Chapter 2
Fundamentals

ANSWERS TO MULTIPLE-CHOICE TYPE QUESTIONS

2.1	b	**2.19**	a	
2.2	a	**2.20**	A. c	
2.3	c		B. a	
2.4	a		C. b	
2.5	b	**2.21**	a	
2.6	c	**2.22**	a	
2.7	a	**2.23**	b	
2.8	c	**2.24**	a	
2.9	a	**2.25**	a	
2.10	c	**2.26**	b	
2.11	a	**2.27**	a	
2.12	b	**2.28**	b	
2.13	b	**2.29**	a	
2.14	c	**2.30**	(i)	c
2.15	a		(ii)	b
2.16	b		(iii)	a
2.17	A. a		(iv)	d
	B. b	**2.31**	a	
	C. a	**2.32**	a	
2.18	c			

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2.1 (a) $\overline{A}_1 = 6\angle 30° = 6[\cos 30° + j\sin 30°] = 5.20 + j3$

(b) $\overline{A}_2 = -4 + j5 = \sqrt{16 + 25} \angle \tan^{-1}\dfrac{5}{-4} = 6.40\angle 128.66° = 6.40e^{j128.66°}$

(c) $\overline{A}_3 = (5.20 + j3) + (-4 + j5) = 1.20 + j8 = 8.01\angle 81.50°$

(d) $\overline{A}_4 = (6\angle 30°)(6.40\angle 128.66°) = 38.414\angle 158.658° = -35.78 + j13.98$

(e) $\overline{A}_5 = (6\angle 30°)/(6.40\angle -128.66°) = 0.94\angle 158.66° = 0.94e^{j158.66°}$

2.2 (a) $\overline{I} = 500\angle -30° = 433.01 - j250$

(b) $i(t) = 4\sin(\omega t + 30°) = 4\cos(\omega t + 30° - 90°) = 4\cos(\omega t - 60°)$

$\overline{I} = (4)\angle -60° = 2.83\angle -60° = 1.42 - j2.45$

(c) $\overline{I} = (5/\sqrt{2})\angle -15° + 4\angle -60° = (3.42 - j0.92) + (2 - j3.46)$

$= 5.42 - j4.38 = 6.964\angle -38.94°$

2.3 (a) $V_{\max} = 400\,\text{V}; I_{\max} = 100\,\text{A}$

(b) $V = 400/\sqrt{2} = 282.84\,\text{V}; I = 100/\sqrt{2} = 70.71\,\text{A}$

(c) $\overline{V} = 282.84\angle 30°\,\text{V}; \overline{I} = 70.71\angle -80°\,\text{A}$

2.4 (a) $\overline{I}_1 = 10\angle 0°\dfrac{-j6}{8 + j6 - j6} = 10\dfrac{6\angle -90°}{8} = 7.5\angle -90°\,\text{A}$

$\overline{I}_2 = \overline{I} - \overline{I}_1 = 10\angle 0° - 7.3\angle -90° = 10 + j7.5 = 12.5\angle 36.87°\,\text{A}$

$\overline{V} = \overline{I}_2(-j6) = (12.5\angle 36.87°)(6\angle -90°) = 75\angle -53.13°\,\text{V}$

(b)

(not to scale)
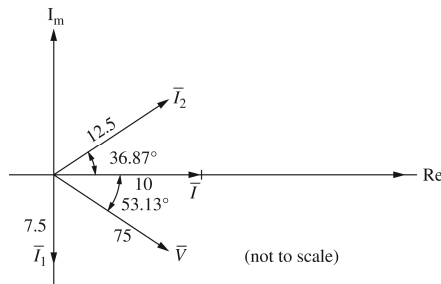
2.5 (a) $\upsilon(t) = 277\sqrt{2}\cos(\omega t + 30°) = 391.7\cos(\omega t + 30°)\,\text{V}$

(b) $\overline{I} = \overline{V}/20 = 13.85\angle 30°\,\text{A}$

$i(t) = 19.58\cos(\omega t + 30°)\,\text{A}$

(c) $\overline{Z} = j\omega L = j(2\pi 60)(10\times 10^{-3}) = 3.771\angle 90°\,\Omega$

$\overline{I} = \overline{V}/\overline{Z} = (277\angle 30°)/(3.771\angle 90°) = 73.46\angle -60°\,\text{A}$

$i(t) = 73.46\sqrt{2}\cos(\omega t - 60°) = 103.9\cos(\omega t - 60°)\,\text{A}$

(d) $\overline{\overline{Z}} = -j\,25\,\Omega$

$\overline{I} = \overline{V}/\overline{Z} = (277\angle 30°)/(25\angle -90°) = 11.08\angle 120°\,\text{A}$

$i(t) = 11.08\sqrt{2}\cos(\omega t + 120°) = 15.67\cos(\omega t + 120°)\,\text{A}$

2.6 (a) $\overline{V} = (75/\sqrt{2})\angle -15° = 53.03\angle -15°$; ω does not appear in the answer.

