

Solutions for Selected Exercises
in
**Statistical Thermodynamics
and Microscale Thermophysics**

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$$\varepsilon = \frac{\hbar^2}{2I} l(l+1) \quad l \geq |m|, \quad m = \pm 1, \pm 2, \dots$$

$$m = \pm 1 \rightarrow l = 1, 2, 3, \dots$$

$$m = \pm 2 \rightarrow l = 2, 3, 4, \dots$$

l	$l(l+1)$
1	2
2	6
3	12

$$\text{lowest} \rightarrow \varepsilon_0 = \frac{(6.63 \times 10^{-34} / 2\pi)^2}{2(6.69 \times 10^{-46})} \cdot 2 = \underline{\underline{1.66 \times 10^{-23} \text{ J}}}$$

similarly, the next two levels are

$$\varepsilon_1 = \underline{\underline{4.99 \times 10^{-23} \text{ J}}}, \quad \varepsilon_2 = \underline{\underline{9.99 \times 10^{-23} \text{ J}}}$$

1.7

From eqn's (1.59) and (1.60)

$$H_0(\alpha^{1/2}x) = 1, \quad \psi_0(x) = \left(\frac{\alpha}{\pi}\right)^{1/4} \exp\left\{-\frac{\alpha x^2}{2}\right\}$$

$$P(x) dx = \psi_0^* \psi_0 dx = \left(\frac{\alpha}{\pi}\right)^{1/2} e^{-\alpha x^2} dx, \quad \alpha = \frac{\sqrt{km}}{\hbar}$$

(a) The energy of the oscillator in the ground state is $(\hbar/2)(k/m)^{1/2}$. The greatest displacement it could have classically is the displacement where all its energy is potential energy.

$$\rightarrow \frac{1}{2} k x_{\max, cl}^2 = \left(\frac{\hbar}{2}\right) \left(\frac{k}{m}\right)^{1/2}$$

$$\rightarrow x_{\max, cl} = \left(\frac{\hbar}{k}\right)^{1/2} \left(\frac{k}{m}\right)^{1/4} = \left(\frac{\hbar^2}{km}\right)^{1/4} = \underline{\underline{\alpha^{-1/2}}}$$

(b) P_c = probability displacement will exceed $x_{\max, cl}$

$$P_c = \int_{-\alpha_c^{-1/2}}^{-\alpha_c^{-1/2}} P(x) dx + \int_{\alpha_c^{-1/2}}^{\infty} P(x) dx = 2 \int_{\alpha_c^{-1/2}}^{\infty} P(x) dx$$

$$= 2 \int_{\alpha_c^{-1/2}}^{\infty} \left(\frac{\alpha}{\pi}\right)^{1/2} e^{-\alpha x^2} dx$$

$$\text{let } \gamma = \sqrt{\alpha} x \rightarrow dx = d\gamma / \sqrt{\alpha}$$

$$P_c = \frac{2}{\sqrt{\pi}} \int_1^{\infty} e^{-\gamma^2} d\gamma = \text{erfc}(1) = \underline{\underline{0.16}}$$

1.10

$$\varepsilon = \frac{3}{2} k_B T = \frac{3}{2} (1.38 \times 10^{-23}) (280) = 5.8 \times 10^{-21} \text{ J}$$

$$\delta \varepsilon = 0.01 \varepsilon = 5.8 \times 10^{-23} \text{ J}$$

$$g = \frac{\pi}{4} \left[\frac{8 m_{\text{He}} V^{2/3}}{h^2} \right]^{3/2} \varepsilon^{1/2} \delta \varepsilon$$

$$= \frac{\pi}{4} \left[\frac{8 (6.64 \times 10^{-27}) (0.1)^{2/3}}{(6.63 \times 10^{-34})^2} \right]^{3/2} (5.8 \times 10^{-21})^{1/2} (5.8 \times 10^{-23})$$

$$g = \underline{\underline{1.46 \times 10^{28}}}$$

1.12



$$m_C = 1.99 \times 10^{-26} \text{ kg} \quad m_O = 2.66 \times 10^{-26} \text{ kg}$$

$$I = \frac{m_O^2 (2r)^2}{2 m_O} = 2 m_O r^2 = 2 (2.66 \times 10^{-26}) (2.77 \times 10^{-10})^2$$

$$I = 4.08 \times 10^{-45} \text{ kg m}^2$$

$$\text{rotational energy} = k_B T = 1.38 \times 10^{-23} (290) = 4.0 \times 10^{-21} \text{ J}$$

$$\varepsilon = \frac{\hbar^2}{2I} \ell(\ell+1)$$

$$\ell(\ell+1) = \frac{2I\varepsilon}{\hbar^2} = \frac{2(4.08 \times 10^{-45})(4.0 \times 10^{-21})}{(1.055 \times 10^{-34})^2} = 2933$$