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CHAPTER 1

Prob. 1.1

(a)

$$\nabla \times \nabla \Phi = \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \frac{\partial \Phi}{\partial x} & \frac{\partial \Phi}{\partial y} & \frac{\partial \Phi}{\partial z} \end{bmatrix}$$

$$= \left(\frac{\partial^2 \Phi}{\partial y \partial z} - \frac{\partial^2 \Phi}{\partial z \partial y} \right) \boldsymbol{a}_x + \left(\frac{\partial^2 \Phi}{\partial x \partial z} - \frac{\partial^2 \Phi}{\partial z \partial x} \right) \boldsymbol{a}_y + \left(\frac{\partial^2 \Phi}{\partial x \partial y} - \frac{\partial^2 \Phi}{\partial y \partial x} \right) \boldsymbol{a}_z = 0$$

(b)

$$\nabla \cdot \nabla \times \boldsymbol{F} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot \begin{bmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_y \end{bmatrix}$$

$$= \frac{\partial}{\partial x} \left(\frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial x} \right) - \frac{\partial}{\partial y} \left(\frac{\partial F_z}{\partial x} - \frac{\partial F_x}{\partial z} \right) + \frac{\partial}{\partial z} \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right)$$

$$= \frac{\partial^2 F_z}{\partial x \partial y} - \frac{\partial^2 F_y}{\partial x \partial z} - \frac{\partial^2 F_z}{\partial y \partial x} + \frac{\partial^2 F_x}{\partial y \partial z} + \frac{\partial^2 F_y}{\partial z \partial x} - \frac{\partial^2 F_x}{\partial z \partial y} = 0$$

(c)

$$\nabla(\nabla \cdot \boldsymbol{F}) - \nabla^2 \boldsymbol{F} = \left[\frac{\partial}{\partial x} \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) - \frac{\partial^2 F_x}{\partial x^2} - \frac{\partial^2 F_x}{\partial y^2} - \frac{\partial^2 F_x}{\partial z^2} \right] \boldsymbol{a}_x$$

$$+ \left[\frac{\partial}{\partial y} \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) - \frac{\partial^2 F_y}{\partial x^2} - \frac{\partial^2 F_y}{\partial y^2} - \frac{\partial^2 F_y}{\partial z^2} \right] \boldsymbol{a}_y$$

$$+ \left[\frac{\partial}{\partial z} \left(\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} \right) - \frac{\partial^2 F_z}{\partial x^2} - \frac{\partial^2 F_z}{\partial y^2} - \frac{\partial^2 F_z}{\partial z^2} \right] \boldsymbol{a}_z$$

$$= \left[\frac{\partial}{\partial y} \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) - \frac{\partial}{\partial z} \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) \right] \boldsymbol{a}_x$$

$$+ \left[\frac{\partial}{\partial z} \left(\frac{\partial F_x}{\partial y} - \frac{\partial F_y}{\partial z} \right) - \frac{\partial}{\partial x} \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y} \right) \right] \boldsymbol{a}_y$$

$$+ \left[\frac{\partial}{\partial x} \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x} \right) - \frac{\partial}{\partial y} \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z} \right) \right] \boldsymbol{a}_z$$

$$= \nabla \times \nabla \times \boldsymbol{F}$$

Prob. 1.2 Let $\boldsymbol{A} = U \nabla V$ and apply Stokes' theorem.

$$\oint_L U \nabla V \cdot d\boldsymbol{l} = \int_S \nabla \times (U \nabla V) \cdot d\boldsymbol{S}$$

$$= \int_S (\nabla U \times \nabla V) \cdot d\boldsymbol{S} + \int_S U (\nabla \times \nabla V) \cdot d\boldsymbol{S}$$

Since $\nabla \times \nabla V = 0$,

$$\oint_L U\nabla V \cdot d\boldsymbol{l} = \int_S (\nabla U \times \nabla V) \cdot d\boldsymbol{S}$$

Similarly, we can show that

$$\oint_L V\nabla U \cdot d\boldsymbol{l} = \int_S (\nabla V \times \nabla U) \cdot d\boldsymbol{S} = -\int_S (\nabla U \times \nabla V) \cdot d\boldsymbol{S}$$

Thus,

$$\oint_L U\nabla V \cdot d\boldsymbol{l} = -\oint_L V\nabla U \cdot d\boldsymbol{l}$$

as required.