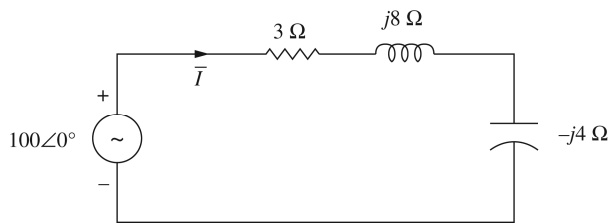(b) $\upsilon(t) = 50\sqrt{2}\cos(\omega t + 10°)$; with $\omega = 377$,

$\upsilon(t) = 70.71\cos(377t + 10°)$

(c) $\overline{A} = A\angle \alpha;\ \overline{B} = B\angle \beta;\ \overline{C} = \overline{A} + \overline{B}$

$c(t) = a(t) + b(t) = \sqrt{2}\,\text{Re}\left[\overline{C}e^{j\omega t}\right]$

The resultant has the same frequency ω.

2.7 (a) The circuit diagram is shown below:

(b) $\overline{\overline{Z}} = 3 + j8 - j4 = 3 + j4 = 5\angle 53.1°\,\Omega$

(c) $\overline{I} = (100\angle 0°)/(5\angle 53.1°) = 20\angle -53.1°\,\text{A}$

The current lags the source voltage by 53.1°
Power Factor $= \cos 53.1° = 0.6$ Lagging

2.8 $\overline{\overline{Z}}_{LT} = j(377)(30.6\times 10^{-6}) = j11.536\,\text{m}\Omega$

$\overline{\overline{Z}}_{LL} = j(377)(5\times 10^{-3}) = j1.885\,\Omega$

$\overline{\overline{Z}}_C = -j\dfrac{1}{(377)(921\times 10^{-6})} = -j2.88\,\Omega$

$\overline{V} = \dfrac{120\sqrt{2}}{\sqrt{2}}\angle -30°\,\text{V}$

The circuit transformed to phasor domain is shown below:
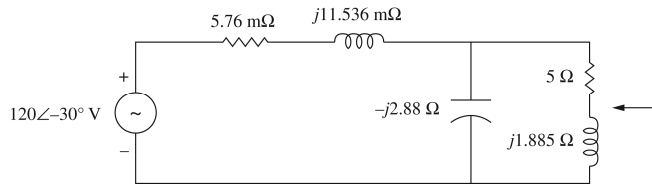
2.9 $KVL:$ $120\angle 0° = (60\angle 0°)(0.1 + j0.5) + \overline{V}_{LOAD}$

$$\therefore \ \overline{V}_{LOAD} = 120\angle 0° - (60\angle 0°)(0.1 + j0.5)$$

$$= 114.1 - j30.0 = 117.9\angle -14.7° \, V \ \leftarrow$$

2.10 (a) $p(t) = \upsilon(t)i(t) = \left[400\cos(\omega t + 30°)\right]\left[100\cos(\omega t - 80°)\right]$

$$= \frac{1}{2}(400)(100)\left[\cos110° + \cos(2\omega t - 50°)\right]$$

$$= -6840.4 + 2\times10^4\cos(2\omega t - 50°)\,W$$

  (b) $P = VI\cos(\delta - \beta) = (282.84)(70.71)\cos(30° + 80°)$

$$= -6840\,W \quad \text{Absorbed}$$

$$= +6840\,W \quad \text{Delivered}$$

  (c) $Q = VI\sin(\delta - \beta) = (282.84)(70.71)\sin110°$

$$= 18.79\,\text{kVAR Absorbed}$$

  (d) The phasor current $(-\overline{I}) = 70.71\angle -80° + 180° = 70.71\angle100°$ A leaves the positive
     terminal of the generator.
     The generator power factor is then $\cos(30° - 100°) = 0.3420$ leading

2.11 (a) $p(t) = \upsilon(t)i(t) = 391.7\times19.58\cos^2(\omega t + 30°)$

$$= 0.7669\times10^4\left(\frac{1}{2}\right)\left[1 + \cos(2\omega t + 60°)\right]$$

$$= 3.834\times10^3 + 3.834\times10^3\cos(2\omega t + 60°)\,W$$

$$P = VI\cos(\delta - \beta) = 277\times13.85\cos0° = 3.836\,\text{kW}$$

$$Q = VI\sin(\delta - \beta) = 0\,\text{VAR}$$

   Source Power Factor $= \cos(\delta - \beta) = \cos(30° - 30°) = 1.0$

  (b) $p(t) = \upsilon(t)i(t) = 391.7\times103.9\cos(\omega t + 30°)\cos(\omega t - 60°)$

$$= 4.07\times10^4\left(\frac{1}{2}\right)\left[\cos90° + \cos(2\omega t - 30°)\right]$$

$$= 2.035\times10^4\cos(2\omega t - 30°)\,W$$

$$P = VI\cos(\delta - \beta) = 277\times73.46\cos(30° + 60°) = 0\,W$$

$$Q = VI\sin(\delta - \beta) = 277\times73.46\sin90° = 20.35\,\text{kVAR}$$

$$pf = \cos(\delta - \beta) = 0 \text{ Lagging}$$

(c) $p(t) = \upsilon(t)i(t) = 391.7 \times 15.67\cos(\omega t + 30°)\cos(\omega t + 120°)$

$$= 6.138 \times 10^3 \left(\frac{1}{2}\right)\left[\cos(-90°) + \cos(2\omega t + 150°)\right] = 3.069 \times 10^3 \cos(2\omega t + 150°)\,\text{W}$$

