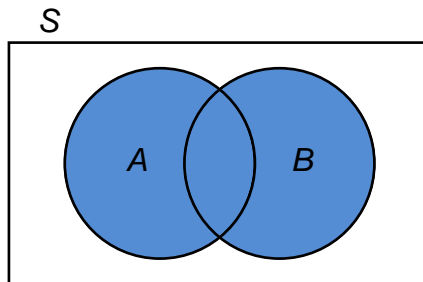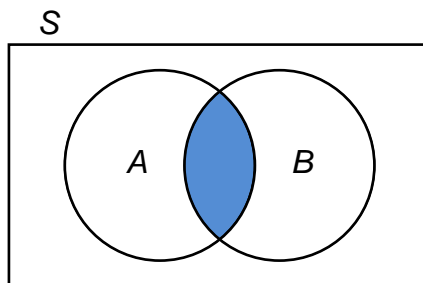## Solutions
Chapter 3

Practice Problem for Sections 3.2 and 3.3

1. (a) $A \cup B$ (b) $A \cap B$ (c) $\bar{A} \cap \bar{B}$ or $\overline{(A \cup B)}$ (d) $(A \cap \bar{B}) \cup (\bar{A} \cap B)$ (e) $\overline{(A \cap B)}$.

2. A and B are any events in S, then we have the following:.
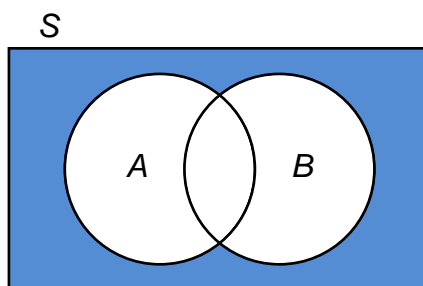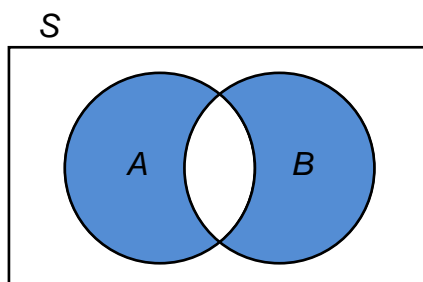
(a) $A \cup B$
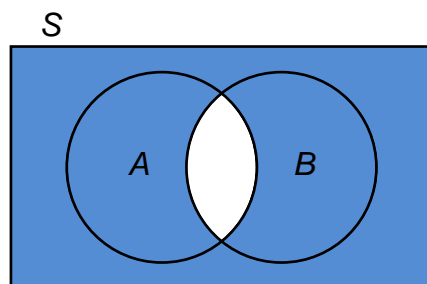

(b) $A \cap B$

(c) $\bar{A} \cap \bar{B}$ or $\overline{(A \cup B)}$

(d) $(A \cap \bar{B}) \cup (\bar{A} \cap B)$

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(e) $\overline{(A \cap B)}$.

S

3. (a) S = { HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}.
   (b) S = {(H,1) (H,2) (H,3) (H,4) (H,5) (H,6) (T,1) (T,2) (T,3) (T,4) (T,5) (T,6)}
   (c) S = {(1,1) (1,2) $\cdots$ (1,6) (2,1) (2,2) $\cdots$ (2,6) $\cdots$ (6,6)}
   (d) S = {BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG}
   (e) S = {(HH,1) (HT,1) (TH,1) (TT,1) (HH,2) (HT,2) (TH,2) (TT,2) $\cdots$ (HH,6) (HT,6) (TH,6) (TT,6)}

4. S = {(1,2) $\cdots$ (1,6) (2,1) (2,3) $\cdots$ (2,6) $\cdots$ (6,5)}

   Let A be the event that the total of points on two dice is ten. Then
   $$A = \{(4, 6)\ (6, 4)\}$$
   Total number of points in S is 30. Thus,

   $$P(A) = \frac{2}{30} = \frac{1}{15}.$$

5. Let C, E, and M be the events that are defined as follows:

   C : The selected student is a Chemical Engineering Major
   E : The selected student is a Electrical Engineering Major
   M : The selected student is a Mechanical Engineering Major.