Prob. 1.3
Using divergence theorem,

$$\int_S (U\nabla V) \cdot d\boldsymbol{S} = \int_v \nabla \cdot (U\nabla V) dv$$

But $\nabla \cdot (U\boldsymbol{A}) = U\nabla \cdot \boldsymbol{A} + \boldsymbol{A} \cdot \nabla U$, where $\boldsymbol{A} = \nabla V$

$$\int_S (U\nabla V) \cdot d\boldsymbol{S} = \int_v (U\nabla \cdot \nabla V + \nabla V \cdot \nabla U) dv$$

$$= \int_v (U\nabla^2 V + \nabla U \cdot \nabla V) dv$$

Prob. 1.4 If $\boldsymbol{J} = 0 = \rho_v$, then Maxwell's equations become

$$\nabla \cdot \boldsymbol{B} = 0 \qquad (1)$$

$$\nabla \cdot \boldsymbol{D} = 0 \qquad (2)$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad (3)$$

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} \qquad (4)$$

Since $\nabla \cdot \nabla \times \boldsymbol{A} = 0$ for any vector field $\boldsymbol{A}$,

$$\nabla \cdot \nabla \times \boldsymbol{E} = -\frac{\partial \nabla \cdot \boldsymbol{B}}{\partial t} = 0$$

$$\nabla \cdot \nabla \times \boldsymbol{H} = \frac{\partial \nabla \cdot \boldsymbol{D}}{\partial t} = 0$$

Showing that (1) and (2) are incorporated in (3) and (4). Thus Maxwell's equations can be reduced to curl equations (3) and (4).

Prob. 1.5 If $\boldsymbol{J} \neq 0 \neq \rho_v$,

$$\nabla \cdot \varepsilon \boldsymbol{E} = \rho_v$$
$$\nabla \cdot \mu \boldsymbol{H} = 0$$
$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t}$$
$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \varepsilon \frac{\partial \boldsymbol{E}}{\partial t}$$

$$\nabla \times \nabla \times \boldsymbol{E} = -\mu \frac{\partial}{\partial t} \nabla \times \boldsymbol{H} = -\mu \frac{\partial \boldsymbol{J}}{\partial t} - \mu\varepsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$
$$\nabla(\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{J}}{\partial t} - \frac{1}{c^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$
$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2} \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = \nabla(\rho_v / \varepsilon) + \mu \frac{\partial \boldsymbol{J}}{\partial t}$$

Similarly,

$$\nabla \times \nabla \times \boldsymbol{H} = \nabla \times \boldsymbol{J} + \varepsilon \frac{\partial}{\partial t} \nabla \times \boldsymbol{E}$$
$$\nabla(\nabla \cdot H) - \nabla^2 H = \nabla \times J - \mu\varepsilon \frac{\partial^2 H}{\partial t^2}$$

or

$$\nabla^2 \boldsymbol{H} - \frac{1}{c^2} \frac{\partial^2 \boldsymbol{H}}{\partial t^2} = -\nabla \times \boldsymbol{J}$$

It is assumed that the medium is free space so that the medium is homogeneous and $u = \frac{1}{\sqrt{\mu\varepsilon}} = c.$

Prob. 1.6

(a)