$P = VI\cos(\delta - \beta) = 277 \times 11.08\cos(30° - 120°) = 0\,\text{W}$

$Q = VI\sin(\delta - \beta) = 277 \times 11.08\sin(-90°)$

$= -3.069\,\text{kVAR Absorbed} = +3.069\,\text{kVAR Delivered}$

$pf = \cos(\delta - \beta) = \cos(-90°) = 0\,\text{Leading}$

2.12 (a) $p_R(t) = (359.3\cos\omega t)(35.93\cos\omega t)$

$\qquad = 6455 + 6455\cos 2\omega t\,\text{W}$

(b) $p_x(t) = (359.3\cos\omega t)\left[14.37\cos(\omega t + 90°)\right]$

$\qquad = 2582\cos(2\cot + 90°)$

$\qquad = -2582\sin 2\omega t\,\text{W}$

(c) $P = V^2/R = \left(359.3/\sqrt{2}\right)^2 \big/ 10 = 6455\,\text{W Absorbed}$

(d) $Q = V^2/X = \left(359.3/\sqrt{2}\right)^2 \big/ 25 = 2582\,\text{VARS Delivered}$

(e) $(\beta - \delta) = \tan^{-1}(Q/P) = \tan^{-1}(2582/6455) = 21.8°$

$\qquad$ Power factor $= \cos(\delta - \beta) = \cos(21.8°) = 0.9285\,\text{Leading}$

2.13 $\bar{Z} = R - jx_c = 10 - j25 = 26.93\angle -68.2°\,\Omega$

$i(t) = (359.3/26.93)\cos(\omega t + 68.2°)$

$\qquad = 13.34\cos(\omega t + 68.2°)\,\text{A}$

(a) $p_R(t) = \left[13.34\cos(\omega t + 68.2°)\right]\left[133.4\cos(\omega t + 68.2°)\right]$

$\qquad = 889.8 + 889.8\cos\left[2(\omega t + 68.2°)\right]\text{W}$

(b) $p_x(t) = \left[13.34\cos(\omega t + 68.2°)\right]\left[333.5\cos(\omega t + 68.2° - 90°)\right]$

$\qquad = 2224\sin\left[2(\omega t + 68.2°)\right]\text{W}$

(c) $P = I^2R = \left(13.34/\sqrt{2}\right)^2 10 = 889.8\,\text{W}$

(d) $Q = I^2X = \left(13.34/\sqrt{2}\right)^2 25 = 2224\,\text{VARS}$

(e) $pf = \cos\left[\tan^{-1}(Q/P)\right] = \cos\left[\tan^{-1}(2224/889.8)\right]$

$\qquad = 0.3714\,\text{Leading}$

2.14 (a) $\overline{I} = 2\angle 0°\,\text{kA}$

$$\overline{V} = \overline{Z}\,\overline{I} = (3\angle -45°)(2\angle 0°) = 6\angle -45°\,\text{kV}$$

$$\upsilon(t) = 6\sqrt{2}\cos(\omega t - 45°)\,\text{kV}$$

$$p(t) = \upsilon(t)i(t) = \left[6\sqrt{2}\cos(\omega t - 45°)\right]\left[2\sqrt{2}\cos\omega t\right]$$

$$= 24\left(\frac{1}{2}\right)\left[\cos(-45°) + \cos(2\omega t - 45°)\right]$$

$$= 8.49 + 12\cos(2\omega t - 45°)\,\text{MW}$$

(b) $P = VI\cos(\delta - \beta) = 6 \times 2\cos(-45° - 0°) = 8.49\,\text{MW Delivered}$

(c) $Q = VI\sin(\delta - \beta) = 6 \times 2\sin(-45° - 0°)$

$$= -8.49\,\text{MVAR Delivered} = +8.49\,\text{MVAR Absorbed}$$

(d) $pf = \cos(\delta - \beta) = \cos(-45° - 0°) = 0.707\,\text{Leading}$

2.15 (a) $\overline{I} = \left[\left(4/\sqrt{2}\right)\angle 60°\right]\Big/(2\angle 30°) = \sqrt{2}\,\angle 30°\,\text{A}$

$$i(t) = 2\cos(\omega t + 30°)\,\text{A with } \omega = 377\,\text{rad/s}$$

$$p(t) = \upsilon(t)i(t) = 4\left[\cos 30° + \cos(2\omega t + 90°)\right]$$

$$= 3.46 + 4\cos(2\omega t + 90°)\,\text{W}$$

(b) $\upsilon(t)$, $i(t)$, and $p(t)$ are plotted below.

