Problem 2.1 The tangential velocity component of the cylinder is $R\dot{\phi}$, and the tangential velocity component of the contact point is $r\dot{\theta}$. If there is no slipping, these two components must be equal. Thus $R\dot{\phi} = r\dot{\theta}$.

Problem 2.2 The tangential velocity component of the cylinder is $R\dot{\phi}$, and the tangential velocity component of the contact point is $r\dot{\theta}$. If there is no slipping, these two components must be equal. Thus $R\dot{\phi} = r\dot{\theta}$.

Problem 2.3 Let θ be the angle of the line BA measured from the vertical. That is, $\theta = 0$ when point A is in contact with the surface. Then $\omega = \dot{\theta}$ and $\alpha = \ddot{\theta}$.

Let s be the linear displacement of point B. Then

$$s = R\theta$$

$$v_B = R\dot{\theta} = R\omega$$

$$a_B = R\ddot{\theta} = R\alpha$$

Establish an xy coordinate system whose origin is located at the point of contact of point A with the surface at time $t = 0$. The coordinate x is positive to the left and y is positive upward. Then the xy coordinates of point A are

$$x = s - R\,\sin\,\theta = R(\theta - \sin\,\theta) \qquad y = R - R\,\cos\,\theta$$

$$\dot{x} = R\dot{\theta}(1 - \cos\,\theta) = v_B(1 - \cos\,\theta) \qquad \dot{y} = R\dot{\theta}\,\sin\,\theta = v_B\,\sin\,\theta$$

So the velocity of point A has the components $\dot{x}$ and $\dot{y}$. The magnitude of the velocity is

$$v_A = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{v_B^2(1 - \cos\,\theta)^2 + v_B^2\sin^2\theta} = v_B\sqrt{2(1 - \cos\,\theta)}$$

To find the acceleration, differentiate again to obtain

$$\ddot{x} = \dot{v}_B(1 - \cos\,\theta) + v_B\dot{\theta}\,\sin\,\theta = a_B(1 - \cos\,\theta) + R\omega^2\,\sin\,\theta$$

$$\ddot{y} = \dot{v}_B\sin\,\theta + v_B\dot{\theta}\,\cos\,\theta = a_B\sin\,\theta + R\omega^2\,\cos\,\theta$$

or

$$\ddot{x} = R\alpha(1 - \cos\,\theta) + R\omega^2\,\sin\,\theta$$

$$\ddot{y} = R\alpha\sin\,\theta + R\omega^2\,\cos\,\theta$$

The acceleration magnitude is

$$a_A = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{2R^2\alpha^2 + R^2\omega^4 + 2R^2\alpha\omega^2 - 2R^2\alpha^2\,\cos\,\theta}$$

Problem 2.4 Let x be the horizontal displacement of the vehicle measured to the left from directly underneath the pulley. Let y be the height of the block above the ground. Let D be the length of the hypotenuse of the triangle whose sides are x and 3 m. Then, from the Pythagorean theorem,

$$D^2 = x^2 + 3^2 \qquad (1)$$

Differentiate this to obtain

$$D\dot{D} = x\dot{x} \qquad (2)$$

The total cable length is 10 m, so $D + 3 - y = 10$ or

$$D = y + 7 \qquad (3)$$

This gives

$$x^2 = D^2 - 9 = (y + 7)^2 - 9 \qquad (4)$$

and

$$\dot{y} = \dot{D} = \frac{x\dot{x}}{D} = \frac{x\dot{x}}{7 + y} \qquad (5)$$

Substituting the given values of $y = 2$, $\dot{x} = 0.2$ into Equations (4) and (5), we obtain $x = \sqrt{72} = 6\sqrt{2}$ and

$$\dot{y} = \frac{6\sqrt{2}(0.2)}{9} = 0.1885 \ \text{ m/s}$$

This is the velocity of the block after it has been raised 2 m.

To obtain the acceleration, differentiate Equations (2) and (5) to obtain

$$\dot{D}^2 + \dot{D}\ddot{D} = \dot{x}^2 + \dot{x}\ddot{x}$$

$$\ddot{y} = \ddot{D}$$

Solve for $\ddot{D}$:

$$\ddot{D} = \frac{\dot{x}^2 + \dot{x}\ddot{x} - \dot{D}^2}{\dot{D}} = \frac{(0.2)^2 + 0 - (0.1885)^2}{0.1885} = 0.0237 \ \text{ m/s}^2$$

Since $\ddot{y} = \ddot{D}$, the acceleration of the block after it has been raised 2 m is 0.0237 m/s^2.

Problem 2.5 Only one of the pulleys translates, so $v_2 = v_1/2$.

Problem 2.6 Let v_C denote the velocity of the middle pulley. Then $v_C = v_A/2$ and $v_B = v_C/2$. Thus, $v_B = (v_A/2)/2 = v_A/4$.