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}$$
$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t}$$
$$\nabla \times \nabla \times \boldsymbol{H} = \nabla \times \boldsymbol{J} + \frac{\partial}{\partial t}(\nabla \times \boldsymbol{D}) = \nabla \times \boldsymbol{J} + \varepsilon \frac{\partial}{\partial t}(\nabla \times \boldsymbol{E}) = \nabla \times \boldsymbol{J} - \frac{\partial}{\partial t}(\mu \frac{\partial \boldsymbol{H}}{\partial t})$$
$$\nabla \times \nabla \times \boldsymbol{H} + \mu\varepsilon \frac{\partial^2 \boldsymbol{H}}{\partial t^2} = \nabla \times \boldsymbol{J}$$

(b) $\nabla \times \nabla \times \boldsymbol{E} = -\mu \frac{\partial}{\partial t}(\nabla \times \boldsymbol{H}) = -\mu \frac{\partial}{\partial t}(\boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}) = -\mu \frac{\partial \boldsymbol{J}}{\partial t} - \mu\varepsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$

$$\nabla \times \nabla \times \boldsymbol{E} + \mu\varepsilon \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = -\mu \frac{\partial \boldsymbol{J}}{\partial t}$$

Prob. 1.7

$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} \quad (1)$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \quad (2)$$

Dotting both sides of (2) with **E** gives

$$\boldsymbol{E} \cdot (\nabla \times \boldsymbol{H}) = \boldsymbol{E} \cdot \boldsymbol{J} + \boldsymbol{E} \cdot \frac{\partial \boldsymbol{D}}{\partial t} \quad (3)$$

But for any arbitrary vectors **A** and **B**,

$$\nabla \cdot (\boldsymbol{A} \times \boldsymbol{B}) = \boldsymbol{B} \cdot (\nabla \times \boldsymbol{A}) - \boldsymbol{A} \cdot (\nabla \times \boldsymbol{B})$$

Applying this to the left-hand side of (3) by letting $\boldsymbol{A} \equiv \boldsymbol{H}$ and $\boldsymbol{B} \equiv \boldsymbol{E}$, we get

$$\boldsymbol{H} \cdot (\nabla \times \boldsymbol{E}) + \nabla \cdot (\boldsymbol{H} \times \boldsymbol{E}) = \boldsymbol{E} \cdot \boldsymbol{J} + \frac{1}{2} \frac{\partial}{\partial t} (\boldsymbol{D} \cdot \boldsymbol{E}) \quad (4)$$

From (1),

$$\boldsymbol{H} \cdot (\nabla \times \boldsymbol{E}) = \boldsymbol{H} \cdot (-\frac{\partial \boldsymbol{B}}{\partial t}) = -\frac{1}{2} \frac{\partial}{\partial t} (\boldsymbol{B} \cdot \boldsymbol{H})$$

Substituting this into (4) gives

$$-\frac{1}{2} \frac{\partial}{\partial t} (\boldsymbol{B} \cdot \boldsymbol{H}) - \nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) = \boldsymbol{J} \cdot \boldsymbol{E} + \frac{1}{2} \frac{\partial}{\partial t} (\boldsymbol{D} \cdot \boldsymbol{E})$$

Rearranging terms and then taking the volume integral of both sides,

$$\int_v \nabla \cdot (\boldsymbol{E} \times \boldsymbol{H}) dv = -\frac{1}{2} \frac{\partial}{\partial t} \int_v (\boldsymbol{E} \cdot \boldsymbol{D} + \boldsymbol{H} \cdot \boldsymbol{B}) dv - \int_v \boldsymbol{J} \cdot \boldsymbol{E} dv$$

Using the divergence theorem, we get

$$\oint (\boldsymbol{E} \times \boldsymbol{H}) \cdot d\boldsymbol{S} = -\frac{\partial W}{\partial t} - \int_v \boldsymbol{J} \cdot \boldsymbol{E} dv$$

Or

$$\frac{\partial W}{\partial t} = -\oint_S (\boldsymbol{E} \times \boldsymbol{H}) \cdot d\boldsymbol{S} - \int_v \boldsymbol{E} \cdot \boldsymbol{J} dv$$

as required.

