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CHAPTER 1

MATLAB as our computation tool Solutions

1.1

(d) See Fig. 1.1.

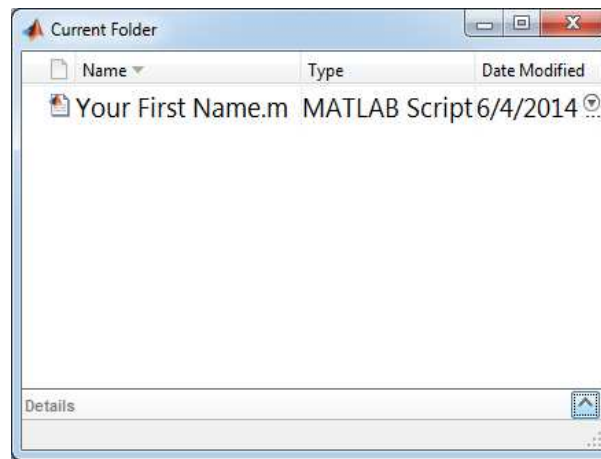


Figure 1.1: Screen shot for Problem 1.1(d)

1.2

- (a) `>> z = [1/6 3/4 2/9]`
- (b) `format` command can be used to change the format and the spacing of the numeric output display in the command window.
- (c) To print the complete contents of the Command Window, select **File** → **Print**. To print only a selection, first highlight the selection in the Command Window and then select **File** → **Print Selection**.

- (d) `format rat`, `format short`, `format long`, `format short e` are some format commands. More options are available in MATLAB help. `format loose`, `format compact` can be used to change spacing of the printed output.
- (e) `% clear workspace`
`clear;`
`clc;`
`% define array`
`z = [1/6 3/4 2/9];`
`% show ans in the command window`
`display(z);`
- `format compact`
`display(z);`
- `format rat`
`display(z);`
- `format long`
`display(z);`
- `format short e`
`display(z);`
-

1.3

- (a) $A = [1 \ 2; 3 \ 4; 5 \ 6; 7 \ 8]$
 $B = [1 \ 2 \ 3 \ 4; 5 \ 6 \ 7 \ 8]$
- (b) Highlight the desired lines, right click and select **Create M-file**. See the M file in part (f).
- (c) `clear;clc;`
`A = [`
`1 2;`
`3 4;`
`5 6;`
`7 8`
`];`
`B = [`
`1 2 3 4;`
`5 6 7 8`
`];`
`display(A*B);`
`display(B*A);`
- Note that $AB \neq BA$. Matrix multiplication is generally not commutative.

- (e) See M file in part (f).

```
A = [
0 1;
9 -1;
3 2;
1 0
```

```

];
B = [
    3    -1     0     2;
    5     4     2     1
];
disp(A*B);
disp(B*A);

```

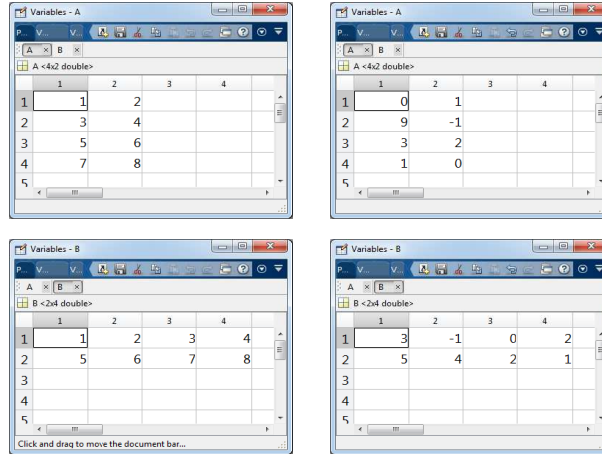


Figure 1.2: Screen shot for 1.3(f)

- (f) See Fig. 1.2 for screen shots of the array editor before and after modification. The M-file before modification is given below:

```

A=[1 2;3 4;5 6;7 8]
B=[1 2 3 4;5 6 7 8]
AB=A*B
BA=B*A

```

The M-file after modification of A and B is given below:

```

A=[0 1;9 -1;3 2;1 0]
B=[3 -1 0 2;5 4 2 1]
AB=A*B
BA=B*A

```

1.4

- (a) `clear;clc;close all`
`n = 100;`
`x = 0:10/n:10;`
- (b) `y1 = sin(x);`
`y2 = exp(x);`
`y3 = x.^2+2*x+1;`
`y4 = x.^3+5;`