(c) The instantaneous power has an average value of 3.46 W, and the frequency is <u>twice</u> that of the voltage or current.
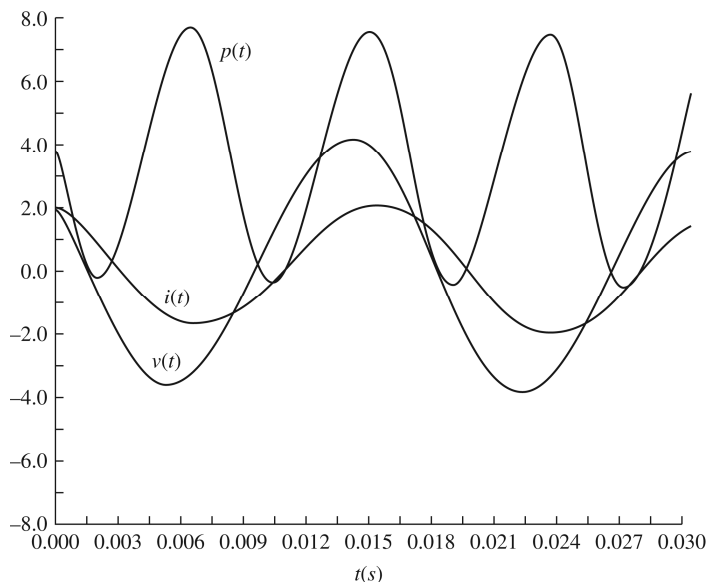
2.16 (a) $\bar{Z} = 10 + j120\pi \times 0.04 = 10 + j15.1 = 18.1\angle56.4°\,\Omega$

$pf = \cos56.4° = 0.553$ Lagging

(b) $\bar{V} = 120\angle0°\,\text{V}$

The current supplied by the source is

$\bar{I} = (120\angle0°)/(18.1\angle56.4°) = 6.63\angle-56.4°\,\text{A}$

The real power absorbed by the load is given by

$P = 120 \times 6.63 \times \cos56.4° = 440\,\text{W}$

which can be checked by $I^2R = (6.63)^2\,10 = 440\,\text{W}$

The reactive power absorbed by the load is

$Q = 120 \times 6.63 \times \sin36.4° = 663\,\text{VAR}$

(c) Peak Magnetic Energy $= W = LI^2 = 0.04(6.63)^2 = 1.76\,\text{J}$

$Q = \omega W = 377 \times 1.76 = 663\,\text{VAR}$ is satisfied.

2.17 (a) $\bar{S} = \bar{V}\bar{I}^* = \bar{Z}\bar{I}\bar{I}^* = \bar{Z}|\bar{I}|^2 = j\omega LI^2$

$Q = \text{Im}[\bar{S}] = \omega LI^2$ ←

(b) $\upsilon(t) = L\dfrac{di}{dt} = -\sqrt{2}\omega L\,I\sin(\omega t + \theta)$

$p(t) = \upsilon(t)\cdot i(t) = -2\omega L\,I^2\sin(\omega t + \theta)\cos(\omega t + \theta)$

$\qquad = -\omega L\,I^2\sin2(\omega t + \theta)$ ←

$\qquad = -Q\sin2(\omega t + \theta)$ ←

Average real power P supplied to the inductor $= 0$ ←

Instantaneous power supplied (to sustain the changing energy in the magnetic field) has a maximum value of Q. ←

2.18 (a) $\bar{S} = \bar{V}\bar{I}^* = \bar{Z}\bar{I}\bar{I}^* = \text{Re}\left[\bar{Z}I^2\right] + j\,\text{Im}\left[\bar{Z}I^2\right]$

$\qquad = P + jQ$

$\therefore P = \bar{Z}I^2\cos\angle\bar{Z}; Q = \bar{Z}I^2\sin\angle\bar{Z}$ ←

(b) Choosing $i(t) = \sqrt{2}\,I\cos\omega t,$

Then $\upsilon(t) = \sqrt{2}\bar{Z}\,I\cos(\omega t + \angle\bar{Z})$

$\therefore p(t) = \upsilon(t)\cdot i(t) = \bar{Z}\,I^2\cos(\omega t + \angle\bar{Z})\cdot\cos\omega t$

$\qquad = \bar{Z}\,I^2\left[\cos\angle\bar{Z} + \cos(2\omega t + \angle\bar{Z})\right]$

$\qquad = \bar{Z}\,I^2\left[\cos\angle\bar{Z} + \cos2\omega t\cos\angle\bar{Z} - \sin2\omega t\sin\angle\bar{Z}\right]$

$\qquad = P(1 + \cos2\omega t) - Q\sin2\omega t$ ←

(c) $\bar{Z} = R + j\omega L + \dfrac{1}{j\omega C}$

From part (a), $P = RI^2$ and $Q = Q_L + Q_C$

where $Q_L = \omega L I^2$ and $Q_C = -\dfrac{1}{\omega C}I^2$

which are the reactive powers into L and C, respectively.

Thus $p(t) = P(1 + \cos 2\omega t) - Q_L \sin 2\omega t - Q_C \sin 2\omega t$ ←

If $\omega^2 LC = 1$, $\quad Q_L + Q_C = Q = 0$

$\qquad$ Then $\quad p(t) = P(1 + \cos 2\omega t)$ $\Big\}$ ←

2.19 (a) $\bar{S} = \bar{V}\bar{I}^* = \left(\dfrac{150}{\sqrt{2}}\angle 10°\right)\left(\dfrac{5}{\sqrt{2}}\angle -50°\right)^* = 375\angle 60°$

