

2.1  $v = \sqrt{2} \times 120 \cos(\omega t + 30^\circ) \Rightarrow v = 120 \angle 30^\circ \text{ V}$   
 $i = \sqrt{2} \times 10 \cos(\omega t - 30^\circ) \Rightarrow I = 10 \angle -30^\circ \text{ A}$

(a)  $p(t) = |V||I| [\cos \phi + \cos(2\omega t + \angle V + \angle I)]$   
 $= 600 + 1200 \cos 2\omega t \text{ W}$

$S = VI^* = 1200 \angle 60^\circ = P + jQ \Rightarrow P = 600 \text{ W}, Q = 1039 \text{ VAR}$

(b)  $Z = V/I = 12 \angle 60^\circ = 6 + j10.39 = R + jX \Rightarrow R = 6, X = 10.39$

2.2 (a) Using (2.3) we find  $P_{\max} = 1707 = |V||I|(\cos \phi + 1)$   
 and  $P_{\min} = -293 = |V||I|(\cos \phi - 1)$ . Then, since  $|V| = 100$ , we  
 get  $|I| = 10$  and  $\cos \phi = \pm 45^\circ$ . Pick  $\phi = 45^\circ \Rightarrow Z = 10 \angle 45^\circ =$   
 $7.07 + j7.07 = R + jX \Rightarrow R = 7.07, X = 7.07$

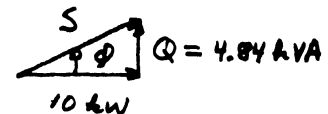
(b)  $S = VI^* = Z|I|^2 = (7.07 + j7.07) 10^2 \Rightarrow P = 707, Q = 707$

(c) For simplicity assume  $i(t) = \sqrt{2}|I| \cos \omega t$ . Then  
 $p_L(t) = v_L(t) i(t) = L \frac{di}{dt} i = -2\omega L |I|^2 \cos \omega t \sin \omega t = -\omega L |I|^2 \sin 2\omega t$   
 $P_{L\max} = \omega L |I|^2 = 707 = Q$ . Thus  $P_{\max} = Q$ . The same!

2.3 0.7 PF lagging  $\Rightarrow \phi = 45.57^\circ, Q = 5.10 \text{ MVAR}$   
 0.9 PF lagging  $\Rightarrow \phi = 25.84^\circ, Q = 2.42 \text{ MVAR}$ . Capacitor must  
 supply  $5.10 - 2.42 = 2.68 \text{ MVAR}$ .

2.4 0.707 PF lagging  $\Rightarrow S_{3\phi} = 200 + j200 \text{ kVA}$ . Cap supplies  
 $50 \text{ kVAR}$ . Resultant  $S_{3\phi} = 200 + j150 \text{ kVA} \Rightarrow PF = 0.80$ .  
 $|S| = \frac{|S_{3\phi}|}{3} = \frac{250 \times 10^3}{3} = |V||I| = \frac{440}{\sqrt{3}} |I| \Rightarrow |I| = 328 \text{ A}$

2.5 0.9 PF lagging  $\Rightarrow \phi = 25.84^\circ$



(a)  $S = 10 + j4.84 \text{ kVA}$

(b)  $10 \times 10^3 = 416 \times |I| \times 0.9 \Rightarrow |I| = 26.71 \text{ A}$

(c) Using (2.3), (or first principles) we get

$p(t) = 10 \times 10^3 + 11.11 \times 10^3 \cos(2\omega t + 25.84^\circ)$

Note: the average value of  $p(t)$  is  $10 \text{ kW}$

2.6 Because system is balanced  $V_{ab} = 208 \angle 120^\circ$ ,  $V_{bc} = 208 \angle 0^\circ$ .  
 Using (2.17) or Fig 2.11,  $V_{an} = 120 \angle 90^\circ \Rightarrow V_{bn} = 120 \angle -30^\circ$ ,  
 $V_{cn} = 120 \angle -150^\circ$ . Using per phase analysis,  $I_a = 12 \angle 105^\circ \Rightarrow$   
 $I_b = 12 \angle -15^\circ$ ,  $I_c = 12 \angle -135^\circ$ .

