Chapter 1 - Introduction to Composites

Exercise Set

1.1 A combination of materials that provides a range of properties not available by using any of the materials alone. Combined on a macroscopic level, the components are not soluble in each other. Commonly, a reinforcing phase is embedded in a continuous, binding matrix.

1.2 There may be various reasons to include straw in unfired clay bricks:

- As a binder,
- To reduce shrinkage during curing,
- To prevent cracking, and
- As random discontinuities to limit crack propagation.

1.3 Prehistoric man used tools made of natural composites: stone, wood, and bone.

Man-made composites appear in history as follows:

Ceramics
Vegetable fiber reinforced clays
Glass
Laminated wood
Laminated metals
Resin-soaked linens for mummification
Concrete
Paper
Printing ink
Synthetic rubber
Plastics
Reinforced concrete
Synthetic fibers
Ceramic superconductors
Glass fiber composites
Carbon fiber composites
Advanced composites

1.4 Naturally found composites:

Bone - Calcium compounds bound by collagen
Wood - Cellulose fibers bound by lignin
Skin - Laminate consisting of epidermis, basement membrane, and dermis.
Layers consist of keratin with collagen fibers
Teeth - Laminated structure of enamel over a dentin core

1.5 Fuel savings per unit mass per year -

$$V_m := \left(\frac{360 \cdot gal}{lbm \cdot yr}\right) \cdot \left(\frac{1 \cdot lbm}{0.453 \cdot kg}\right) \qquad V_m = 794.7 \cdot \frac{gal}{kg \cdot yr}$$