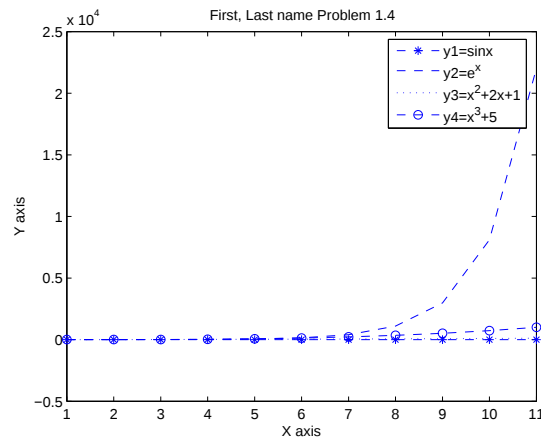


Figure 1.3: Plot of the 4 functions for Problem 1.4 (e)

```
(c) save loadmat.mat
(d) load loadmat.mat
(e) h = sprintf('%1.0f points', n);
    figure(1);
    hold on;
    plot(x,y1,'o-');
    xlabel('x');
    ylabel('y1');
    legend(h);
    title('Plot of y1=sinx');
    figure(2);
    plot(x,y2,':');
    xlabel('x');
    ylabel('y2');
    legend(h);
    title('Plot of y2=e^x');
    figure(3);
    plot(x,y3,'-');
    xlabel('x');
    ylabel('y3');
    legend(h);
    title('Plot of y3=x^2+2*x+1');
    figure(4);
    plot(x,y4,'*-');
    xlabel('x');
    ylabel('y4');
    legend(h);
    title('Plot of y4=x^3+5');
    See the plot in Fig. 1.3.
```

(f) The number of elements in arrays x and y govern the smoothness of the curve. As we include more and more points in the array, the curve obtained will be smoother. See the plots in Figure

1.4.

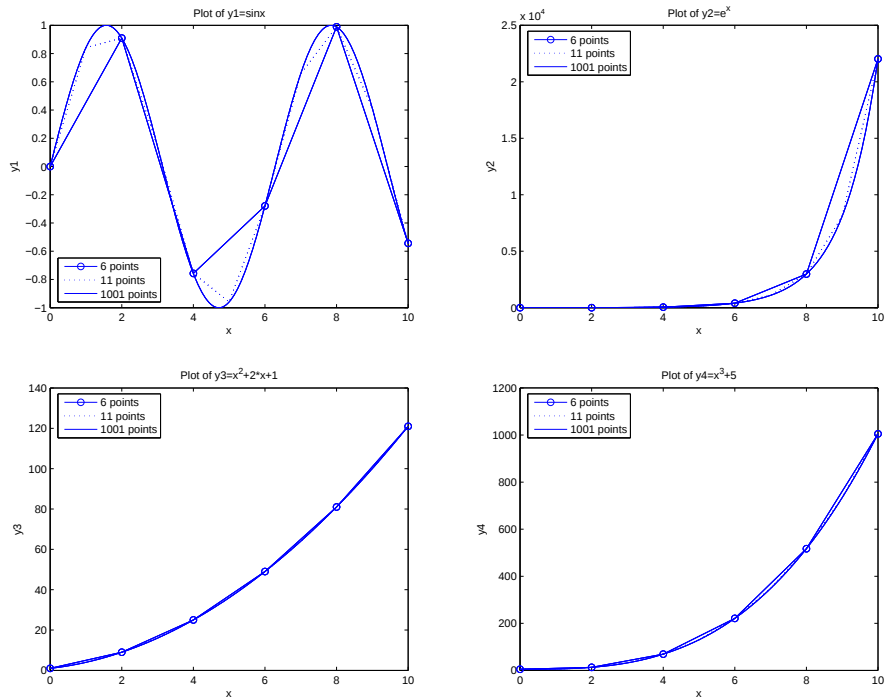


Figure 1.4: Problem 1.4

1.5

- (a) $A = \begin{bmatrix} 2 & 4 & 6 \\ 3 & 5 & 1 \\ 7 & 5 & 9 \end{bmatrix};$
 $B = \begin{bmatrix} 1 & 3 & 6 \end{bmatrix};$
 $C = \begin{bmatrix} 5 & 7 & 2 & 0 \end{bmatrix};$
 $F = \begin{bmatrix} 3 & 17 & 12 & -2 \end{bmatrix};$
 $H = \begin{bmatrix} 5 & 7 & 4 & -2 & 3 & 12 & -6 & 14 \end{bmatrix};$
 $D = \begin{bmatrix} A & B \end{bmatrix};$