$\qquad = 187.5 + j324.8$

$\quad P = \operatorname{Re}\bar{S} = 187.5\,\text{W Absorbed}$

$\quad Q = \operatorname{Im}\bar{S} = 324.8\,\text{VAR S Absorbed}$

(b) $pf = \cos(60°) = 0.5$ Lagging

(c) $Q_S = P\tan Q_S = 187.5\tan\left[\cos^{-1}0.9\right] = 90.81\,\text{VAR S}$

$\quad Q_C = Q_L - Q_S = 324.8 - 90.81 = 234\,\text{VAR S}$
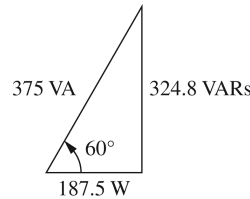
2.20 $\bar{Y}_1 = \dfrac{1}{\bar{Z}_1} = \dfrac{1}{20\angle 30°} = 0.05\angle -30° = (0.0433 - j0.025)\text{S} = G_1 - jB_1$
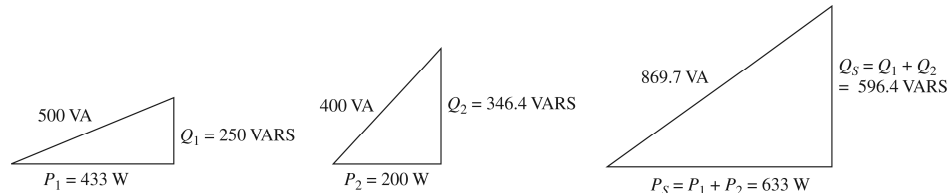
$\bar{Y}_2 = \dfrac{1}{\bar{Z}_2} = \dfrac{1}{25\angle 60°} = 0.04\angle -60° = (0.02 - j0.03464)\text{S} = G_2 + jB_2$

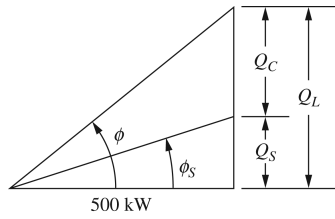$P_1 = V^2 G_1 = (100)^2\,0.0433 = 433\,\text{W Absorbed}$

$Q_1 = V^2 B_1 = (100)^2\,0.025 = 250\,\text{VAR S Absorbed}$

$P_2 = V^2 G_2 = (100)^2\,0.02 = 200\,\text{W Absorbed}$

$Q_2 = V^2 B_2 = (100)^2\,0.03464 = 346.4\,\text{VAR S Absorbed}$

2.21 (a)

$$\phi_L = \cos^{-1} 0.6 = 53.13°$$
$$Q_L = P\tan\phi_L = 500\tan 53.13° = 666.7\,\text{kVAR}$$
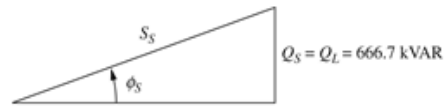$$\phi_S = \cos^{-1} 0.9 = 25.84°$$
$$Q_S = P\tan\phi_S = 500\tan 25.84° = 242.2\,\text{kVAR}$$
$$Q_C = Q_L - Q_S = 666.7 - 242.2 = 424.5\,\text{kVAR}$$
$$S_C = Q_C = 424.5\,\text{kVA}$$

(b) The synchronous motor absorbs $P_m = \dfrac{375}{0.9} = 416.7\,\text{kW}$ and $Q_m = 0\,\text{kVAR}$

$P_S = P + P_m = 500 + 416.7 = 916.7$ kW

Source $\text{PF} = \cos\left[\tan^{-1}\left(666.7/916.7\right)\right] = 0.809$ Lagging

2.22 (a) $\overline{Y}_1 = \dfrac{1}{\overline{Z}_1} = \dfrac{1}{(4+j5)} = \dfrac{1}{6.4\angle 51.34°} = 0.16\angle -51.34°$
$$= (0.1 - j0.12)\text{S}$$

$$\overline{Y}_2 = \dfrac{1}{\overline{Z}_2} = \dfrac{1}{10} = 0.1\text{S}$$

$$P = V^2(G_1 + G_2) \Rightarrow V = \sqrt{\dfrac{P}{G_1+G_2}} = \sqrt{\dfrac{1000}{(0.1+0.1)}} = 70.71\text{ V}$$

$$P_1 = V^2 G_1 = (70.71)^2\, 0.1 = 500\,\text{W}$$

$$P_2 = V^2 G_2 = (70.71)^2\, 0.1 = 500\,\text{W}$$

(b) $\overline{Y}_{eq} = \overline{Y}_1 + \overline{Y}_2 = (0.1 - j0.12) + 0.1 = 0.2 - j0.12$
$$= 0.233\angle -30.96°\text{ S}$$

$$I_S = V\, Y_{eq} = 70.71(0.233) = 16.48\,\text{A}$$

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2.23 $\overline{S} = \overline{V}\,\overline{I}^* = (120\angle 0°)(15\angle -30°) = 1800\angle -30°$

$$= 1558.85 - j900$$

$P = \mathrm{Re}\,\overline{S} = 1558.85\,\mathrm{W}\ \text{Delivered}$

$Q = \mathrm{Im}\,\overline{S} = -900\,\mathrm{VARS}\ \text{Delivered} = +900\,\mathrm{VARS}\ \text{Absorbed}$

2.24 $\overline{S}_1 = P_1 + jQ_1 = 10 + j0;\ \overline{S}_2 = 10\angle\cos^{-1}0.9 = 9 + j4.359$

