CHAPTER 1

Section 1.1 Solutions --

1. Solve for x: $\frac{1}{2}=\frac{x}{360^\circ}$ $360^\circ = 2x$, so that $\boxed{x=180^\circ}$.	**2.** Solve for x: $\frac{1}{4}=\frac{x}{360^\circ}$ $360^\circ = 4x$, so that $\boxed{x=90^\circ}$.
3. Solve for x: $-\frac{1}{3}=\frac{x}{360^\circ}$ $360^\circ = -3x$, so that $\boxed{x=-120^\circ}$. (Note: The angle has a negative measure since it is a clockwise rotation.)	**4.** Solve for x: $-\frac{2}{3}=\frac{x}{360^\circ}$ $720^\circ = 2(360^\circ) = -3x$, so that $\boxed{x=-240^\circ}$. (Note: The angle has a negative measure since it is a clockwise rotation.)
5. Solve for x: $\frac{5}{6}=\frac{x}{360^\circ}$ $1800^\circ = 5(360^\circ) = 6x$, so that $\boxed{x=300^\circ}$.	**6.** Solve for x: $\frac{7}{12}=\frac{x}{360^\circ}$ $2520^\circ = 7(360^\circ) = 12x$, so that $\boxed{x=210^\circ}$.
7. Solve for x: $-\frac{4}{5}=\frac{x}{360^\circ}$ $1440^\circ = 4(360^\circ) = -5x$, so that $\boxed{x=-288^\circ}$. (Note: The angle has a negative measure since it is a clockwise rotation.)	**8.** Solve for x: $-\frac{5}{9}=\frac{x}{360^\circ}$ $1800^\circ = 5(360^\circ) = -9x$, so that $\boxed{x=-200^\circ}$. (Note: The angle has a negative measure since it is a clockwise rotation.)
9. **a)** complement: $90^\circ - 18^\circ = \boxed{72^\circ}$ **b)** supplement: $180^\circ - 18^\circ = \boxed{162^\circ}$	**10.** **a)** complement: $90^\circ - 39^\circ = \boxed{51^\circ}$ **b)** supplement: $180^\circ - 39^\circ = \boxed{141^\circ}$
11. **a)** complement: $90^\circ - 42^\circ = \boxed{48^\circ}$ **b)** supplement: $180^\circ - 42^\circ = \boxed{138^\circ}$	**12.** **a)** complement: $90^\circ - 57^\circ = \boxed{33^\circ}$ **b)** supplement: $180^\circ - 57^\circ = \boxed{123^\circ}$
13. **a)** complement: $90^\circ - 89^\circ = \boxed{1^\circ}$ **b)** supplement: $180^\circ - 89^\circ = \boxed{91^\circ}$	**14.** **a)** complement: $90^\circ - 75^\circ = \boxed{15^\circ}$ **b)** supplement: $180^\circ - 75^\circ = \boxed{105^\circ}$

15. Since the angles with measures $(4x)^\circ$ and $(6x)^\circ$ are assumed to be complementary, we know that $(4x)^\circ + (6x)^\circ = 90^\circ$. Simplifying this yields $(10x)^\circ = 90^\circ$, so that $x = 9$. So, the two angles have measures $\boxed{36^\circ \text{ and } 54^\circ}$.

16. Since the angles with measures $(3x)^\circ$ and $(15x)^\circ$ are assumed to be supplementary, we know that $(3x)^\circ + (15x)^\circ = 180^\circ$. Simplifying this yields $(18x)^\circ = 180^\circ$, so that $x = 10$. So, the two angles have measures $\boxed{30^\circ \text{ and } 150^\circ}$.

17. Since the angles with measures $(8x)^\circ$ and $(4x)^\circ$ are assumed to be supplementary, we know that $(8x)^\circ + (4x)^\circ = 180^\circ$. Simplifying this yields $(12x)^\circ = 180^\circ$, so that $x = 15$. So, the two angles have measures $\boxed{60^\circ \text{ and } 120^\circ}$.