$D =$

2	4	6
3	5	1
7	5	9
1	3	6

- (b) $E = \begin{bmatrix} D & C \end{bmatrix};$
 $E =$
- | | | | |
|---|---|---|---|
| 2 | 4 | 6 | 5 |
| 3 | 5 | 1 | 7 |
| 7 | 5 | 9 | 2 |
| 1 | 3 | 6 | 0 |

```
(c) det(E)
ans =
    470

(d) inv(E)
ans =
   -0.0702   -0.0064    0.1979   -0.2255
   -0.4191    0.3255   -0.0915    0.5021
    0.2213   -0.1617    0.0128   -0.0468
    0.2979   -0.0638   -0.0213   -0.2553

(e) E'
ans =
     2     3     7     1
     4     5     5     3
     6     1     9     6
     5     7     2     0

(h) E*E
ans =
    63    73   100    50
    35    63    74    52
    94   104   140    88
    53    49    63    38

F*F
not possible, matrix dimensions do not agree.

H*H
not possible, matrix dimensions do not agree.

E*F
ans =
   136
    92
   210
   126

F*E
not possible, matrix dimensions do not agree.

H*E
ans =
    57    69    61    82
    14    84    60    87

E*H
not possible, matrix dimensions do not agree.

F*H
not possible, matrix dimensions do not agree.

H*F
ans =
   186
   113
```

1.6

```
clear;clc;
A=[2 4 6; 3 5 1; 7 5 9];
[V, e] = eig(A);
display(e);      % eigenvalue (diagonal entries)
display(V);      % eigenvector (per column)
```

The diagonal elements of d denote the eigen values, and the columns of v denote the eigen vectors of A .

1.7

```
(a) clear;clc;
A=[12 14 16 40; 32 15 11 1; 7 25 19 10];
B=[9 1 36 4; 19 0 -31 2];
C=[7; 5; 7; 2; 0];
F=[16; 3; 17; 12;-2];
H=[5 7 4 -2 -1; -9 3 12 -6 14];
D=[A;B];
D =
    12    14    16    40
    32    15    11     1
     7    25    19    10
     9     1     3     6
    19     0   -31     2

(b) >> E=[D C]
E =
    12    14    16    40     7
    32    15    11     1     5
     7    25    19    10     7
     9     1     3     6     2
    19     0   -31     2     0

(c) >> det(E)
ans =
   -676510

(d) >> inv(E)
ans =
    0.0152    0.0482   -0.0354   -0.0496   -0.0016
    0.0355    0.0497    0.0003   -0.2495    0.0120
    0.0122    0.0304   -0.0236   -0.0360   -0.0331
    0.0450    0.0132   -0.0297   -0.0868    0.0013
   -0.2394   -0.3269    0.2838    1.1622    0.0468

(e) >> E'
ans =
    12    32     7     9    19
    14    15    25     1     0
    16    11    19     3   -31
```

```

      40      1      10      6      2
      7      5      7      2      0

(h) >> E*E
ans =
    1197         818         553         908         346
    1045         949         734        1421         378
    1240         958         561         569         327
     253         222         168         431         101
      29        -507        -279         462         -80

FF not possible, matrix dimensions do not agree.
HH not possible, matrix dimensions do not agree.

>> E*F
ans =
    972
    746
    616
    266
   -199

FE not possible, matrix dimensions do not agree.

>> H*E
ans =
    275    273    258    233    94
    284    213   -335   -245    24

EH not possible, matrix dimensions do not agree.
FH not possible, matrix dimensions do not agree.

>> H*F
ans =
    147
   -31
  
```

1.8

```

% clear workspace
clear;clc;
% see the values in the VARIABLE editor (workspace)
%% part 1
A=[2 4 6; 3 5 1; 7 5 9];
[V1,e1]=eig(A);
%% part 2
A=[12 14 16 40; 32 15 11 1; 7 25 19 10];
B=[9 1 36 4; 19 0 -31 2];
C=[7; 5; 7; 2; 0];
F=[16; 3; 17; 12;-2];
H=[5 7 4 -2 -1; -9 3 12 -6 14];
D=[A;B];
E=[D C];
  