$\overline{S}_3 = \dfrac{7.5}{0.85\times 0.95}\angle -\cos^{-1}0.95 = 9.288\angle -18.19° = 8.824 - j2.899$

$\overline{S}_S = \overline{S}_1 + \overline{S}_2 + \overline{S}_3 = 27.82 + j1.460 = 27.86\angle 3.00°$

$P_S = \mathrm{Re}(\overline{S}_S) = 27.82\,\mathrm{kW}$

$Q_S = \mathrm{Im}(\overline{S}_S) = 1.460\,\mathrm{kVAR}$

$S_S = \left|\overline{S}_S\right| = 27.86\,\mathrm{kVA}$

2.25 $\overline{S}_R = \overline{V}_R\overline{I}^* = R\overline{I}\,\overline{I}^* = I^2R = (20)^2\,3 = 1200 + j0$

$\overline{S}_L = \overline{V}_L\overline{I}^* = (jX_L\overline{I})\overline{I}^* = jX_L I^2 = j8(20)^2 = 0 + j3200$

$\overline{S}_C = \overline{V}_C\overline{I}^* = (-j\overline{I}X_C)\overline{I}^* = -jX_C I^2 = -j4(20)^2 = 0 - j1600$

Complex power absorbed by the total load $\overline{S}_{LOAD} = \overline{S}_R + \overline{S}_L + \overline{S}_C = 2000\angle 53.1°$

Power Triangle:
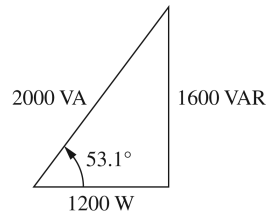
Complex power delivered by the source is

$\overline{S}_{SOURCE} = \overline{V}\,\overline{I}^* = (100\angle 0°)(20\angle -53.1°)^* = 2000\angle 53.1°$

The complex power delivered by the source is equal to the total complex power absorbed by the load.

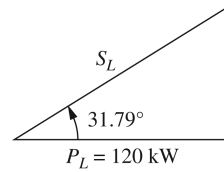**2.26** (a) The problem is modeled as shown in figure below:
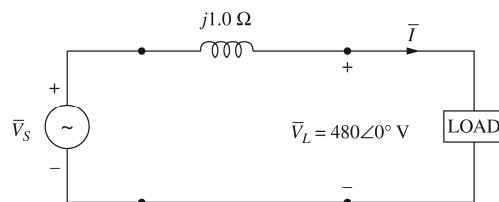
$P_L = 120\,\mathrm{kW}$

$pf_L = 0.85\,\text{Lagging}$

$\theta_L = \cos^{-1}0.85 = 31.79°$

Power triangle for the load:

$$\overline{S}_L = P_L + jQ_L = 141.18\angle 31.79° \text{ kVA}$$
$$I = S_L / V = 141,180 / 480 = 294.13 \text{ A}$$

$$Q_L = P_L \tan(31.79°)$$
$$= 74.364 \text{ kVAR}$$

Real power loss in the line is zero.

Reactive power loss in the line is $Q_{LINE} = I^2 X_{LINE} = (294.13)^2 1$
$$= 86.512 \text{ kVAR}$$

$$\therefore \overline{S}_S = P_S + jQ_S = 120 + j(74.364 + 86.512) = 200.7\angle 53.28° \text{ kVA}$$

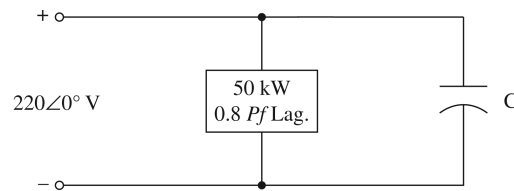The input voltage is given by $V_S = S_S / I = 682.4 \text{ V (rms)}$

The power factor at the input is $\cos 53.28° = 0.6$ Lagging

(b) Applying KVL, $\overline{V}_S = 480\angle 0° + j1.0(294.13\angle -31.79°)$
$$= 635 + j250 = 682.4\angle 21.5° \text{ V (rms)}$$
$$(pf)_S = \cos(21.5° + 31.79°) = 0.6 \text{ Lagging}$$

2.27 The circuit diagram is shown below:

$$P_{old} = 50 \text{ kW}; \cos^{-1} 0.8 = 36.87°; \theta_{OLD} = 36.87°; Q_{old} = P_{old} \tan(\theta_{old})$$
$$= 37.5 \text{ kVAR}$$

$$\therefore \overline{S}_{old} = 50,000 + j37,500$$

$$\theta_{new} = \cos^{-1} 0.95 = 18.19°; \overline{S}_{new} = 50,000 + j50,000 \tan(18.19°)$$
$$= 50,000 + j16,430$$

Hence $\overline{S}_{cap} = \overline{S}_{new} - \overline{S}_{old} = -j21,070 \text{ VA}$

$$\therefore C = \frac{21,070}{(377)(220)^2} = 1155 \mu\text{F} \leftarrow$$

2.28 $\bar{S}_1 = 15 + j6.667$

$\bar{S}_2 = 3(0.96) - j3\left[\sin\left(\cos^{-1} 0.96\right)\right] = 2.88 - j0.84$