18. Since the angles with measures $(3x+15)^\circ$ and $(10x+10)^\circ$ are assumed to be complementary, we know that $(3x+15)^\circ + (10x+10)^\circ = 90^\circ$. Simplifying this yields $(13x+25)^\circ = 90^\circ$, so that $(13x)^\circ = 65^\circ$ and thus, $x = 5$. So, the two angles have measures $\boxed{30^\circ \text{ and } 60^\circ}$.

19. Since $\alpha + \beta + \gamma = 180^\circ$, we know that $\underbrace{117^\circ + 33^\circ}_{=150^\circ} + \gamma = 180^\circ$ and so, $\boxed{\gamma = 30^\circ}$.

20. Since $\alpha + \beta + \gamma = 180^\circ$, we know that $\underbrace{110^\circ + 45^\circ}_{=155^\circ} + \gamma = 180^\circ$ and so, $\boxed{\gamma = 25^\circ}$.

21. Since $\alpha + \beta + \gamma = 180^\circ$, we know that $\underbrace{(4\beta) + \beta + (\beta)}_{=6\beta} = 180^\circ$ and so, $\beta = 30^\circ$.

Thus, $\boxed{\alpha = 4\beta = 120^\circ \text{ and } \gamma = \beta = 30^\circ}$.

22. Since $\alpha + \beta + \gamma = 180^\circ$, we know that $\underbrace{(3\beta) + \beta + (\beta)}_{=5\beta} = 180^\circ$ and so, $\beta = 36^\circ$.

Thus, $\boxed{\alpha = 3\beta = 108^\circ \text{ and } \gamma = \beta = 36^\circ}$.

23. $\alpha = 180^\circ - (53.3^\circ + 23.6^\circ) = \boxed{103.1^\circ}$

24. $\beta = 180^\circ - (105.6^\circ + 13.2^\circ) = \boxed{61.2^\circ}$

25. Since this is a right triangle, we know from the Pythagorean Theorem that $a^2 + b^2 = c^2$. Using the given information, this becomes $4^2 + 3^2 = c^2$, which simplifies to $c^2 = 25$, so we conclude that $\boxed{c = 5}$.

26. Since this is a right triangle, we know from the Pythagorean Theorem that $a^2 + b^2 = c^2$. Using the given information, this becomes $3^2 + 3^2 = c^2$, which simplifies to $c^2 = 18$, so we conclude that $\boxed{c = \sqrt{18} = 3\sqrt{2}}$.

27. Since this is a right triangle, we know from the Pythagorean Theorem that $a^2 + b^2 = c^2$. Using the given information, this becomes $6^2 + b^2 = 10^2$, which simplifies to $36 + b^2 = 100$ and then to, $b^2 = 64$, so we conclude that $\boxed{b = 8}$.

28. Since this is a right triangle, we know from the Pythagorean Theorem that $a^2+b^2=c^2$. Using the given information, this becomes $a^2+7^2=12^2$, which simplifies to $a^2=95$, so we conclude that $\boxed{a=\sqrt{95}}$.

29. Since this is a right triangle, we know from the Pythagorean Theorem that $a^2+b^2=c^2$. Using the given information, this becomes $8^2+5^2=c^2$, which simplifies to $c^2=89$, so we conclude that $\boxed{c=\sqrt{89}}$.

30. Since this is a right triangle, we know from the Pythagorean Theorem that $a^2+b^2=c^2$. Using the given information, this becomes $6^2+5^2=c^2$, which simplifies to $c^2=61$, so we conclude that $\boxed{c=\sqrt{61}}$.

31. Since this is a right triangle, we know from the Pythagorean Theorem that $a^2+b^2=c^2$. Using the given information, this becomes $7^2+b^2=11^2$, which simplifies to $b^2=72$, so we conclude that $\boxed{b=\sqrt{72}=6\sqrt{2}}$.

32. Since this is a right triangle, we know from the Pythagorean Theorem that $a^2+b^2=c^2$. Using the given information, this becomes $a^2+5^2=9^2$, which simplifies to $a^2=56$, so we conclude that $\boxed{a=\sqrt{56}=2\sqrt{14}}$.

33. Since this is a right triangle, we know from the Pythagorean Theorem that $a^2+b^2=c^2$. Using the given information, this becomes $a^2+\left(\sqrt{7}\right)^2=5^2$, which simplifies to $a^2=18$, so we conclude that $\boxed{a=\sqrt{18}=3\sqrt{2}}$.