```

```
% this is what E should be in the workspace
% E = [12 14 16 40 7; 32 15 11 1 5; 7 25 19 10 7; 9 1 3 6 2; 19 0 -31 2 0]
[V2,e2]=eig(E);
```

1.9

```
(a) clear;clc;
A=[12 16 4; 23 1 21; 9 10 1];
B=[2 7 14; 3 11 2; -9 10 12];
X = A^(-1);
Y = B^(-1);
>> A*B
ans =
    36    300    248
   -140    382    576
    39    183    158
```

```
(b) >>B*A
ans =
    311    179    169
    307     79    245
    230    -14    186
```

(c) $AB \neq BA$. Matrix multiplication is not generally commutative.

```
(d) >> X=inv(A)
X =
   -0.2025    0.0233    0.3217
    0.1609   -0.0233   -0.1550
    0.2141    0.0233   -0.3450
```

```
(e) >> Y=inv(B)
Y =
    0.0678    0.0339   -0.0847
   -0.0327    0.0908    0.0230
    0.0781   -0.0502    0.0006
```

```
(f) >> A*X
ans =
    1.0000    0.0000         0
         0    1.0000   -0.0000
   -0.0000    0.0000    1.0000
```

```
(g) >> B*Y
ans =
    1.0000    0.0000    0.0000
    0.0000    1.0000   -0.0000
         0         0    1.0000
```

(h) Parts (g) and (h) have the same answers, the identity matrix. It is true because $AX = AA^{-1} = I$ and $BY = BB^{-1} = I$

1.10

- (a) `clear;clc;`
`A=[12 16 4; 23 1 21; 9 10 1];`
`B=[2 7 14;3 11 2; -9 10 12];`
`C=[43 12; 13 12];`
`D=[1 2 3; 4 5 6];`
- (1) `>> A+B`
`ans =`

14	23	18
26	12	23
0	20	13
- (2) A+C not possible, A and C not the same order.
- (3) A+D not possible, A and D not the same order.
- (4) B+C not possible, B and C not the same order.
- (5) B+D not possible, B and D not the same order.
- (6) C+D not possible, C and D not the same order.

(c) See above

- (1) `>> A+B`
`ans =`

14	23	18
26	12	23
0	20	13
- (2) `>> B+A`
`ans =`

14	23	18
26	12	23
0	20	13
- (3) A+B and B+A are equal. Matrix addition is commutative.
-

1.11

- (a) `clear;clc;`
`A=[12 16 4; 23 1 21; 9 10 1];`
`B=[2 7 14;3 11 2; -9 10 12];`
`>> D=A*B`
`D =`

36	300	248
-140	382	576
39	183	158

- (b) `>> E=A'`
`ans =`
 12 23 9
 16 1 10
 4 21 1
- (c) `>> F=B'`
`F =`
 2 3 -9
 7 11 10
 14 2 12
- (d) `>> G=D'`
`G =`
 36 -140 39
 300 382 183
 248 576 158
- (e) (1) `>> E*F`
`ans =`
 311 307 230
 179 79 -14
 169 245 186
- (2) `>> F*E`
`ans =`
 36 -140 39
 300 382 183
 248 576 158
- (f) Part (2), $F*E$ is the same as matrix D.
- (g) Recall the property $(AB)'=B'A'$

1.12

- (a) `>> a=[3 4;4 2];`
`>> b=[12;10];`
`>> x=inv(a)*b`
`x =`
 1.6000
 1.8000
- (b) `>> a=[3 4;4 0];`
`>> b=[12;10];`
`>> x=inv(a)*b`
`x =`
 2.5000
 1.1250
- (c) `>> a=[-4 1;4 3];`
`>> b=[14;10];`
`>> x=inv(a)*b`
`x =`

-2
6

```
(d) >> a=[13 12;-4 7;11 -13];  
>> b=[-6;-73;157];  
>> x=pinv(a)*b  
x =  
    6.0000  
   -7.0000
```