2.7  $S = VI^* = V(YV)^* = Y^* |V|^2 = Y_C^* + Y_L^* + Y_R^*$   
 $= -j5 + j10 + 0.1 = 0.1 + j5$

2.8 (a) Using loop or nodal analysis we find, after much work,  
 $I_a = 0.9123 \angle -90.351^\circ$ ,  $I_b = 0.9123 \angle -209.65^\circ$ ,  $I_c = 0.9929 \angle 30^\circ$ .

(b) Using per phase analysis  $I_a = 1 \angle 0^\circ / j1.1 = 0.9091 \angle -90^\circ$ ,  
 then,  $I_b = 0.9091 \angle -210^\circ$ ,  $I_c = 0.9091 \angle 30^\circ$ .

2.9 Proceeding by analogy with  $3\phi$ , we note

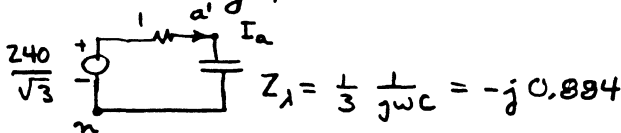
$$E_{ab} = E_{an} - E_{bn} = E_{an}(1 - e^{-j\pi/2}) = \sqrt{2} E_{an} e^{j\pi/4}$$

Thus  $E_{an} = \frac{1}{\sqrt{2}} E_{ab} e^{-j\pi/4}$ , and  $E_{an}, E_{bn}, E_{cn}, E_{dn}$  form  
 a pos. seq. set of  $4\phi$  voltages. Doing per phase (phase a)  
 analysis we have

$$\frac{1}{\sqrt{2}} \angle 45^\circ \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} \begin{array}{c} \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \\ \text{---} \text{---} \text{---} \end{array} -j0.5 \Rightarrow I_a = \frac{\frac{1}{\sqrt{2}} \angle -45^\circ}{j0.5} = \sqrt{2} \angle -135^\circ$$

Then  $I_b = \sqrt{2} \angle -225^\circ$ ,  $I_c = \sqrt{2} \angle -315^\circ$ ,  $I_d = \sqrt{2} \angle -405^\circ$

2.10 Using per phase circuit we find  $I_a = 103.8 \angle 41.5^\circ$ .



Then  $|I_b| = 103.8 \text{ A}$

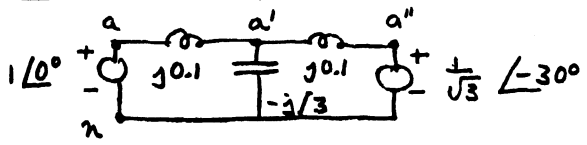
$$V_{a'n} = Z_l I_a = 91.76 \angle -48.5^\circ \Rightarrow |V_{a'b'}| = \sqrt{3} \cdot 91.76 = 158.9 \text{ V}$$

$$S_{load} = V_{a'n} I_a^* = 9524 \angle -90^\circ$$

$$S_{load}^{3\phi} = 3 S_{load} = 28574 \angle -90^\circ \text{ W}$$

2.11 Assume pos. seq. operatin.  $V_{a''b''} = V_{a''n} - V_{b''n} = \sqrt{3} V_{a''n} e^{j\pi/6} \Rightarrow V_{a''n} = \frac{1}{\sqrt{3}} V_{a''b''} e^{-j\pi/6} = \frac{1}{\sqrt{3}} \angle -30^\circ$

Per Phase Ckt

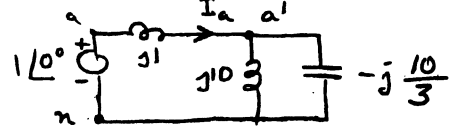


Using superposition & voltage divider law we get

$V_{a'n} = 0.899 \angle -10.89^\circ \Rightarrow V_{b'n} = 0.899 \angle -130.89^\circ$  and  
 $V_{c'n} = 0.899 \angle -250.89^\circ$ . Then  $V_{a'b'c'} = 1.557 \angle 19.11^\circ$