34. Since this is a right triangle, we know from the Pythagorean Theorem that $a^2+b^2=c^2$. Using the given information, this becomes $5^2+b^2=10^2$, which simplifies to $b^2=75$, so we conclude that $\boxed{b=\sqrt{75}=5\sqrt{3}}$.

35. If $x=10$ in., then the hypotenuse of this triangle has length $\boxed{10\sqrt{2}\approx 14.14 \text{ in.}}$

36. If $x=8$ m, then the hypotenuse of this triangle has length $\boxed{8\sqrt{2}\approx 11.31 \text{ m}}$.

37. Let x be the length of a leg in the given $45^\circ-45^\circ-90^\circ$ triangle. If the hypotenuse of this triangle has length $2\sqrt{2}$ cm, then $\sqrt{2}\,x=2\sqrt{2}$, so that $x=2$. Hence, the length of each of the two legs is $\boxed{2 \text{ cm}}$.

38. Let x be the length of a leg in the given $45^\circ-45^\circ-90^\circ$ triangle. If the hypotenuse of this triangle has length $\sqrt{10}$ ft., then $\sqrt{2}\,x=\sqrt{10}$, so that $x=\dfrac{\sqrt{10}}{\sqrt{2}}=\sqrt{\dfrac{10}{2}}=\sqrt{5}$.

Hence, the length of each of the two legs is $\boxed{\sqrt{5} \text{ ft.}}$

39. The hypotenuse has length $\sqrt{2}\left(4\sqrt{2}\right)\text{in.} = 8\,\text{in.}$	**40.** Since $\sqrt{2}x = 6\text{m} \Rightarrow x = \frac{6\sqrt{2}}{2} = 3\sqrt{2}\text{m}$, each leg has length $3\sqrt{2}$ m.

41. Since the lengths of the two legs of the given $30^\circ - 60^\circ - 90^\circ$ triangle are x and $\sqrt{3}\,x$, the shorter leg must have length x. Hence, using the given information, we know that $x = 5\,\text{m}$. Thus, the two legs have lengths 5 m and $5\sqrt{3} \approx 8.66$ m, and the hypotenuse has length 10 m.

42. Since the lengths of the two legs of the given $30^\circ - 60^\circ - 90^\circ$ triangle are x and $\sqrt{3}\,x$, the shorter leg must have length x. Hence, using the given information, we know that $x = 9\,\text{ft}$. Thus, the two legs have lengths 9 ft. and $9\sqrt{3} \approx 15.59$ ft., and the hypotenuse has length 18 ft.

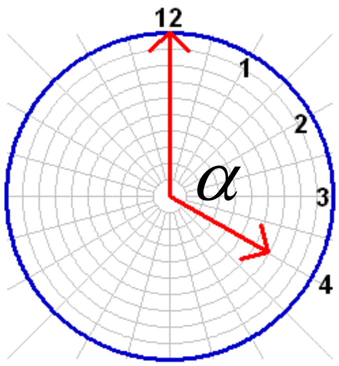
43. The length of the longer leg of the given triangle is $\sqrt{3}x = 12$ yards. So, $x = \frac{12}{\sqrt{3}} = \frac{12\sqrt{3}}{3} = 4\sqrt{3}$. As such, the length of the shorter leg is $4\sqrt{3} \approx 6.93$ yards, and the hypotenuse has length $8\sqrt{3} \approx 13.9$ yards.

44. The length of the longer leg of the given triangle is $\sqrt{3}x = n$ units. So, $x = \frac{n}{\sqrt{3}} = \frac{n\sqrt{3}}{3}$. As such, the length of the shorter leg is $\frac{n\sqrt{3}}{3}$ units, and the hypotenuse has length $\frac{2n\sqrt{3}}{3}$ units.