Note the usage of the command `pinv(a)` in this problem. Since `a` is not square, we use its pseudoinverse.

```
(e) >> a=[2 3 -1;4 -2 1;1 5 -2];  
>> b=[8;5;9];  
>> x=inv(a)*b  
x =  
     2  
     1  
    -1
```

```
(f) >> a=[4 -8 3;-1 2 -5;3 -6 1];  
>> b=[16;-21;7];  
>> x=pinv(a)*b  
x =  
    0.2000  
   -0.4000  
    4.0000
```

```
(g) >> a=[2 3 1 -11;5 -2 5 -4;1 -1 3 -3;3 4 -7 2];  
>> b=[1;5;3;-7];  
>> x=pinv(a)*b  
x =  
   -0.0000  
   -0.4286  
    0.7143  
   -0.1429
```

1.13

```
% Generating 25 elements in A using for loop logic  
for i=1:1:25  
    A(i)=i;  
end
```

1.14

```
%Test if two numbers are greater than zero  
function fun(n,m)
```

```
if ((n>0) && (m>0))
    display('Both');
elseif ((n<=0) && (m<=0))
    display('None');
else
    display('Other');
end
```

1.15

```
(a) a=ones(1,50);
(b) for i=1:1:50
    if rem(i,2)==0 a(i)=2;
    end
end
(c) for i=1:1:50
    if rem(i,3)==0 a(i)=3;
    end
end
```

The final output:

```
a =
     1     2     3     2     1     3     1     2     3     2
     1     3     1     2     3     2     1     3     1     2
     3     2     1     3     1     2     3     2     1     3
     1     2     3     2     1     3     1     2     3     2
     1     3     1     2     3     2     1     3     1     2
```

1.16 See Fig. 1.5.

```
% clear workspace
clear;clc;

% contour plots
[x,y]=meshgrid(1:0.5:2,3:0.5:5);
z = (3*x.^2+4*y.^2);

% subplots of contour plots
subplot(2,3,1)
[c,h] = contour(x,y,z,4);
clabel(c,h)
xlabel('x')
ylabel('Function value')
title('Two Contours')
```

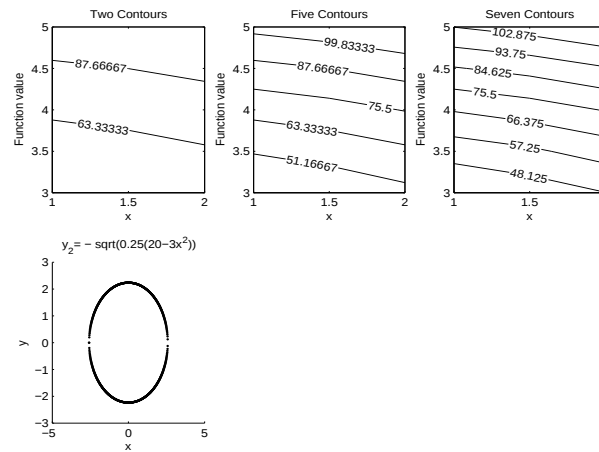


Figure 1.5: Contour Plots and Scatter plot for Problem 1.16

```
subplot(2,3,2)
[c2,h2] = contour(x,y,z,7);
clabel(c2,h2)
xlabel('x')
ylabel('Function value')
title('Five Contours')

subplot(2,3,3)
[c3,h3] = contour(x,y,z,9);
clabel(c3,h3)
xlabel('x')
ylabel('Function value')
title('Seven Contours')

% scatter plot
x1 = -sqrt(20/3):0.01:sqrt(20/3);
y1 = sqrt(1/4*(20-3*x1.^2));
y2 = -sqrt(1/4*(20-3*x1.^2));
% subplot of scatter plots
subplot(2,3,4)
scatter(x1,y1,'.')
xlabel('x')
ylabel('y')
title('y_1=sqrt(0.25(20-3x^2))')
hold on
scatter(x1,y2,'.')
xlabel('x')
ylabel('y')
title('y_2= -sqrt(0.25(20-3x^2))')
```

1.17

See Fig. 1.6.