Prob. 1.8

$$\nabla \cdot \boldsymbol{E} = 0, \qquad \nabla \cdot \boldsymbol{H} = 0$$

$$\nabla \times \boldsymbol{E} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & E_y & 0 \end{vmatrix} = -\frac{\partial E_y}{dz} \boldsymbol{a}_x + \frac{\partial E_x}{dz} \boldsymbol{a}_y$$

$$= -10k \sin(\omega t + kz) \boldsymbol{a}_x - 20k \cos(\omega t - kz) \boldsymbol{a}_y$$

$$\boldsymbol{H} = -\frac{1}{\mu_o}\int \nabla\times\boldsymbol{E}\,\partial t$$
$$= \frac{k}{\omega\mu_o}\left[-10\cos(\omega t - kz)\boldsymbol{a}_x + 20\sin(\omega t - kz)a_y\right]$$

which is the given ***H***. Since all of Maxwell's equations are satisfied by the fields, they are genuine EM fields.

Prob. 1.9

$$\nabla\times\boldsymbol{E} = -\mu\frac{\partial\boldsymbol{H}}{\partial t} \quad\rightarrow\quad \frac{\partial\boldsymbol{H}}{\partial t} = -\frac{1}{\mu_o\varepsilon_o}\nabla\times\varepsilon_o\boldsymbol{E}$$

$$\frac{\partial\boldsymbol{H}}{\partial t} = -\frac{1}{\mu_o\varepsilon_o}\nabla\times\boldsymbol{D} = -\frac{1}{\mu_o\varepsilon_o}\begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ D_x(z,t) & 0 & 0\end{vmatrix}$$
$$= -\frac{1}{\mu_o\varepsilon_o}\frac{\partial D_x}{\partial z}\boldsymbol{a}_y \quad = \frac{\beta}{\mu_o\varepsilon_o}D_o\sin(\omega t + \beta z)\boldsymbol{a}_y$$
$$\boldsymbol{H} = -\frac{D_o}{\beta}\cos(\omega t - \beta z)\boldsymbol{a}_y$$

Prob. 1.10

$$\nabla\times\boldsymbol{H}_s = j\omega\boldsymbol{E}_s = \frac{1}{\rho}\frac{d}{d\rho}(\rho H_\phi)\boldsymbol{a}_z$$
$$= \frac{H_o}{\rho}\left(\frac{1}{\rho}\rho^{1/2}e^{-j\beta\rho} - j\beta\rho^{1/2}e^{-j\beta\rho}\right)\boldsymbol{a}_z$$
$$= \frac{H_o}{\sqrt{\rho}}\left(\frac{1}{\rho^2} - j\beta\right)e^{-j\beta\rho}\boldsymbol{a}_z$$
$$\boldsymbol{E}_s = \frac{\nabla\times\boldsymbol{H}_s}{j\omega\varepsilon} = \frac{1}{j\omega\varepsilon}\frac{H_o}{\sqrt{\rho}}\left(\frac{1}{\rho^2} - j\beta\right)e^{-j\beta\rho}\boldsymbol{a}_z$$

Prob. 1.11

It is evident that $\nabla\cdot\boldsymbol{E} = 0$ and $\nabla\cdot\boldsymbol{H} = 0$ are satisfied.

$$\nabla\times\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t} \quad\longrightarrow\quad \nabla\times\boldsymbol{E}_s = -j\omega\mu\boldsymbol{H}_s$$

$$\nabla\times\boldsymbol{E}_s = \begin{vmatrix}\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ E_oe^{-j\beta z} & 0 & 0\end{vmatrix} = -j\beta E_oe^{-j\beta z}\boldsymbol{a}_y$$

i.e. $$-j\omega\mu\frac{E_o}{\eta}e^{-j\beta z}\boldsymbol{a}_y = -j\beta E_oe^{-j\beta z}\boldsymbol{a}_y$$

$$\beta = \frac{\omega\mu}{\eta} \qquad (1)$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t} \longrightarrow \nabla \times \boldsymbol{H}_s = (\sigma + j\omega\varepsilon)\boldsymbol{E}_s$$