   Selecting of three students can be considered a three step process, which is, first selecting one student and recording his/her major, then selecting the second and recording his/her major, and then selecting the third student and recording his/her major. Since each step has three possible outcomes, the total number sample points in the sample space are $3 \times 3 \times 3 = 27$, that is

   S = {MMM, MMC, MME, MCM, MCC, MCE, MEM, MCE, MEE, CMM, CMC,
   CME, CCM, CCC, CCE, CEM, CCE, CEE, EMM, EMC, EME, ECM, ECC,

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ECE, EEM, ECE, EEE }

Now suppose that A is the event that at the most one of the three selected students is an Electrical Engineering major. Then, we have

A = {MMM, MMC, MME, MCM, MCC, MCE, MEM, MCE, CMM, CMC, CME, CCM, CCC, CCE, CEM, CCE, EMM, EMC, ECM, ECC}

Hence

P(A) = 20/27.

6. Let D and G be the events that are defined as follows

D : The chip is defective

G : The chip is not defective

Then the sample space S is consist of 16 sample points, that is

S = {GGGG, GGGD, GGDG, GDGG, GGGD, GGDD, $\cdots$, DDDD}

(a) 4/16 or 1/4.

(b) 1/16.

(c) 6/16 or 3/8.

7.

(a) $A \cap B \cap C = \{4\}$.

(b) $(A \cap B) \cup (C \cap D) = \{1, 4, 5, 7\}$.

(c) $A \cap (B \cup C \cup D) = \{1, 3, 4, 7\}$.

(d) $\bar{A} \cap \bar{B} = \overline{(A \cup B)} = \overline{\{1,2,3,4,6,7,8,9\}} = \{5\}$.

(e) $\varnothing$.

(f) $\bar{A} \cap \bar{B} \cap \bar{C} \cap \bar{D} = \overline{(A \cup B \cup C \cup D)} = \{ \} = \varnothing$.

8.

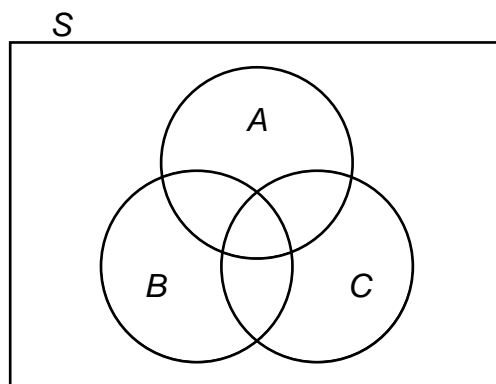(a) $\bar{A} = \{x | 3 < x \le 4, 7 \le x < 10\}$.

(b) $\overline{(A \cup B)} = \{x | 3 < x \le 4, 9 \le x < 10\}$.

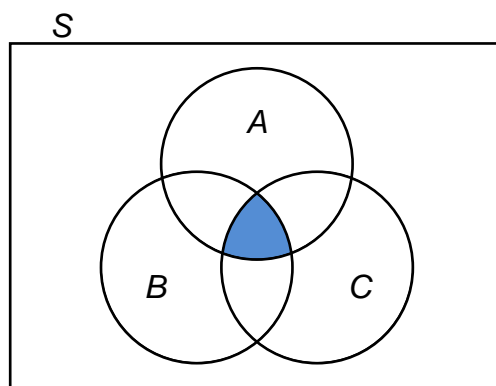(c) $A \cup B = \{x | 4 < x < 9\}$.

(d) $A \cap B = \{x | 5 < x < 7\}$

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9.

A, B and C are events shown in the following Venn diagram.

(a) $A \cap B \cap C$: The patient is diagnosed with liver cancer who needs a liver transplant and the hospital finds a matching liver on time.