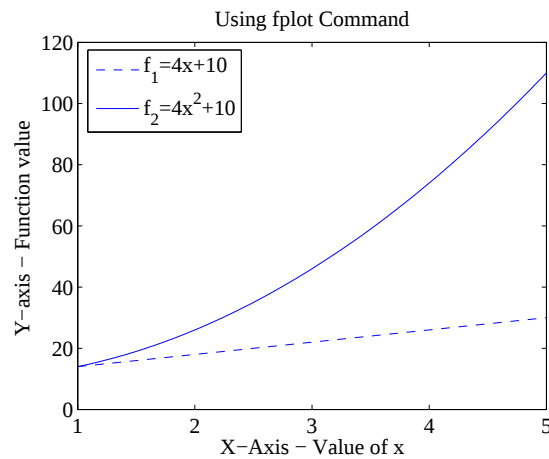


Figure 1.6: Plot of the 2 functions for Problem 1.17

```
% clear workspace
clear;clc;close all
% plot within specified limits
fplot('4*x+10',[1 5],':')
hold on
fplot('4*x^2+10',[1 5])
set(gca,'FontName','Times','FontSize',15)
xlabel('X-Axis - Value of x')
ylabel('Y-axis - Function value ')
legend('f_1=4x+10','f_2=4x^2+10',2)
title('Using fplot Command')
```

1.18

See Figs. 1.7 and 1.8.

```
% clear workspace
clear;clc;close all
fplot('4*x+10',[0 3.5],'-')
hold on
fplot('4*x^2+10',[0 3.5],':')
hold on
fplot('sin(x)',[0 3.5],'-.')
legend('y_1=4x+10','y_2=4x^2+10','y_3=sinx',2)
xlabel('X-Axis -Value of x')
ylabel('Y-Axis - Function value')
title('Problem 1.19 -Part 1')
figure(2)
% create subplots
```

```
subplot(3,1,1);
fplot('4*x+10',[0 3.5],'-');
xlabel('Value of x')
ylabel('Function value')
title('Plot 1 - y_1=4x+10')
legend('y_1=4x+10',2)

subplot(3,1,2);
fplot('4*x^2+10',[0 3.5],':');
xlabel('Value of x')
ylabel('Function value')
title('Plot 2 - y_2=4x^2+10')
legend('y_2=4x^2+10',2)

subplot(3,1,3);
fplot('sin(x)',[0 3.5],'-');
xlabel('Value of x')
ylabel('Function value')
title('Plot 3 - y_3=sinx')
legend('y_3=sinx',2)
```

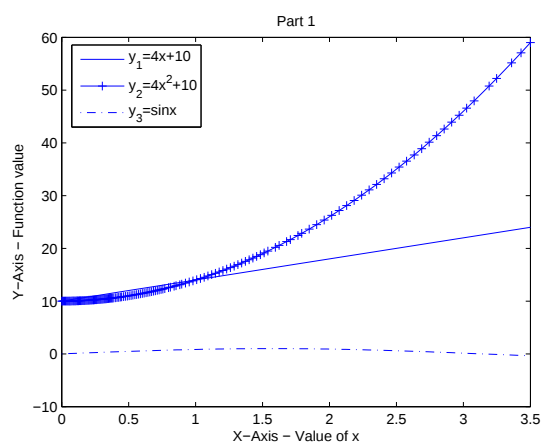


Figure 1.7: Plot of the 3 functions for Problem 1.18

1.19

See Figs. 1.9 and 1.10.

```
% clear workspace
clear;clc;close all
% define peak function
Z=peaks;
% mesh plot
figure(1)
```

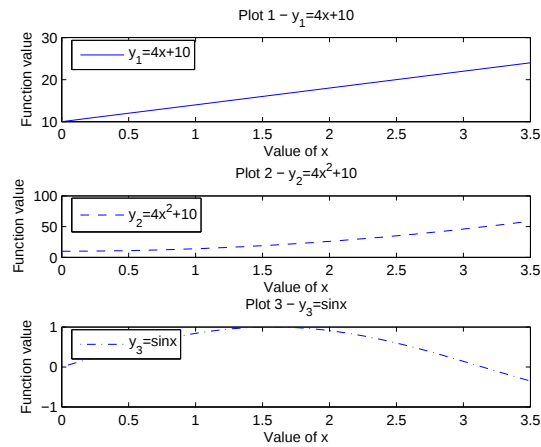


Figure 1.8: Subplots of the 3 functions for Problem 1.18

```
mesh(Z)
xlabel('X - axis')
ylabel('Y - axis')
zlabel('Z - axis')
title('3D plot of peaks using mesh')
legend('legend')
text(0,60,10,'First name')
text(0,0,0,'Last name')
% surface plot
figure(2)
surf(Z)
xlabel('X - axis')
ylabel('Y - axis')
zlabel('Z -axis')
title('3D plot of peaks using surf')
legend('legend')
text(0,60,10,'First name')
text(0,0,0,'')
```