$$\nabla \times \boldsymbol{H}_s = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & \frac{E_o}{\eta} e^{-j\beta z} & 0 \end{vmatrix} = \frac{j\beta E_o}{\eta} e^{-j\beta z} \boldsymbol{a}_x$$

$$(\sigma + j\omega\varepsilon) E_o e^{-j\beta z} \boldsymbol{a}_x = \frac{j\beta E_o}{\eta} e^{-j\beta z} \boldsymbol{a}_x$$

$$\beta = -j\eta(\sigma + j\omega\varepsilon) \qquad (2)$$

From (1) and (2),

$$\frac{\omega\mu}{\eta} = -j\eta(\sigma + j\omega\varepsilon) \longrightarrow \eta^2 = \frac{j\omega\mu}{\sigma + j\omega\varepsilon}$$

Thus,

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\varepsilon}}, \qquad \beta = \frac{\omega\mu}{\eta}$$

Prob. 1.12

The surface current density is

$$\boldsymbol{K} = \boldsymbol{a}_n \times \boldsymbol{H} = \boldsymbol{a}_n \times (H_x, 0, H_z)$$

At x = 0, $\boldsymbol{a}_n = \boldsymbol{a}_x$, $\quad \boldsymbol{K} = H_z(0, y, z, t)\boldsymbol{a}_y$

i.e. $\quad \boldsymbol{K} = H_o \cos(\omega t - \beta z)\boldsymbol{a}_y$

At x = a, $\boldsymbol{a}_n = -\boldsymbol{a}_x$

$$\boldsymbol{K} = -H_z(a, y, z, t)\boldsymbol{a}_y = -H_o \cos(\pi)\cos(\omega t - \beta z)a_y$$

i.e. $\quad \boldsymbol{K} = H_o \cos(\omega t - \beta z)\boldsymbol{a}_y$

At y =0, $\boldsymbol{a}_n = \boldsymbol{a}_y$

$$\boldsymbol{K} = -H_z(x, 0, z, t)\boldsymbol{a}_x + H_x(x, 0, z, t)\boldsymbol{a}_z$$

$$= -H_o \cos(\pi x / a)\cos(\omega t - \beta z)\boldsymbol{a}_x - \frac{\beta a}{\pi} H_o \sin(\pi x / a)\sin(\omega t - \beta z)\boldsymbol{a}_z$$

At y =b, $\boldsymbol{a}_n = -\boldsymbol{a}_y$

$$\boldsymbol{K} = -\boldsymbol{a}_y \times (H_x, 0, H_z) = H_x(x, b, z, t)\boldsymbol{a}_z - H_x(x, b, z, t)\boldsymbol{a}_x$$

$$= H_o \cos(\pi x / a)\cos(\omega t - \beta z)\boldsymbol{a}_x - \frac{\beta a}{\pi} H_o \sin(\pi x / a)\sin(\omega t - \beta z)\boldsymbol{a}_z$$

Prob. 1.13

(a) $$\nabla^2 \boldsymbol{E} = \mu_o \varepsilon_o \frac{\partial^2 \boldsymbol{E}}{\partial t^2}$$

For **E**, $\nabla^2 \boldsymbol{E} = \frac{\partial^2}{\partial z^2} \cos(\omega t - \beta z)\boldsymbol{a}_x = \frac{\partial}{\partial z}[\beta \sin(\omega t - \beta z)\boldsymbol{a}_x = -\beta^2 \cos(\omega t - \beta z)\boldsymbol{a}_x$

$$\mu_o \varepsilon_o \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = -\omega \mu_o \varepsilon_o \cos(\omega t - \beta z)\boldsymbol{a}_x$$

Thus, $-\beta^2 = -\omega^2 \mu_o \varepsilon_o \quad \longrightarrow \quad \beta = \omega\sqrt{\mu_o \varepsilon_o}$