2.12 Assume pos. seq..

Per Phase Ckt



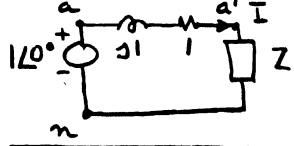
Combining parallel elements we have  $Z_{||} = -j5$ .  $I_a = 0.25 \angle 90^\circ$   
 $V_{a'n} = -j5 \cdot j0.25 = 1.25 \angle 0^\circ$   
 $V_{a'b'c'} = 2.165 \angle 30^\circ \Rightarrow I_{cap} = 2.165 \angle 120^\circ$   
 $I_{load} = 3 V_{a'n} I_a^* = 0.3125 \angle -90^\circ$

2.13

(a)  $V_{bc} = 208 \angle -120^\circ$ ,  $V_{ca} = 208 \angle 120^\circ$   
 $V_{an} = \frac{208}{\sqrt{3}} \angle -30^\circ \Rightarrow I_a = 1.20 \angle -90^\circ$   
 Then  $I_b = 1.20 \angle -210^\circ$ ,  $I_c = 1.20 \angle -330^\circ$

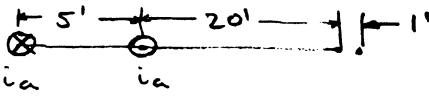
(b)  $V_{bc} = 208 \angle 120^\circ$ ,  $V_{ca} = 208 \angle -120^\circ$   
 $V_{an} = \frac{208}{\sqrt{3}} \angle 30^\circ \Rightarrow I_a = 1.20 \angle -30^\circ$   
 $I_b = 1.20 \angle 90^\circ$ ,  $I_c = 1.20 \angle -150^\circ$

2.14 Per Phase Ckt. Problem reduces to picking Z so that  $|V_{a'n}| > |V_{an}|$ . It helps to draw some phasor diagrams.



I. $Z = j\omega L$	II $Z = R$	III $Z = -j \frac{1}{\omega C}$
<p>drop in <math>j\Omega</math></p> <p>drop in <math>j\Omega</math></p> <p>Clearly <math> V_{a'n}  &lt;  V_{an} </math></p>	<p>drop in <math>j\Omega</math></p> <p>drop in <math>1\Omega</math></p> <p>Clearly <math> V_{a'n}  &lt;  V_{an} </math></p>	<p>For example <math>Z = -j2</math></p> <p><math>I = \frac{1}{\sqrt{2}} \angle 45^\circ</math></p> <p><math>V_{a'n} = \sqrt{2} \angle -45^\circ</math></p> <p><math> V_{a'n}  &gt;  V_{an} </math></p> <p>This is O.K.</p>

2.15 Since  $E_a + E_b + E_c = 0$ , neutrals are at same potential.  
 $Z = E_a / I_a = \sqrt{2} \angle 55^\circ$ . For each  $Z$ ,  $S = VI^* = |V|^2 / Z^*$ . Thus  
 $S^{3\phi} = \frac{(\sqrt{2})^2 + 1^2 + 1^2}{\sqrt{2} \angle -55^\circ} = 2\sqrt{2} \angle 55^\circ$ .

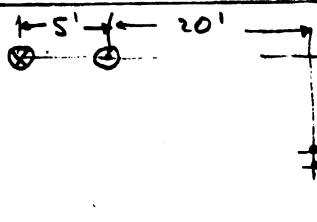
3.1   $\frac{\mu_0}{2\pi} = 2 \times 10^{-7}$ ,  $|I_a| = 100$

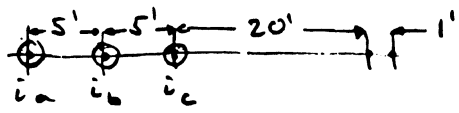
(Upward) flux linkages of telephone wires

$$\lambda = 2 \times 10^{-7} \left[ i_a \ln \frac{26}{25} - i_a \ln \frac{21}{20} \right] = -0.00957 \times 2 \times 10^{-7} i_a$$

$$v_{tel} = \frac{d\lambda}{dt} \Rightarrow V_{tel} = j\omega \times (-0.00957 \times 2 \times 10^{-7}) \times 100 \text{ V/meter}$$