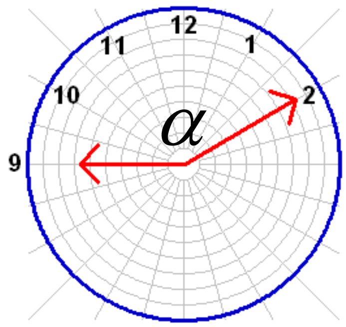
45. The length of the hypotenuse is $2x = 10$ inches. So, $x = 5$. Thus, the length of the shorter leg is 5 inches, and the length of the longer leg is $5\sqrt{3} \approx 8.66$ inches.

46. The length of the hypotenuse is $2x = 8$ cm. So, $x = 4$. Thus, the length of the shorter leg is 4 cm, and the length of the longer leg is $4\sqrt{3} \approx 6.93$ cm.

47. For simplicity, we assume that the minute hand is on the 12.
Let $\alpha =$ measure of the desired angle, as indicated in the diagram below.
Since the measure of the angle formed using two rays emanating from the center of the clock out toward consecutive hours is always $\frac{1}{12}(360^\circ) = 30^\circ$, it immediately follows that $\alpha = 4\cdot(-30^\circ) = \boxed{-120^\circ}$ (Negative since measured *clockwise*.)



48. For simplicity, we assume that the minute hand is on the 9.
Let $\alpha =$ measure of the desired angle, as indicated in the diagram below.
Since the measure of the angle formed using two rays emanating from the center of the clock out toward consecutive hours is always $\frac{1}{12}(360^\circ) = 30^\circ$, it immediately follows that $\alpha = 5\cdot(-30^\circ) = \boxed{-150^\circ}$. (Negative since measured *clockwise*.)



49. The key to solving this problem is setting up the correct proportion.
Let x = the measure of the desired angle.
From the given information, we know that since 1 complete revolution corresponds to 360°, we obtain the following proportion:

$$\frac{360^\circ}{30 \text{ minutes}} = \frac{x}{12 \text{ minutes}}$$

Solving for x then yields

$$x = \left(12 \cancel{\text{ minutes}}\right)\left(\frac{360^\circ}{30 \cancel{\text{ minutes}}}\right) = \boxed{144^\circ}.$$

50. The key to solving this problem is setting up the correct proportion.
Let x = the measure of the desired angle.
From the given information, we know that since 1 complete revolution corresponds to 360°, we obtain the following proportion:

$$\frac{360^\circ}{30 \text{ minutes}} = \frac{x}{5 \text{ minutes}}$$

Solving for x then yields

$$x = \left(5 \cancel{\text{ minutes}}\right)\left(\frac{360^\circ}{30 \cancel{\text{ minutes}}}\right) = \boxed{60^\circ}.$$

51. We know that 1 complete revolution corresponds to 360°.
Let x = time (in minutes) it takes to make 1 complete revolution about the circle.
Then, we have the following proportion:

$$\frac{270^\circ}{45 \text{ minutes}} = \frac{360^\circ}{x}$$

Solving for x then yields

$$270^\circ x = 360^\circ (45 \text{ minutes})$$
$$x = \frac{360^\circ (45 \text{ minutes})}{270^\circ} = 60 \text{ minutes}.$$

So, it takes $\boxed{\text{one hour}}$ to make one complete revolution.

52. We know that 1 complete revolution corresponds to 360°.
Let x = time (in minutes) it takes to make 1 complete revolution about the circle.
Then, we have the following proportion:

$$\frac{72^\circ}{9 \text{ minutes}} = \frac{360^\circ}{x}$$

Solving for x then yields

$$72^\circ x = 360^\circ (9 \text{ minutes})$$
$$x = \frac{360^\circ (9 \text{ minutes})}{72^\circ} = 45 \text{ minutes}.$$

So, it takes $\boxed{\text{45 minutes}}$ to make one complete revolution.

53. Let d = distance (in feet) the dog runs along the hypotenuse. Then, from the Pythagorean Theorem, we know that

$$30^2 + 80^2 = d^2$$
$$7,300 = d^2$$
$$85 \approx \sqrt{7,300} = d$$

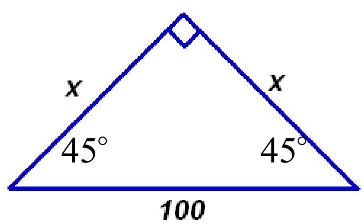
So, $d \approx \boxed{85 \text{ feet}}$.