(b) $A \cap (B \cup C)$:
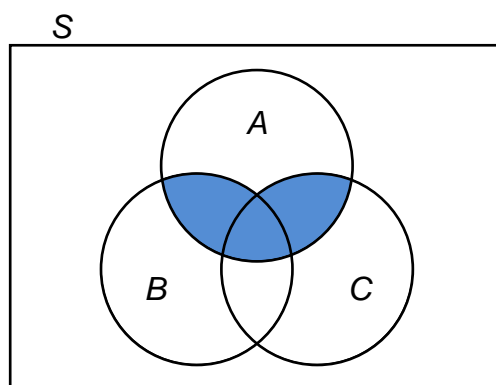
The patient is diagnosed with liver cancer and the patient needs a liver transplant but the hospital does not find the matching liver on time

or:

the patient is diagnosed with liver cancer and does need a liver transplant but the hospital finds a matching liver

or:

the patient is diagnosed with liver cancer who needs a liver transplant and the hospital finds a matching liver on time.

(c) $\bar{A} \cap \bar{B} = (\overline{A \cup B})$: The patient is not diagnosed with liver cancer and does not need a liver transplant.

(d) $(\bar{A} \cap \bar{B} \cap \bar{C}) = (\overline{A \cup B \cup C})$: The patient is not diagnosed with liver cancer and does not need a liver transplant and the hospital does not find a matching liver.
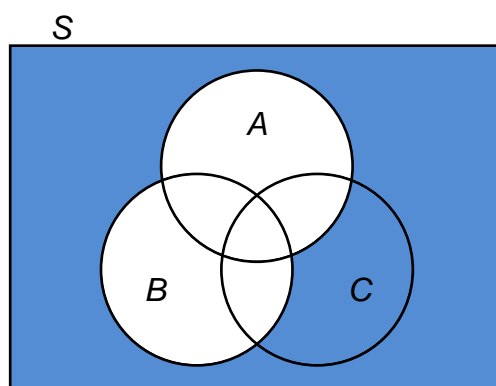
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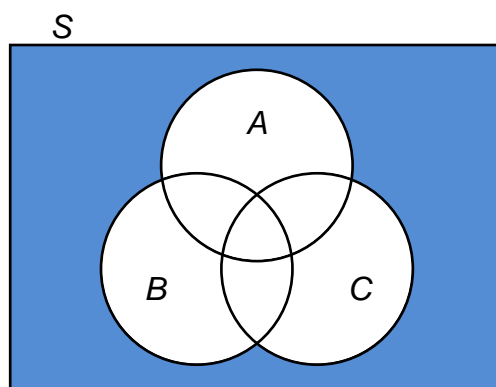
10. In this problem the sample space has 16 sample points, that is

{DDDD, DDDN, DDND, DNDD, NDDD, DDNN, DNDN, NDDN, DNND, NDND,
  NNDD, DNNN, NDNN, NNDN, NNND, NNNN}

11. In this problem the sample space has 32 sample points, that is

{CCCCC, CCCCN, CCCNC, CCNCC, CNCCC, NCCCC, CCCNN, $\cdots$, NNNNC, NNNNN}

12. $\{X \mid X = 1, 2, 3, \cdots\}$

13. (a) {HHT, HTH, THH, HHH}   (b) {HTT, THT, TTH, TTT} (c) {HHT, HTH, THH}

(d) {TTT}
Since each possible outcome is equally likely, the probabilities for the events in parts (a) - (d) are
given by 1/4 , 1/4 , 3/16 , and 1/16 respectively.

14. (a) {(1,6) (2,5) (3,4) (4,3) (5,2) (6,1)}

   (b) {(1,1) (1,2) (2,1) (1,3) (2,2) (3,1) (1,4) (2,3) (3,2) (4,1)}

   (c) In this part the sum is either 2, 4, 6, 8 10, 12, or 9. Thus, the sample space is

     {(1,1) (1,3) (2,2) (3,1) (1,5) (2,4) (3,3) (4,2) (5,1) (2,6) (3,5) (4,4) (5,3) (6,2) (4,6) (5,5)
       (6,4) (6,6) (3,6) (4,5) (5,4) (6,3)}

(d) Since each possible outcome is equally likely, the probabilities for the events in parts (a) - (c)
are given by 6/36, 10/36, and 22/36 respectively.