Since 1 mile = 1.609 km,  $|V_{tel}| = 0.116 \text{ V/mile}$

3.2   $\lambda = 2 \times 10^{-7} i_a \left[ \ln \frac{\sqrt{25^2 + 11^2}}{\sqrt{25^2 + 10^2}} - \ln \frac{\sqrt{20^2 + 11^2}}{\sqrt{20^2 + 10^2}} \right]$   
 $= -0.066294 \times 2 \times 10^{-7} i_a$   
 $\Rightarrow |V_{tel}| = 0.076 \text{ V/mile}$

3.3 

$$\lambda = 2 \times 10^{-7} \left[ i_a \ln \frac{31}{30} + i_b \ln \frac{26}{25} + i_c \ln \frac{21}{20} \right]$$

$$= 2 \times 10^{-7} \left[ 0.0328 i_a + 0.0392 i_b + 0.0488 i_c \right]$$

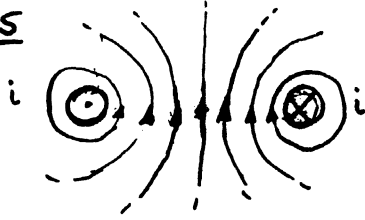
$$V_{tel} = j\omega \times 2 \times 10^{-7} \left[ 0.0328 I_a + 0.0392 I_b + 0.0488 I_c \right]$$

$$|V_{tel}| = 377 \times 2 \times 10^{-7} \times 100 \times \left| 0.0328 + 0.0392 \angle -120^\circ + 0.0488 \angle 120^\circ \right|$$

$$= 1048 \times 10^{-7} \text{ V/meter} = 0.1692 \text{ V/mile}$$

3.4 The telephone wires are transposed every 1000 ft. Cancellation occurs in all but 720' of line  $\Rightarrow 0.0158 \text{ V}$ .

3.5



The flux linkages due to the current in the left conductor is  $\approx$

$$\lambda_1 = \frac{\mu_0}{2\pi} i \left[ \frac{\mu_r}{4} + \ln \frac{D}{r} \right] = i \frac{\mu_0}{2\pi} \ln \frac{D}{r}$$

Here we are neglecting partial flux linkages of the far (right) conductor. The current in the right conductor contributes an equal number of flux linkages. Thus (at least approximately),

$$l = 2 \lambda_1 / i = \frac{\mu_0}{\pi} \ln \frac{D}{r} = 4 \times 10^{-7} \ln \frac{D}{r} \text{ H/meter}$$

3.6 If each conductor is hollow then there are no partial flux linkages and in (3.13) the term involving  $\mu_r$  is absent. Then  $r' = r$  and

$$l = \frac{\mu_0}{2\pi} \ln \frac{D}{r} \quad \text{H/m}$$

3.7 The hint is misleading. A better hint for combining the 7 (unequal) inductances  $l_1, l_2, \dots, l_7$  would be to use the average inductance i.e. let

$$l_a = \frac{l_{av}}{7} = \frac{l_1 + l_2 + \dots + l_7}{7^2}$$

Returning to the problem, paralleling the steps which lead to (3.27) we have

$$\begin{aligned} \lambda_1 &= \frac{\mu_0}{2\pi} \left[ \frac{i_a}{7} \left( \ln \frac{1}{d_{11}} + \ln \frac{1}{d_{12}} + \dots + \ln \frac{1}{d_{17}} \right) \right. \\ &\quad + \frac{i_b}{7} \left( \ln \frac{1}{d_{18}} + \ln \frac{1}{d_{19}} + \dots + \ln \frac{1}{d_{1,21}} \right) \\ &\quad \left. + \frac{i_c}{7} \left( \ln \frac{1}{d_{1,15}} + \ln \frac{1}{d_{1,16}} + \dots + \ln \frac{1}{d_{1,21}} \right) \right] \\ &\approx \frac{\mu_0}{2\pi} i_a \ln \frac{D}{(d_{11} \dots d_{17})^{1/7}} \Rightarrow l_1 = 7 \times \frac{\mu_0}{2\pi} \ln \frac{D}{(d_{11} \dots d_{17})^{1/7}} \end{aligned}$$