54. Let d = distance (in feet) the dog runs along the hypotenuse. Then, from the Pythagorean Theorem, we know that

$$25^2 + 100^2 = d^2$$
$$10,625 = d^2$$
$$103 \approx \sqrt{10,625} = d$$

So, $d \approx \boxed{103 \text{ feet}}$.

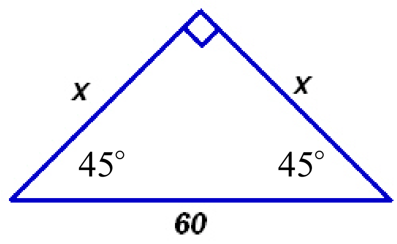
55. Consider the following triangle *T*.



Since T is a $45^\circ - 45^\circ - 90^\circ$ triangle, the two legs (i.e., the sides opposite the angles with measure 45°) have the same length. Call this length *x*. Since the hypotenuse of such a triangle has measure $\sqrt{2}x$, we have that $\sqrt{2}x = 100$, so that $x = \frac{100}{\sqrt{2}} = \frac{100\sqrt{2}}{2} = 50\sqrt{2}$.

So, since lights are to be hung over both legs and the hypotenuse, the couple should buy $50\sqrt{2} + 50\sqrt{2} + 100 = 100 + 100\sqrt{2} \approx \boxed{241 \text{ feet}}$ of Christmas lights.

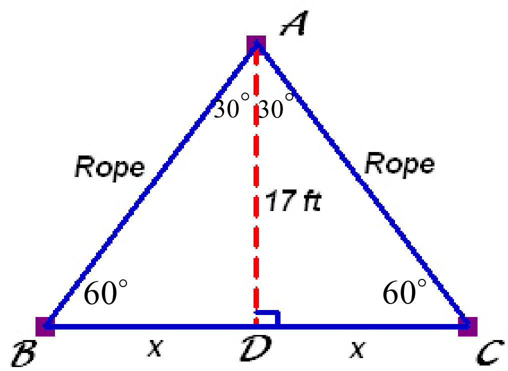
56. Consider the following triangle *T*.



Since *T* is a $45^\circ - 45^\circ - 90^\circ$ triangle, the two legs (i.e., the sides opposite the angles with measure 45°) have the same length. Call this length *x*. Since the hypotenuse of such a triangle has measure $\sqrt{2}x$, we have that $\sqrt{2}x = 60$, so that $x = \frac{60}{\sqrt{2}} = \frac{60\sqrt{2}}{2} = 30\sqrt{2}$.

So, since lights are to be hung over both legs and the hypotenuse, the couple should buy $30\sqrt{2} + 30\sqrt{2} + 60 = 60 + 60\sqrt{2} \approx \boxed{145 \text{ feet}}$ of Christmas lights.

57. Consider the following diagram:



The dashed line segment AD represents the TREE and the vertices of the triangle ABC represent STAKES. Also, note that the two right triangles ADB and ADC are congruent (using the Side-Angle-Side Postulate from Euclidean geometry).

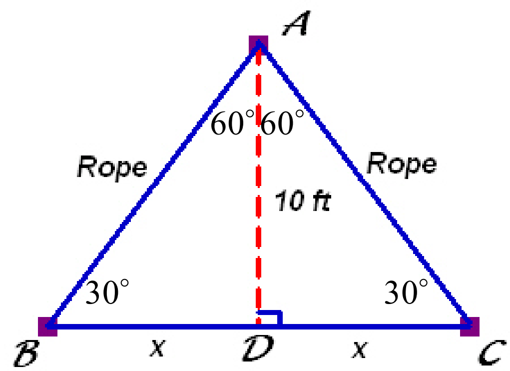
Let x = distance between the base of the tree and one staked rope (measured in feet). For definiteness, consider the right triangle ADC. Since it is a $30^\circ - 60^\circ - 90^\circ$ triangle, the side opposite the 30°-angle (namely DC) is the shorter leg, which has length x feet. Then, we know that the hypotenuse must have length $2x$. Thus, by the Pythagorean Theorem, it follows that:

$$\begin{aligned} x^2 + 17^2 &= (2x)^2 \\ x^2 + 289 &= 4x^2 \\ 289 &= 3x^2 \\ \frac{289}{3} &= x^2 \\ 9.8 \approx \sqrt{\frac{289}{3}} &= x \end{aligned}$$

So, the ropes should be staked approximately $\boxed{9.8 \text{ feet}}$ from the base of the tree.