(b)

$$\nabla \times \boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} \quad \longrightarrow \quad \boldsymbol{H} = -\mu_o \int \nabla \times \boldsymbol{E} dt$$

$$\nabla \times \boldsymbol{E} = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \cos(\omega t - \beta z) & 0 & 0 \end{vmatrix} = \beta \sin(\omega t - \beta z)\boldsymbol{a}_y$$

$$\boldsymbol{H} = -\mu_o \int \beta \sin(\omega t - \beta z) dt \boldsymbol{a}_y = \frac{\mu_o \beta}{\omega} \cos(\omega t - \beta z)\boldsymbol{a}_y$$

Prob. 1.14

$$\nabla \times \boldsymbol{H} = \frac{\partial \boldsymbol{D}}{\partial t} = \varepsilon \frac{\partial \boldsymbol{E}}{\partial t} \quad \rightarrow \quad \nabla \times \boldsymbol{H}_s = j\omega\varepsilon \boldsymbol{E}_s$$

$$\nabla \times \boldsymbol{H}_s = \frac{1}{r \sin\theta} \frac{IL}{4\pi r}(2\sin\theta\cos\theta)\left(\frac{1}{r} + j\beta\right)e^{-j\beta r}\boldsymbol{a}_r - \frac{IL\sin\theta}{4\pi r}\left[-j\beta\left(\frac{1}{r} + j\beta\right)e^{-j\beta r} - \frac{1}{r^2}e^{-j\beta r}\right]\boldsymbol{a}_\theta$$

$$\boldsymbol{E}_s = \frac{IL}{4\pi r j\omega\varepsilon} e^{-j\beta r}\left[2\cos\theta\left(\frac{1}{r^2} + \frac{j\beta}{r^2}\right)\boldsymbol{a}_r - \sin\theta\left(\beta^2 - \frac{j\beta}{r} - \frac{1}{r^2}\right)\boldsymbol{a}_\theta\right]$$

Prob. 1.15

$$E_s = \frac{20(e^{jk_x x} - e^{-jk_x x})}{2j} \frac{\left(e^{jk_y y} - e^{-jk_y y}\right)}{2j}$$
$$= j5\left[e^{j(k_x x + k_y y)} + e^{j(k_x x - k_y y)} - e^{j(k_x x - k_y y)} - e - ^{j(k_x x + k_y y)}\right]$$

which consists of four plane waves.

$$\nabla \times \boldsymbol{E}_s = -j\omega\mu_o \boldsymbol{H}_s$$

Or

$$\boldsymbol{H}_s = \frac{j}{\omega\mu_o}\nabla\times\boldsymbol{E}_s = \frac{j}{\omega\mu_o}\begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & E_{sz}(x,y) \end{vmatrix}$$

$$= \frac{j}{\omega\mu_o}\left(\frac{\partial E_{sz}}{\partial y}\boldsymbol{a}_x - \frac{\partial E_{sz}}{\partial x}\boldsymbol{a}_y\right)$$

$$= -\frac{20}{\omega\mu_o}\left[k_y\sin(k_x x)\sin(k_y y)\boldsymbol{a}_x + k_x\cos(k_x x)\cos(k_y y)\boldsymbol{a}_y\right]$$

Prob. 1.16

(a) $I = \text{Re}(I_s e^{j\omega t}) = \sin\pi x\cos\pi y\cos(\omega t - z)$

(b) $V = \text{Re}(20e^{-j2x}e^{-j90^o}e^{j\omega t} - 10e^{-j4x}e^{j\omega t})$

$$V_s = 20e^{-j2x}e^{-j90^o} - 10e^{-j4x} = -j20e^{-j2x} - 10e^{-j4x}$$

Prob. 1.17

(a) $\boldsymbol{A} = \text{Re}(\boldsymbol{A}_s e^{j\omega t}) = \cos(\omega t - 2z)\boldsymbol{a}_x - \sin(\omega t - 2z)\boldsymbol{a}_y$