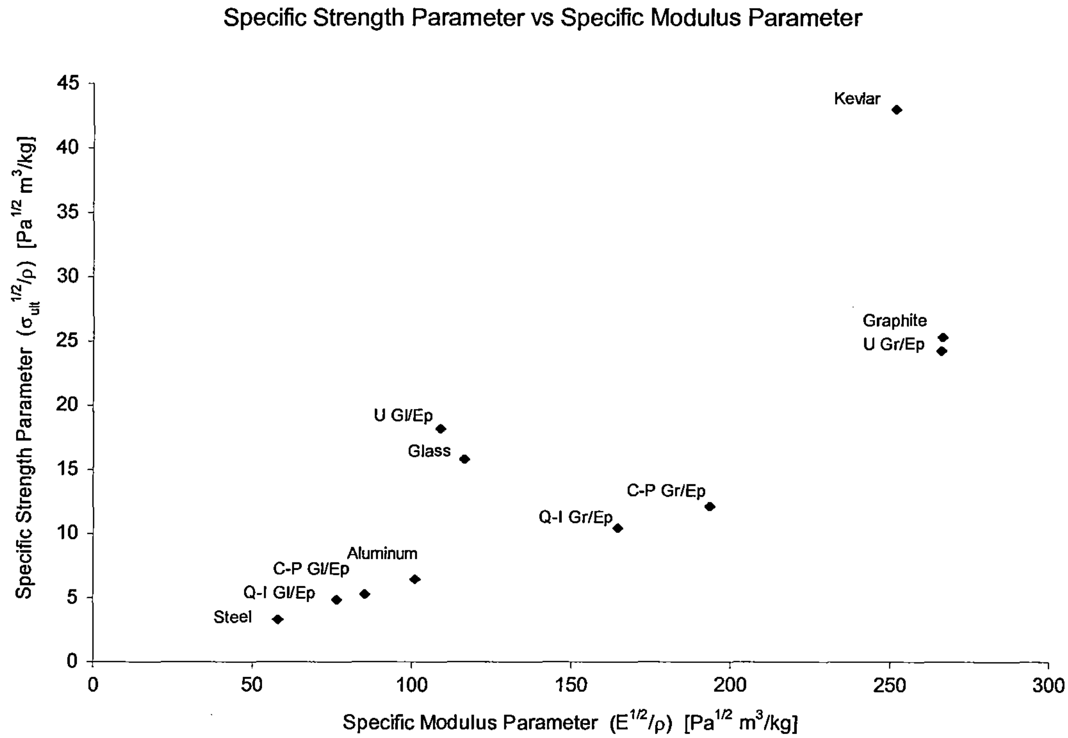
Yearly fuel savings -

$$V_s := 300 \cdot kg \cdot V_m \qquad V_s = 238411 \cdot \frac{gal}{yr}$$

1.6

Term and Definition	SI Units	USCS Units
Coefficient of Thermal Expansion - Change in length per unit length per unit temperature change.	m/m/°C	in/in/°F
Coefficient of Moisture Expansion - Change in length per unit length per unit mass of moisture content change per unit mass.	m/m/kg/kg	in/in/lbm/lbm
Thermal Conductivity - Amount of heat energy conducted through a body per unit area per unit time due to a temperature gradient.	W/m^2	BTU/s - in^2
Young's Modulus - Slope of linear portion of stress-strain curve.	Pa	psi
Ultimate Strength - Failure stress.	Pa	psi
Poisson's Ratio - Ratio of transverse strain to strain in the direction of applied normal load.	Non - dim	Non - dim
Specific Modulus - Ratio of Young's modulus to density.	Pa - m^3/kg	psi - in^3/lbm
Specific Strength - Ratio of ultimate strength to density.	Pa - m^3/kg	psi - in^3/lbm
Density - Mass per unit volume.	kg/m^3	lbm/in^3
Specific Gravity - Ratio of material density to density of water.	Non - dim	Non - dim
Ductility - Measure of material to undergo large strains before failure.	Non - dim	Non - dim
Fracture toughness - Measure of material to resist crack growth.	Pa $\sqrt{m}$	psi $\sqrt{in}$
Specific Heat - Heat required to change the temperature of a unit mass a unit temperature.	J/kg - °C	BTU/lbm - °F

1.7 From Tables 1.1, 1.2 and 1.9 -



1.8 Given:

$N := 100$ $\quad$ $d_1 := 10 \cdot \mu m$

$l := 10 \cdot mm$ $\quad$ $d_2 := 5 \cdot \mu m$

Interfacial area for d_1 -

$$A_1 := N \cdot \pi \cdot d_1 \cdot l \qquad A_1 = 3.142 \cdot 10^{-5} \cdot m^2$$

Fiber volume -

$$V := \frac{N \cdot \pi \cdot d_1^{\,2} \cdot l}{4} \qquad V = 7.854 \cdot 10^{-11} \cdot m^3$$

Fiber count at constant volume for d_2 -

$$\frac{n \cdot \pi \cdot d_2^{\,2} \cdot l}{4} = V$$

Solving for n yields

$$n := 4 \cdot \frac{V}{\left[\pi \cdot \left(d_2^{\,2} \cdot l\right)\right]} \qquad n = 400$$

Interfacial area for d_2 -

$$A_2 := n \cdot \pi \cdot d_2 \cdot l \qquad A_2 = 6.283 \cdot 10^{-5} \cdot m^2$$

Interfacial increase at constant volume -

$$\Delta A := \frac{A_2 - A_1}{A_1} \qquad \Delta A = 100 \cdot \%$$

1.9 Given:

$E_{Steel} := 30 \cdot Msi$ $\qquad d_{Steel} := 0.01 \cdot in$

$E_{Aluminum} := 10 \cdot Msi$ $\qquad d_{Aluminum} := 0.02 \cdot in$

Flexibility comparison -

$$\frac{F_{Steel}}{F_{Aluminum}} = \frac{E_{Aluminum} \cdot d_{Aluminum}^4}{E_{Steel} \cdot d_{Steel}^4}$$

$$\frac{E_{Aluminum} \cdot d_{Aluminum}^4}{E_{Steel} \cdot d_{Steel}^4} = 5.333$$

$$F_{Steel} = 5.333 \cdot F_{Aluminum}$$

The steel rod is 5.333 times more flexible than the aluminum rod.

1.10 Limitations in using modern composites:

- High cost of fabrication,
- Complex mechanical characterization,
- Repair difficulties,
- Difficult flaw detection,
- Low combination of strength and fracture toughness, and
- May not meet all criteria for use.