58. Using the computations from Problem 57, we observe that since the length of the hypotenuse is $2x$, and $x = \sqrt{\frac{289}{3}}$, it follows that the length of each of the two ropes should be $2\sqrt{\frac{289}{3}} \approx 19.6299$ feet. Thus, one should have $2 \times 19.6299 \approx \boxed{39.3 \text{ feet}}$ of rope in order to have such stakes support the tree.

59. Consider the following diagram:



The dashed line segment AD represents the TREE and the vertices of the triangle ABC represent STAKES. Also, note that the two right triangles ADB and ADC are congruent (using the Side-Angle-Side Postulate from Euclidean geometry).

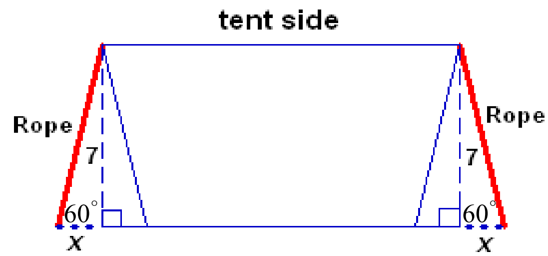
Let x = distance between the base of the tree and one staked rope (measured in feet). For definiteness, consider the right triangle ADC. Since it is a $30^\circ - 60^\circ - 90^\circ$ triangle, the side opposite the 30°-angle (namely AD) is the shorter leg, which has length 10 feet. Then, we know that the hypotenuse must have length 2(10) = 20 feet. Thus, by the Pythagorean Theorem, it follows that:

$$x^2 + 10^2 = 20^2$$
$$x^2 + 100 = 400$$
$$x^2 = 300$$
$$x = \sqrt{300} \approx 17.3 \text{ feet}$$

So, the ropes should be staked approximately $\boxed{17.3 \text{ feet}}$ from the base of the tree.

60. Using the computations from Problem 59, we observe that since the length of the hypotenuse is 20 feet, it follows that the length of each of rope tied from tree to the stake in this manner should be 20 feet in length. Hence, for four stakes, one should have $4 \times 20 \approx \boxed{80 \text{ feet}}$ of rope.

61. The following diagram is a view from one of the four sides of the tented area – note that the actual length of the side of the tent which we are viewing (be it 40 ft. or 20 ft.) does not affect the actual calculation since we simply need to determine the value of *x*, which is the amount beyond the length or width of the tent base that the ropes will need to extend in order to adhere the tent to the ground.



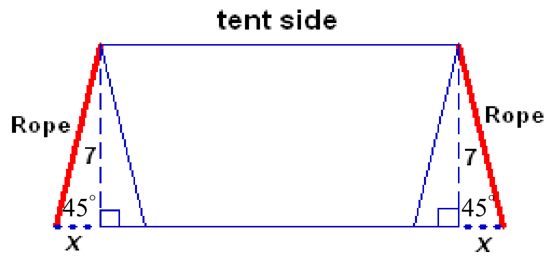
Now, solving this problem is very similar to solving Problem 57. The two right triangles labeled in the diagram are congruent. So, we can focus on the leftmost one, for definiteness. The side opposite the angle with measure 30° is the shorter leg, the length of which is *x*. So, the hypotenuse has length 2*x*. From the Pythagorean Theorem, it then follows that

$$x^2 + 7^2 = (2x)^2$$
$$49 = 3x^2$$
$$4.0 \approx \tfrac{7}{\sqrt{3}} \approx \sqrt{\tfrac{49}{3}} = x$$

Hence, along any of the four edges of the tent, the staked rope on either side extends approximately 4 feet beyond the actual dimensions of the tent. As such, the actual footprint of the tent is approximately $\left(40+2(4)\right)\text{ft.} \times \left(20+2(4)\right)\text{ft.}$, which is

$\boxed{48\,\text{ft.} \times 28\,\text{ft.}}$

62. The following diagram is a view from one of the four sides of the tented area – note that the actual length of the side of the tent which we are viewing (be it 80 ft. or 40 ft.) does not affect the actual calculation since we simply need to determine the value of *x*, which is the amount beyond the length or width of the tent base that the ropes will need to extend in order to adhere the tent to the ground.