(b) $\boldsymbol{B} = \text{Re}(\boldsymbol{B}_s e^{j\omega t}) = -10\sin x\sin\omega t\boldsymbol{a}_x - 5\cos(\omega t - 12z - 45^o)\boldsymbol{a}_z$

(c) $C = \text{Re}(C_s e^{j\omega t}) = 2\cos 2x\sin(\omega t - 3x) + e^{3x}\cos(\omega t - 4x)$

Prob. 1.18 Assuming the time factor $e^{j\omega t}$, equation

$$\nabla^2\boldsymbol{E} - \mu\varepsilon\frac{\partial^2\boldsymbol{E}}{\partial t^2} = \mu\frac{\partial\boldsymbol{J}}{\partial t}$$

becomes

$$\nabla^2\boldsymbol{E}_s + \omega^2\mu\varepsilon\boldsymbol{E}_s = j\omega\mu\sigma\boldsymbol{E}_s$$

or

$$\nabla^2\boldsymbol{E}_s - j\omega\mu(\sigma + j\omega\varepsilon)\boldsymbol{E}_s = 0$$

For conducting medium, $\sigma \gg \omega\varepsilon$ so that

$$\nabla^2\boldsymbol{E}_s - j\omega\mu\sigma\boldsymbol{E}_s = 0$$

Prob. 1.19

Let $\boldsymbol{A} = \text{Re}(\boldsymbol{A}_s e^{j\omega t})$, $\quad V = \text{Re}(V_s e^{j\omega t})$, etc

$$\frac{\partial \mathbf{A}}{\partial t} \quad \rightarrow \quad j\omega \mathbf{A}_s$$

$$\frac{\partial^2 \mathbf{A}}{\partial t^2} \quad \rightarrow \quad -\omega^2 \mathbf{A}_s$$

Similarly,

$$\frac{\partial^2 V}{\partial t^2} \quad \rightarrow \quad -\omega^2 V_s$$

Substituting these into eqs. (1.42) and (1.43), we obtain

$$\nabla^2 V_s + \omega^2 \mu\varepsilon V_s = -\frac{\rho_{vs}}{\varepsilon}$$

$$\nabla^2 \mathbf{A}_s + \omega^2 \mu\varepsilon \mathbf{A}_s = -\mu \mathbf{J}_s$$

But $k^2 = \omega^2 \mu\varepsilon$. Thus,

$$\nabla^2 V_s + k^2 V_s = -\frac{\rho_{vs}}{\varepsilon}$$

$$\nabla^2 \mathbf{A}_s + k^2 \mathbf{A}_s = -\mu \mathbf{J}_s$$

Prob. 1.20

(a) a = 1, b = 2, c = 0,

$$b^2 - 4ac = -16$$

Hence, it is elliptic.

(b) $a = y^2 + 1, \quad b = 0, \quad c = x^2 + 1,$

$$b^2 - 4ac = -4(x^2+1)(y^2+1) < 0$$

Hence it is elliptic.

(c) $a = 1, \quad b = -2\cos x, \quad c = -(3 + \sin^2 x)$

$$b^2 - 4ac = 4\cos^2 x + 12 + 4\sin^2 x = 16 > 0$$

Hence it is hyperbolic.

(d) $a = x^2, \quad b = -2xy, \quad c = y^2,$

$$b^2 - 4ac = 4x^2y^2 - 4x^2y^2 = 0$$

Hence it is parabolic.

Prob. 1.21

(a) $a = \alpha, \quad b = 0, \quad c = 0, \qquad b^2 - 4ac = 0;$ i.e. it is parabolic.

(b) $a = 1, \quad b = 0, \quad c = 0, \qquad b^2 - 4ac = -4;$ i.e. it is elliptic.

(c) $\left(\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2}\right)^2 = 0$

$a = 1, \quad b = 0, \quad c = 1, \qquad b^2 - 4ac = -4;$ i.e. it is elliptic.