1.11 Material selection parameters:

1. Strength
2. Toughness
3. Formability
4. Joinability
5. Corrosion resistance
6. Cost

1.12 Composite classification:

Reinforcement geometry - Particulate
Flake
Fiber

Matrix type - Polymer
Metal
Ceramic
Carbon

1.13 From Tables 1.6, 1.8, and 1.9

Specific Moduli:

$$SM_{Graphite.} := 765\cdot\frac{Msi\cdot in^3}{lbm}$$

$$SM_{Kevlar} := 355.5\cdot\frac{Msi\cdot in^3}{lbm}$$

$$SM_{SGlass} := 137.9\cdot\frac{Msi\cdot in^3}{lbm}$$

Specific Strengths:

$$SS_{Graphite} := 3479\cdot\frac{ksi\cdot in^3}{lbm}$$

$$SS_{Kevlar} := 9823\cdot\frac{ksi\cdot in^3}{lbm}$$

$$SS_{SGlass} := 7396\cdot\frac{ksi\cdot in^3}{lbm}$$

Coefficients of Thermal Expansion:

$$CTE_{Graphite} := -0.30\cdot\frac{\mu in}{\frac{in}{°F}}$$

$$CTE_{Kevlar} := -1.111\cdot\frac{\mu in}{\frac{in}{°F}}$$

$$CTE_{SGlass} := 3.1\cdot\frac{\mu in}{\frac{in}{°F}}$$

The Graphite fiber has the highest specific modulus indicating it is the stiffest per unit mass. The Kevlar 49 fiber has the highest specific strength of the lot, indicating it is the strongest per unit mass. The graphite fiber has the lowest coefficient of thermal expansion giving the greatest dimensional stability.

1.14 Pultrusion is a manufacturing method for polymer matrix composites. The reinforcement fibers are pulled through a resin bath where they are saturated with the matrix material. The wetted fibers continue to be pulled through a heated forming die where they cure and emerge as continuous finished parts.

1.15 Reasons for epoxy being the most common matrix material in production use:

1. High strength
2. Low viscosity
3. Low flow rates
4. Low volatility
5. Low shrink rates
6. Available in many grades
7. Good adhesion

1.16 Applications of polymer matrix composites:

1. Satellite antennas
2. Cable ducting

3. Deep-sea submersibles
4. Automotive intake manifolds
5. Electrical junction boxes
6. Transformer insulation
7. Trolley pantograph arms
8. Pipes and fittings
9. Portable bathrooms
10. Minesweeper hulls

1.17 Comparison of metal matrix and polymer matrix composites:

Characteristic	MMC	PMC
Elastic properties	High	Low
Service temperature	High	Low
Moisture Sensitivity	Low	High
Thermal and Electric conductivity	High	Low
Wear, Fatigue, and Flaw resistance	High	Low
Processing temperature	High	Low

1.18 Applications of metal matrix composites:

1. Pump impellers
2. Piston sleeves/inserts
3. Mobile military bridges

1.19 Liquid-phase fabrication is a process used in the manufacture of metal matrix composites. The reinforcement is in the form of short fibers, whiskers, flakes, or long directed fibers. The shorter reinforcements are usually incorporated by stirring into the molten matrix prior to casting or extrusion. Continuous reinforcement are placed into a mold and the molten matrix is forced into the mold. Continuous fibers are coated with the matrix material by vapor deposition to aid in wetting.

1.20 Ceramic matrix composite applications:

1. Journal bearings
2. Piston crowns
3. Aluminum can forming punches

1.21 Chemical vapor infiltration is a process used to manufacture ceramic matrix composites. The reinforcement fibers are preformed to near finished shape and placed in a reactor vessel. The ceramic matrix is "grown" within the fiber network by introducing the matrix components to the reactor in gaseous form. The gases react on the fibers to form the ceramic matrix.

1.22 Applications of carbon-carbon composites:

1. Ramjet combustion liners
2. Radioactive isotope containment
3. Bone fracture internal fixation

1.23 Chemical vapor deposition is a method used for manufacturing carbon-carbon composites. The carbon matrix is deposited on a heated carbon substrate by pyrolysis of a hydrocarbon.