Now, solving this problem is very similar to solving Problem 53. The two right triangles labeled in the diagram are congruent. So, we can focus on the leftmost one, for definiteness. Since this is a $45^\circ - 45^\circ - 90^\circ$ triangle, the lengths of the two legs must be equal. So, $x = 7$. Hence, along any of the four edges of the tent, the staked rope on either side must extend 7 feet beyond the actual dimensions of the tent. As such, the actual footprint of the tent is approximately $\left(40 + 2(7)\right)\text{ft.} \times \left(80 + 2(7)\right)\text{ft.}$, which is $\boxed{54\text{ ft.} \times 94\text{ ft.}}$.

63. The corner is not 90° because $10^2 + 15^2 \neq 20^2$.

64. $x^2 + 8^2 = 17^2 \;\Rightarrow\; x^2 = 225 \;\Rightarrow\; x = 15\text{ ft.}$

65. The speed is $\frac{1700}{60} = \frac{170}{6}$ revolutions per second. Since each revolution corresponds to 360°, the engine turns $\left(\frac{170}{6}\right)\left(360^\circ\right) = 10,200^\circ$ each second.

66. The speed is $\frac{300,000^\circ}{15} = 20,000^\circ$ per second. Since each revolution corresponds to 360°, this amounts to $\frac{500}{9}$ revolutions per second. Since 1 minute = 60 seconds, the speed is $\frac{500}{9} \times 60 = \frac{10,000}{3} \approx 3,333.3$ RPMs.

67. In a $30^\circ - 60^\circ - 90^\circ$ triangle, the length opposite the 60°-angle has length $\sqrt{3} \times (\text{shorter leg})$, not $2 \times (\text{shorter leg})$. So, the side opposite the 60°-angle has length $10\sqrt{3} \approx 17.3$ inches.

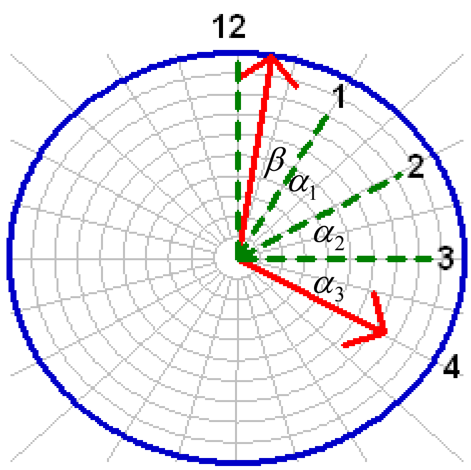
68. The length of the hypotenuse must be positive. Hence, the length must be $5\sqrt{2}$ cm.

69. False. Each of the three angles of an equilateral triangle has measure 60°. But, in order to apply the Pythagorean theorem, one of the three angles must have measure 90°.

70. False. Since the Pythagorean theorem doesn't apply to equilateral triangles, and equilateral triangles are also isosceles (since at least two sides are congruent), we conclude that the given statement is false.