1.20

(a) getSquared.m

```
% clear workspace
clear;clc;
%% for part c
% define input
xinput = 5;
% Call the function
xoutput = getSquared(xinput);
% Display result on screen
```

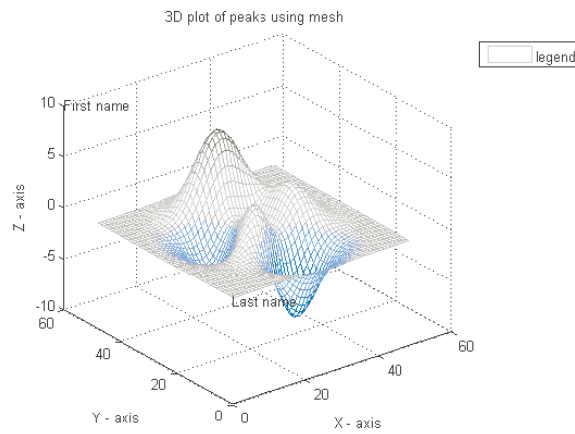


Figure 1.9: 3-D plot using mesh for Problem 1.19

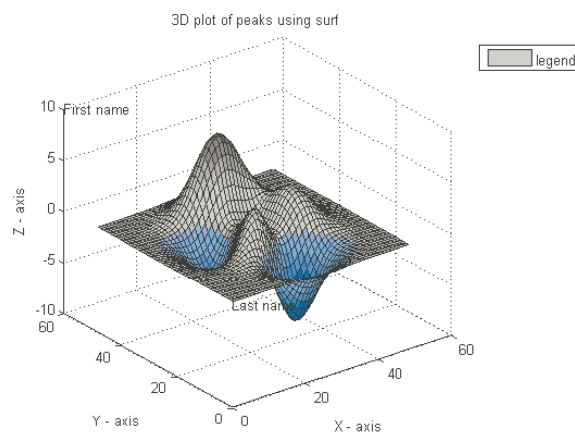


Figure 1.10: 3-D plot using surf for Problem 1.19

```
fprintf('Input = %d, Output = %d\n',xinput,xoutput);
```

(b) getSquared.m

```
% This function returns the squared value of input
function y = getSquared(x)
y = x^2;
```

(c) % define input

```
xinput = 5;      % non-negative input
% xinput = -5;   % negative input
% Call the function
xoutput = getSquaredSpecial(xinput);
% Display result on screen
fprintf('Input = %d, Output = %d\n',xinput,xoutput);
```

(d) getsquaredspecial.m

```
% This function checks whether the input >= 0
% if yes, return the squared value of x
% otherwise, return -1
function y = getSquaredSpecial(x)
if x >= 0
    y = x^2;
else
    f = -1;
end
```

1.21

```
(a,c) % This function calculates the sum
% of the first x integers from 1
function y = sumHundred(x)
% initialize sum
sum = 0;
% check if x is between 1 and 100
if x < 1 || x > 100
    disp('Input not between 1 and 100');
    y = -1;
else
% Calculate sum of the first x integers
    for i = 1:1:x
        sum = sum + i;
    end
    y = sum;
end

(d) % This function calculates the sum of all even integers from 1 to x
function y = sumEvenHundred(x)
% initialize sum
sum = 0;
if x < 1 || x > 100
    disp('Input not between 1 and 100')
    y = -1;
else
% Calculate sum of the first inpt integers
    for i = 1:1:x
        % Check if i is even
        if rem(i,2) == 0
            sum = sum + i;
        else
            continue;
        end
    end
    y = sum;
end
```

1.22

```
(a) % Function to calculate the sum of
    % two input numbers
    function opt = hiddenSum(a,b) opt = a + b;

(b) clear;clc;
    %% part b
    % script to test the function "hiddenSum"
    % specify inputs
    x = 23; y = 54;
    z = hiddenSum(x,y);
    display(z);

(c) (1) function hiddenSum_c(a,b)
    global fc
    fc = a+b;

    (2) % script to test the function "hiddenSum_c"
    % specify inputs
    x = 23; y = 54;
    global fc
    hiddenSum_c(x,y);
    z = fc;
    display(z);

(d) (1) function hiddenSum_d
    global fd a b;
    fd = a + b;

    (2) % script to test the function "hiddenSum_d"
    % specify inputs
    x = 23; y = 54;
    global fd a b
    a = x;
    b = y;
    hiddenSum_d();
    z = fd;
    display(z);
```