1.24 Operating temperature limits:

Polymer	750 °F	(400 °C)
Metal	1800 °F	(1000 °C)
Ceramic	2700 °F	(1500 °C)
Carbon	6000 °F	(3315 °C)

1.25

Isotropic	Material that exhibits the same properties in all directions at a given point.
Homogeneous	Material that exhibits the same properties at all points in the body.
Anisotropic	Material that exhibits different properties in all directions at a given point.
Nonhomogeneous	Material that exhibits different properties at all points in the body.
Micromechanics	Study of finding average properties of a single ply of composite material from the properties of the individual constituents.
Macromechanics	Study of the stress-strain relationships of a single ply or laminate made of composite material.
Lamina	Single layer of fibers in a matrix.
Laminate	Stacked lamina bonded to each other.

1.26 An epoxy ply with chopped fibers rolled in a particular direction is homogeneous since its properties are the same at different points. The composite is not isotropic as its elastic properties in the direction of the fibers are different than those perpendicular to the fibers

An unevenly heated bar of steel is a non-homogeneous body that is isotropic. When heated, the bar is still isotropic because it exhibits the same elastic properties in different directions, but different properties at different points. For example, since elastic moduli depend on temperature, they will be different at different points

1.27 No, metal matrix composites may have lower ductility and fracture toughness than monolithic metals.

1.28 Hybrid composite classification:

1. Interply - Combine plies of two or more composite systems.
2. Intraply - Combine two or more fibers in the same ply.
3. Interply-Intraply - Combine more than one fiber in a ply and more than one composite system by ply.
4. Resin Hybrids - Combine two or more matrix materials.

Chapter 2 - Macromechanical Analysis of a Lamina

Exercise Set

2.1 The number of independent elastic constants in three dimensions are:

Anisotropic	21
Monoclinic	13
Orthotropic	9
Transversely Orthotropic	5
Isotropic	2

2.2 Given the following engineering constants for an orthotropic material:

$E_1 := 4\cdot Msi$ $\quad E_2 := 3\cdot Msi$ $\quad E_3 := 3.1\cdot Msi$

$\nu_{12} := 0.2$ $\quad \nu_{23} := 0.4$ $\quad \nu_{31} := 0.6$

$G_{12} := 6\cdot Msi$ $\quad G_{23} := 7\cdot Msi$ $\quad G_{31} := 2\cdot Msi$

Minor Poisson's ratios -

$$\nu_{21} := \frac{E_2}{E_1}\cdot\nu_{12} \qquad \nu_{21} = 0.1500$$

$$\nu_{32} := \frac{E_3}{E_2}\cdot\nu_{23} \qquad \nu_{32} = 0.4133$$

$$\nu_{13} := \frac{E_1}{E_3}\cdot\nu_{31} \qquad \nu_{13} = 0.7742$$

Stiffness matrix, -

$$C := \begin{bmatrix} \frac{E_1\cdot(\nu_{32}\cdot\nu_{23}-1)}{\nu'} & \frac{-E_2\cdot(\nu_{32}\cdot\nu_{13}+\nu_{12})}{\nu'} & \frac{-E_3\cdot(\nu_{13}+\nu_{12}\cdot\nu_{23})}{\nu'} & 0 & 0 & 0 \\ \frac{-E_1\cdot(\nu_{21}+\nu_{31}\cdot\nu_{23})}{\nu'} & \frac{E_2\cdot(\nu_{31}\cdot\nu_{13}-1)}{\nu'} & \frac{-E_3\cdot(\nu_{23}+\nu_{21}\cdot\nu_{13})}{\nu'} & 0 & 0 & 0 \\ \frac{-E_1\cdot(\nu_{31}+\nu_{21}\cdot\nu_{32})}{\nu'} & \frac{-E_2\cdot(\nu_{32}+\nu_{31}\cdot\nu_{12})}{\nu'} & \frac{E_3\cdot(\nu_{21}\cdot\nu_{12}-1)}{\nu'} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_{31} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_{12} \end{bmatrix}$$

where, $\nu' := (\nu_{32}\cdot\nu_{23}+\nu_{31}\cdot\nu_{13}+\nu_{21}\cdot\nu_{12}+\nu_{31}\cdot\nu_{12}\cdot\nu_{23}+\nu_{21}\cdot\nu_{32}\cdot\nu_{13})-1$

$$C = \begin{bmatrix} 13.675 & 6.39 & 10.846 & 0 & 0 & 0 \\ 6.39 & 6.58 & 6.553 & 0 & 0 & 0 \\ 10.846 & 6.553 & 12.316 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{bmatrix}\cdot Msi$$