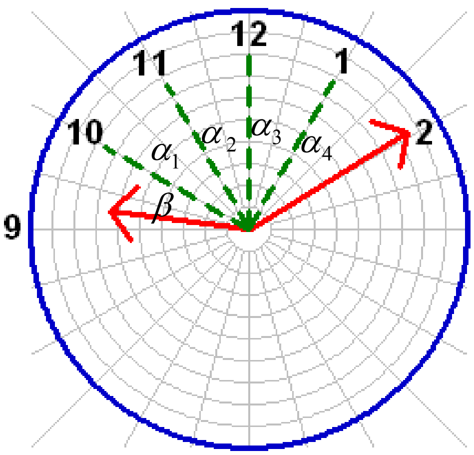
71. True. Since the angles of a right triangle are α°, β°, and 90°, and also we know that $\alpha^\circ + \beta^\circ + 90^\circ = 180^\circ$, it follows that $\alpha^\circ + \beta^\circ = 90^\circ$.	
72. False. The length of the side opposite the 60°-angle is $\sqrt{3}$ times the length of the side opposite the 30°-angle.	
73. True. The sum of the angles $\alpha, \beta, 90^\circ$ must be 180°. Hence, $\alpha + \beta = 90^\circ$, so that α and β are complementary.	
74. False. The legs have the same length *x*, but the hypotenuse has length $\sqrt{2}x$.	
75. True. Angles swept out counterclockwise have a positive measure, while those swept out clockwise have negative measure.	
76. True. Since the sum of the angles $\alpha, \beta, 90^\circ$ must be $180^\circ, \alpha + \beta = 90^\circ$. So, neither angle can be obtuse.	

77. First, note that at **12:00** exactly, both the minute and the hour hands are identically on the **12**. Then, for each minute that passes, the minute hand moves $\frac{1}{60}$ the way around the clock face (i.e., 6°). Similarly, for each minute that passes, the hour hand moves $\frac{1}{60}$ the way between the **12** and the **1;** since there are $\frac{1}{12}(360^\circ) = 30^\circ$ between consecutive integers on the clock face, such movement corresponds to $\frac{1}{60}(30^\circ) = 0.5^\circ$.
Now, when the time is **12:20**, we know that the minute hand is on the **4**, but the hour hand has moved $20 \times 0.5^\circ = 10^\circ$ clockwise from the **12** towards the **1**.
The picture is as follows:



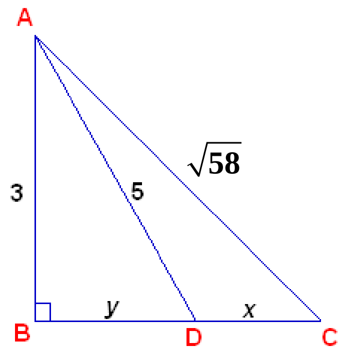
The angle we seek is $\beta + \alpha_1 + \alpha_2 + \alpha_3$. From the above discussion, we know that $\alpha_1 = \alpha_2 = \alpha_3 = 30^\circ$ and $\beta = 20^\circ$. Thus, the angle at time **12:20** is $\boxed{110^\circ}$.

78. First, note that at **9:00** exactly, the minute is identically on the **12** and the hour hand is identically on the **9**. Then, for each minute that passes, the minute hand moves $\frac{1}{60}$ the way around the clock face (i.e., 6°). Similarly, for each minute that passes, the hour hand moves $\frac{1}{60}$ the way between the **9** and the **10;** since there are $\frac{1}{12}(360^\circ) = 30^\circ$ between consecutive integers on the clock face, such movement corresponds to $\frac{1}{60}(30^\circ) = 0.5^\circ$. Now, when the time is **9:10**, we know that the minute hand is on the **2**, but the hour hand has moved $10 \times 0.5^\circ = 5^\circ$ clockwise from the **9** towards the **10**, thereby leaving an angle of 25° between the hour hand and the **10**. The picture is as follows:



The angle we seek is $\beta + \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$. From the above discussion, we know that $\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 30^\circ$ and $\beta = 25^\circ$. Thus, the angle at time **9:10** is $\boxed{145^\circ}$.

79. Consider the following diagram:



Let x = length of $\overline{DC}$ and y = length of $\overline{BD}$.
Ultimately, we need to determine x. We proceed as follows.
First, we find y. Using the Pythagorean Theorem on $\triangle ABD$ yields $3^2 + y^2 = 5^2$, so that $y = 4$. Next, using the Pythagorean Theorem on $\triangle ABC$ yields

$$(y+x)^2 + 3^2 = \left(\sqrt{58}\right)^2$$
$$(4+x)^2 + 9 = 58$$
$$(4+x)^2 = 49$$
$$4 + x = \pm 7 \quad \text{so that} \quad x = \cancel{-11} \ \text{ or } \ 3$$

So, the length of $\overline{DC}$ is $\boxed{3}$.