$\bar{S}_3 = 15 + j0$

$\bar{S}_{TOTAL} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = (32.88 + j5.827)\,\text{kVA}$

(i) Let $\bar{Z}$ be the impedance of a series combination of R and X

Since $\bar{S} = \bar{V}\bar{I}^* = \bar{V}\left(\dfrac{\bar{V}}{\bar{Z}}\right)^* = \dfrac{V^2}{\bar{Z}^*}$, it follows that

$$\bar{Z}^* = \frac{V^2}{\bar{S}} = \frac{(240)^2}{(32.88 + j5.827)10^3} = (1.698 - j0.301)\,\Omega$$

$$\therefore \bar{Z} = (1.698 + j0.301)\,\Omega \quad \leftarrow$$

(ii) Let $\bar{Z}$ be the impedance of a parallel combination of R and X

Then $R = \dfrac{(240)^2}{(32.88)10^3} = 1.7518\,\Omega$

$X = \dfrac{(240)^2}{(5.827)10^3} = 9.885\,\Omega$

$$\therefore \bar{Z} = \left(1.7518 \,\|\, j9.885\right)\Omega \quad \leftarrow$$

2.29   Since complex powers satisfy KCL at each bus, it follows that

$$\bar{S}_{13} = (1 + j1) - (1 - j1) - (0.4 + j0.2) = -0.4 + j1.8 \quad \leftarrow$$

$$\bar{S}_{31} = -\bar{S}_{13}^* = 0.4 + j1.8 \quad \leftarrow$$

Similarly, $\bar{S}_{23} = (0.5 + j0.5) - (1 + j1) - (-0.4 + j0.2) = -0.1 - j0.7 \quad \leftarrow$

$$\bar{S}_{32} = -\bar{S}_{23}^* = 0.1 - j0.7 \quad \leftarrow$$

At Bus 3, $\bar{S}_{G3} = \bar{S}_{31} + \bar{S}_{32} = (0.4 + j1.8) + (0.1 - j0.7) = 0.5 + j1.1 \quad \leftarrow$

2.30   (a) For load 1: $\theta_1 = \cos^{-1}(0.28) = 73.74°$ Lagging

$\bar{S}_1 = 125\angle 73.74° = 35 + j120$

$\bar{S}_2 = 10 - j40$

$\bar{S}_3 = 15 + j0$

$\bar{S}_{TOTAL} = \bar{S}_1 + \bar{S}_2 + \bar{S}_3 = 60 + j80 = 100\angle 53.13°\,\text{kVA} = P + jQ$

$\therefore P_{TOTAL} = 60\,\text{kW}; Q_{TOTAL} = 80\,\text{kVAR}; \text{kVA}_{TOTAL} = S_{TOTAL} = 100\,\text{kVA}. \quad \leftarrow$

Supply $pf = \cos(53.13°) = 0.6$ Lagging $\quad \leftarrow$

(b) $\bar{I}_{TOTAL} = \dfrac{\bar{S}^*}{\bar{V}^*} = \dfrac{100 \times 10^3 \angle -53.13°}{1000\angle 0°} = 100\angle -53.13°\,\text{A}$

At the new *pf* of 0.8 lagging, P_{TOTAL} of 60kW results in the new reactive power Q', such that

$$\theta' = \cos^{-1}(0.8) = 36.87°$$

and $Q' = 60\tan(36.87°) = 45\,\text{kVAR}$

∴ The required capacitor's kVAR is $Q_C = 80 - 45 = 35\,\text{kVAR}$ ←

It follows then $X_C = \dfrac{V^2}{\overline{S}_C^*} = \dfrac{(1000)^2}{j35000} = -j\,28.57\,\Omega$

and $\qquad C = \dfrac{10^6}{2\pi(60)(28.57)} = 92.85\,\mu\text{F}$ ←

The new current is $I' = \dfrac{\overline{S}'^*}{\overline{V}^*} = \dfrac{60,000 - j45,000}{1000\angle 0°} = 60 - j45 = 75\angle -36.87°\,\text{A}$

The supply current, in magnitude, is reduced from 100A to 75A ←

2.31 (a) $\overline{I}_{12} = \dfrac{V_1\angle\delta_1 - V_2\angle\delta_2}{X\angle 90°} = \left(\dfrac{V_1}{X}\angle\delta_1 - 90°\right) - \dfrac{V_2}{X}\angle\delta_2 - 90°$

Complex power $\overline{S}_{12} = \overline{V}_1\overline{I}_{12}^* = V_1\angle\delta_1\left[\dfrac{V_1}{X}\angle 90° - \delta_1 - \dfrac{V_2}{X}\angle 90° - \delta_2\right]$

$\qquad\qquad = \dfrac{V_1^2}{X}\angle 90° - \dfrac{V_1V_2}{X}\angle 90° + \delta_1 - \delta_2$

∴ The real and reactive power at the sending end are

$$P_{12} = \dfrac{V_1^2}{X}\cos 90° - \dfrac{V_1V_2}{X}\cos(90° + \delta_1 - \delta_2)$$

$$= \dfrac{V_1V_2}{X}\sin(\delta_1 - \delta_2) \leftarrow$$

$$Q_{12} = \dfrac{V_1^2}{X}\sin 90° - \dfrac{V_1V_2}{X}\sin(90° + \delta_1 - \delta_2)$$

$$= \dfrac{V_1}{X}\left[V_1 - V_2\cos(\delta_1 - \delta_2)\right] \leftarrow$$

Note: If $\overline{V}_1$ leads $\overline{V}_2$, $\delta = \delta_1 - \delta_2$ is positive and the real power flows from node 1 to node 2. If $\overline{V}_1$ Lags $\overline{V}_2$, δ is negative and power flows from node 2 to node 1.

