_f = \frac{h_f}{L} \quad (2)$$

Combining Equations 1 and 2, and taking $R = D/4$ gives

$$V = 0.849 C_H \left(\frac{D}{4} \right)^{0.63} \left(\frac{h_f}{L} \right)^{0.54}$$

which simplifies to

$$h_f = 6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_H} \right)^{1.85}$$

2.27. Comparing the Hazen-Williams and Darcy-Weisbach equations for head loss gives

$$h_f = 6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_H} \right)^{1.85} = f \frac{L}{D} \frac{V^2}{2g}$$

which leads to

$$f = \frac{134}{C_H^{1.85} D^{0.17} V^{0.15}}$$

For laminar flow, Equation 2.36 gives $f \sim 1/Re \sim 1/V$, and for fully-turbulent flow Equation 2.35 gives $f \sim 1/V^0$. Since the Hazen-Williams formula requires that $f \sim 1/V^{0.15}$, this indicates that the flow must be in the transition regime.

2.28. The Manning equation is given by

$$V = \frac{1}{n} R^{\frac{2}{3}} S_f^{\frac{1}{2}} = \frac{1}{n} \left(\frac{D}{4} \right)^{\frac{2}{3}} \left(\frac{h_f}{L} \right)^{\frac{1}{2}}$$

which re-arranges to give

$$h_f = 6.35 \frac{n^2 L V^2}{D^{\frac{4}{3}}}$$

2.29. Comparing the Manning and Darcy-Weisbach equations gives

$$h_f = 6.35 \frac{n^2 L V^2}{D^{\frac{4}{3}}} = f \frac{L}{D} \frac{V^2}{2g}$$

which leads to

$$f = 125 \frac{n^2}{D^{\frac{1}{3}}}$$

For laminar flow, Equation 2.36 gives $f \sim 1/\text{Re} \sim 1/V$, and for fully-turbulent flow Equation 2.35 gives $f \sim 1/V^0$. Since the Manning equation requires that $f \sim 1/V^0$, this indicates that the flow must be fully turbulent or rough.

2.30. Equating the Hazen-Williams and Manning head loss expressions

$$6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_H} \right)^{1.85} = 6.35 \frac{n^2 L V^2}{D^{\frac{4}{3}}}$$

which re-arranges to give

$$n = \left(1.04 \frac{D^{0.082}}{V^{0.075}} \right) \frac{1}{C_H^{0.93}}$$

2.31. Choose the Darcy-Weisbach equation since this equation is applicable in all flow regimes. The Hazen-Williams and Manning equations are limited to particular flow conditions (transition and fully turbulent respectively).

2.32. (a) The Hazen-Williams roughness coefficient, C_H , can be taken as 110 (Table 2.2), $L = 500$ m, $D = 0.300$ m, $V = 2$ m/s, and therefore the head loss, h_f , is given by Equation 2.82 as

$$h_f = 6.82 \frac{L}{D^{1.17}} \left(\frac{V}{C_H} \right)^{1.85} = 6.82 \frac{500}{(0.30)^{1.17}} \left(\frac{2}{110} \right)^{1.85} = \boxed{8.41 \text{ m}}$$

(b) The Manning roughness coefficient, n , can be taken as 0.013 (approximation from Table 2.2), and therefore the head loss, h_f , is given by Equation 2.85 as

$$h_f = 6.35 \frac{n^2 L V^2}{D^{\frac{4}{3}}} = 6.35 \frac{(0.013)^2 (500) (2)^2}{(0.30)^{\frac{4}{3}}} = \boxed{10.7 \text{ m}}$$