Practice Problems for Section 3.4

1. $\begin{pmatrix} 4 \\ 1 \end{pmatrix} \times \begin{pmatrix} 5 \\ 1 \end{pmatrix} = 20$

2. Suppose that rooms R_1 *and* R_2 are with three twin beds each, rooms R_3, R_4 *and* R_5 are with two twin beds each and the rest of the rooms, that is, rooms $R_6, R_7, R_8,$ *and* R_9 are with one twin bed each. The manager first selects three guest which he can do in $\binom{16}{3} = 560$ ways and assign them one of the two rooms with three twin beds in $\binom{2}{1} = 2$ ways. Thus the total number of ways the manager can fill one of the rooms with three twin beds is $560 \times 2 = 1120$ ways. Now the second room with three twin beds can be assigned in $\binom{13}{3} = 286$ ways. Thus the total number of ways both rooms with twin beds can be assigned are $1120 \times 286 = 320320$. Arguing in the same manner the three rooms with two twin beds each can be filled in

$\binom{10}{2} \times \binom{3}{1} \times \binom{8}{2} \times \binom{2}{1} \times \binom{6}{2} = 113400$ ways. Similarly, the rest of the four rooms with one twin

bed each can be assigned to the remaining four guests in $4! = 24$ ways. Thus, the total number of ways all nine rooms can be assigned to all 16 guests are

$$320320 \times 113400 \times 24 = 871,782,912,000.$$

3.

(a) In case no student can serve in multiple roles, the number of possible committees is

$$P(30,4) = \frac{30!}{26!} = 657,720.$$

(b) In case any student can serve in multiple roles, the number of possible committees is

$$30 \times 30 \times 30 \times 30 = 810,000.$$

4. This experiment can be completed in 15 steps and each step can be performed in 4 ways. Thus, the total number of ways the whole experiment can be carried out in

$$\underbrace{4 \times 4 \times \cdots \times 4}_{(15 \text{ times})} = 4^{15} = 1,073,741,824.$$

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5. A customer can choose the four items, that is, (i) a soup (ii) a salad (iii) an entrée, and (iv) a dessert 4, 3, 10, and 4 ways, respectively. Thus, the total number of ways of choosing the four items simultaneously is $4 \times 3 \times 10 \times 4 = 480$ ways.

6. Referring back to Problem 3, the committee now consists of four members only. Thus, the ordering is not important. In other words, the number of committees that consist of four members is equivalent to just selecting a group of four students from a class of thirty, which is equal to

$$\binom{30}{4} = \frac{30!}{4! \times 26!} = 27,405$$

7. A hand of 13 cards from a well shuffled deck of 52 cards can be dealt in $\binom{52}{13}$ ways. Now a hand of 13 cards consisting of 5 spades, 4 diamond, and the remaining 4 of either club, heart or mixed can be dealt in

$$\binom{13}{5} \times \binom{13}{4} \times \binom{26}{4}$$

ways. Thus, the desired probability is

$$\frac{\binom{13}{5} \times \binom{13}{4} \times \binom{26}{4}}{\binom{52}{13}} = 0.02166.$$

8. **(a)** The word engineering is consisting of 3 e's, 2 g's, 2 *i*'s, 3 *n*'s, and one r. Thus, using Equation (3.4.7), we obtain the total number of permutations

$$\frac{11!}{3! \times 2! \times 2! \times 3!} = 277,200$$

(b) Using the same argument as in part (a) we obtain the total number of permutations

$$\frac{12!}{2! \times 2!} = 119,750,400$$

Since there are 2 *i*'s, 2 o's and rest of the letters are distinct.