(b) Maximum power transfer occurs when $\delta = 90° = \delta_1 - \delta_2$ ←

$$P_{\text{MAX}} = \dfrac{V_1V_2}{X} \leftarrow$$

2.32 4 Mvar minimizes the real power line losses, while 4.5 Mvar minimizes the MVA power flow into the feeder.

2.33

Qcap	MW Losses	Mvar Losses
0	0.42	0.84
0.5	0.4	0.8
1	0.383	0.766
1.5	0.369	0.738
2	0.357	0.714
2.5	0.348	0.696
3	0.341	0.682
3.5	0.337	0.675
4	0.336	0.672
4.5	0.337	0.675
5	0.341	0.682
5.5	0.348	0.696
6	0.357	0.714
6.5	0.369	0.738
7	0.383	0.766
7.5	0.4	0.801
8	0.42	0.84
8.5	0.442	0.885
9	0.467	0.934
9.5	0.495	0.99
10	0.525	1.05

2.34 7.5 Mvars

2.35

(Admittance values are shown)
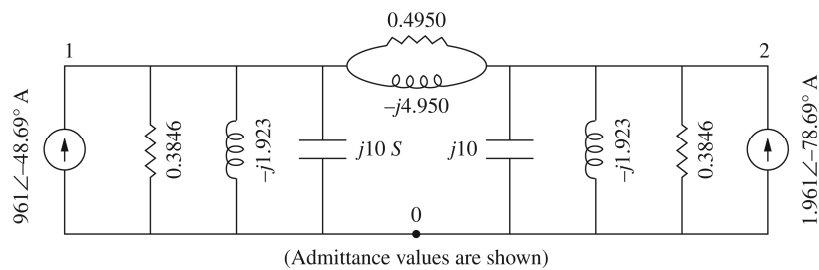
$$\begin{bmatrix} \dfrac{(.3846+.4950)+j(10-1.923-4.950)}{-(.4950-j4.950)} & \dfrac{-(.4950-j4.950)}{(.3846+.4950)+j(10-1.923-4.95)} \end{bmatrix}\begin{bmatrix} \overline{V}_{10} \\ \overline{V}_{20} \end{bmatrix}$$

$$=\begin{bmatrix} 1.961\angle-48.69° \\ 1.961\angle-78.69° \end{bmatrix}$$

$$\begin{bmatrix} 0.8796+j3.127 & -0.4950+j4.950 \\ -0.4950+j4.950 & -0.8796+j3.127 \end{bmatrix}\begin{bmatrix} \overline{V}_{10} \\ \overline{V}_{20} \end{bmatrix}=\begin{bmatrix} 1.961\angle-48.69° \\ 1.961\angle-78.69° \end{bmatrix}$$

2.36 Note that there are two buses plus the reference bus and one line for this problem. After converting the voltage sources in Fig. 2.29 to current sources, the equivalent source impedances are:

$$\overline{Z}_{S1} = \overline{Z}_{S2} = (0.1+j0.5)//(-j0.1) = \frac{(0.1+j0.5)(-j0.1)}{0.1+j0.5-j0.1}$$

$$= \frac{(0.5099\angle78.69°)(0.1\angle-90°)}{0.4123\angle75.96°} = 0.1237\angle-87.27°$$

$$= 0.005882 - j0.1235\ \Omega$$

The rest is left as an exercise to the student.

2.37 After converting impedance values in Figure 2.30 to admittance values, the bus admittance matrix is:

$$\overline{Y}_{bus} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & \left(1+\dfrac{1}{2}+\dfrac{1}{3}+\dfrac{1}{4}-j1\right) & -\left(\dfrac{1}{3}-j1\right) & -\left(\dfrac{1}{4}\right) \\ 0 & -\left(\dfrac{1}{3}-j1\right) & \left(\dfrac{1}{3}-j1+j\dfrac{1}{4}+j\dfrac{1}{2}\right) & -\left(j\dfrac{1}{4}\right) \\ 0 & -\left(\dfrac{1}{4}\right) & -\left(j\dfrac{1}{4}\right) & \left(\dfrac{1}{4}+j\dfrac{1}{4}-j\dfrac{1}{3}\right) \end{bmatrix}$$

Writing nodal equations by inspection:

$$\begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & (2.083-j1) & (-0.3333+j1) & -0.25 \\ 0 & (-0.3333+j1) & (0.3333-j0.25) & -j0.25 \\ 0 & (-0.25) & -j0.25 & (0.25-j0.08333) \end{bmatrix}\begin{bmatrix} \overline{V}_{10} \\ \overline{V}_{20} \\ \overline{V}_{30} \\ \overline{V}_{40} \end{bmatrix}=\begin{bmatrix} 1\angle0° \\ 0 \\ 0 \\ 2\angle30° \end{bmatrix}$$