9. An experiment in which a physician prescribes a drug to a patient can be carried out in three steps, where the first step consists of selecting a manufacturer, the second step is selecting the strength, and the third step is choosing the form of the medication, that is, either a capsule or a tablet. These three steps can respectively be carried out in 4, 5, and 2 ways. Thus, the total number of ways a drug can be prescribed to a patient is equal to

$$4 \times 5 \times 2 = 40.$$

10. The process of manufacturing a car plate is consisting of two steps. The first step is selecting two letters from 26 letters and arranging them in all possible ways and second step is selecting four digits from 10 digits and arranging them in all possible ways. Each of these steps can be carried out in *P(26,2)* and *P(10,4)* ways, respectively. Thus, the total number distinct car plates that the Department of Motor Vehicle can manufacture is equal to

$$P(26,2) \times P(10,4) = \frac{26!}{24!} \times \frac{10!}{6!} = 26 \times 25 \times 10 \times 9 \times 8 \times 7 = 3,276,000.$$

11. There are 6 possible outcomes when a die is rolled, 2 possible outcomes when a coin is tossed and there are 4 possible outcomes when a card is drawn from a well shuffled regular deck of playing cards and its suit is noted. Thus, using the multiplication rule (or, a tree diagram), we obtain the total number of sample points in the sample space equal to

$$6 \times 2 \times 4 = 48.$$

12. There are 10 web sites and each web site may either contain an ad of car or may not contain an ad of a car, that is, each web site results in two possible outcomes. Thus, using the multiplication rule, we obtain the total number of sample points in the sample space equal to

$$2 \times 2 \times \cdots \times 2 = 2^{10} = 1024.$$

Practice Problem for Sections 3.5 and 3.6

1. Let A be the event that an odd number shows up and B be the event that 3 or 5 show up.
Then, we have

$$S = \{1, 2, 3, 4, 5, 6\}$$
$$A = \{1, 3, 5\}$$
$$B = \{3, 5\}$$

We are now interested in finding the conditional probability $P(B|A)$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{2/6}{3/6} = 2/3.$$

2. Let A be the event that at least one head appears and B be the event that two heads appear.

Then, we have

$$S = \{ \text{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT} \}$$
$$A = \{ \text{HHH, HHT, HTH, THH, HTT, THT, TTH} \}$$
$$B = \{ \text{HHT, HTH, THH} \}$$

We are now interested in finding the conditional probability $P(B|A)$.

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{3/8}{7/8} = 3/7.$$

3. Given that A_1, A_2, A_3, A_4 *and* A_5 are mutually exclusive and exhaustive events in a sample space
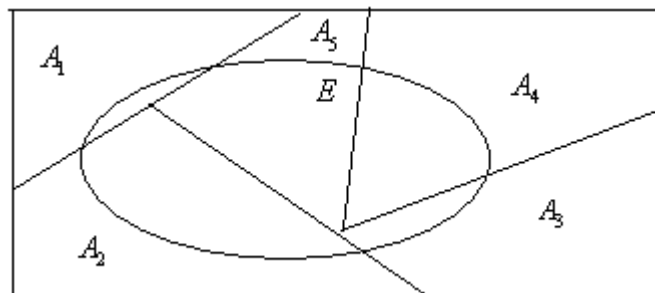
S. Let E be any other event in S. Then we are given that $P(A_1) = .2$, $P(A_2) = .1$, $P(A_3) = .15$

$P(A_4) = .3$, and $P(A_5) = .25$. Further, we are given that $P(E|A_1) = .2$, $P(E|A_2) = .1$, $P(E|A_3) = .35$, $P(E|A_4) = .3$, and $P(E|A_5) = .25$. We now wish to determine the probabilities $P(A_1|E)$,

$P(A_2|E)$, $P(A_3|E)$, $P(A_4|E)$. and $P(A_5|E)$.

Using the given information and the Venn diagram shown below, we have that

$$P(E) = P(E \cap A_1) + P(E \cap A_2) + P(E \cap A_3) + P(E \cap A_4) + P(E \cap A_5)$$

$$= P(E|A_1)P(A_1) + P(E|A_2)P(A_2) + P(E|A_3)P(A_3) + P(E|A_4)P(A_4) + P(E|A_5)P(A_5)$$