Problem 2.7

$$M_O = I_O\alpha = I_O\ddot{\theta}$$

$$I_O = I_{RG} + m_R L^2 + m_C L_C^2$$

$$M_O = -m_R g L \sin\theta - m_C g L_C \sin\theta$$

Thus

$$(I_{RG} + m_R L^2 + m_C L_C^2)\ddot{\theta} = -(m_R L + m_C L_C) g \sin\theta$$

If $m_R \approx 0$ and if $I_{RG} \approx 0$, then

$$m_C L_C^2 \ddot{\theta} = -m_C L_C g \sin\theta$$

or

$$L_C \ddot{\theta} = -g \sin\theta$$

Problem 2.8 The tangential velocity component of the cylinder is $R\dot{\phi}$, and the tangential velocity component of the contact point is $r\dot{\theta}$. If there is no slipping, these two components must be equal. Thus $R\dot{\phi} = r\dot{\theta}$ and $R\ddot{\phi} = r\ddot{\theta}$.

$$KE = \frac{1}{2} I_G \dot{\phi}^2 + \frac{1}{2} M \left[(r - R)\dot{\theta} \right]^2$$

$$PE = Mg\left[(r - R) - (r - R)\cos\theta\right] = Mg(r - R)\left(1 - \cos\theta\right)$$

Because $KE + PE =$ constant, then $d(KE + PE)/dt = 0$ and

$$I_G \dot{\phi}\ddot{\phi} + M(r - R)^2 \dot{\theta}\ddot{\theta} + Mg(r - R)\sin\theta\,\dot{\theta} = 0$$

But $R\dot{\phi} = r\dot{\theta}$ and $\ddot{\phi} = r\ddot{\theta}/R$. Thus

$$I_G \left(\frac{r}{R}\right)^2 \dot{\theta}\ddot{\theta} + M(r - R)^2 \dot{\theta}\ddot{\theta} + Mg(r - R)\sin\theta\,\dot{\theta} = 0$$

Cancel $\dot{\theta}$ and collect terms to obtain

$$\left[I_G \left(\frac{r}{R}\right)^2 + M(r - R)^2 \right] \ddot{\theta} + Mg(r - R)\sin\theta = 0$$

Problem 2.9 The tangential velocity component of the cylinder is $R\dot{\phi}$, and the tangential velocity component of the contact point is $r\dot{\theta}$. If there is no slipping, these two components must be equal. Thus $R\dot{\phi} = r\dot{\theta}$ and $R\ddot{\phi} = r\ddot{\theta}$.

The energy method is easier than the force- moment method here because 1) the motions of the link and cylinder are directly coupled (i.e. if we know the position and velocity of one, we know the position and velocity of the other), 2) the only external force is conservative (gravity), and 3) we need not compute the reaction forces between the link and the cylinder.

$$KE = \frac{1}{2}I_G\dot{\phi}^2 + \frac{1}{2}M\left[(r-R)\dot{\theta}\right]^2 + \frac{1}{2}m\left(\frac{L}{2}\dot{\theta}\right)^2 + \frac{1}{2}\left[I_O + m\left(\frac{L}{2}\right)^2\right]\dot{\theta}^2$$

$$PE = Mg(r-R)\,(1-\cos\,\theta) + mg(r-\frac{L}{2})(1-\cos\,\theta)$$

Because $KE + PE =$ constant, then $d(KE+PE)/dt = 0$, and

$$I_G\dot{\phi}\ddot{\phi} + M(r-R)^2\dot{\theta}\ddot{\theta} + m\frac{L^2}{4}\dot{\theta}\ddot{\theta} + \left[I_O + m\left(\frac{L}{2}\right)^2\right]\dot{\theta}\ddot{\theta} + Mg(r-R)\sin\,\theta\,\dot{\theta} + mg\left(r-\frac{L}{2}\right)\sin\,\theta\,\dot{\theta} = 0$$

But $\dot{\phi} = r\dot{\theta}/R$ and $\ddot{\phi} = r\ddot{\theta}/R$. Substitute these expressions, cancel $\dot{\theta}$, and collect terms to obtain

$$\left[I_G\left(\frac{r}{R}\right)^2 + M(r-R)^2 + m\frac{L^2}{4} + I_O + m\frac{L^2}{4}\right]\ddot{\theta} + \left[Mg(r-R) + mg\left(r-\frac{L}{2}\right)\right]\sin\,\theta = 0$$

Problem 2.10 a) Let point O be the pivot point and G be the center of mass. Let L be the distance from O to G. Treat the pendulum as being composed of three masses:

1) m_1, the rod mass above point O, whose center of mass is 0.03 m above point O;
2) m_2, the rod mass below point O, whose center of mass is 0.045 m below point O, and
3) m_3, the mass of the 4.5 kg block.

Then, summing moments about G gives

$$m_1 g(L + 0.03) - m_2 g(0.045 - L) - m_3 g(0.09 + 0.015 - L) = 0$$

where

$$m_1 = \frac{0.06}{0.15} 1.4 = 0.56 \text{ kg}$$

$$m_2 = \frac{0.09}{0.15} 1.4 = 0.84 \text{ kg}$$

$$m_3 = 4.5 \text{ kg}$$

The factor g cancels out of the equation to give

$$0.56(L + 0.03) - 0.84(0.045 - L) - 4.5(0.105 - L) = 0$$

which gives $L = 0.084$ m.

b) Summing moments about the pivot point O gives

$$I_O \ddot{\theta} = -mgL \sin \theta$$

where m is the total mass. From the parallel-axis theorem, treating the rod as a slender rod, we obtain

$$I_O = \frac{1}{12}(1.4)(0.06 + 0.09)^2 + (1.4)(0.015)^2 + (4.5)(0.09 + 0.015)^2 = 0.0525 \ \ \text{kg} \cdot \text{m}^2$$

and $mgL = (1.4 + 4.5)(9.81)(0.084) = 4.862$ N·m. Thus the equation of motion is

$$0.0525\ddot{\theta} = -4.862 \sin \theta$$

or

$$\ddot{\theta} + 92.61 \sin \theta = 0$$

Problem 2.11 See the figure for the coordinate definitions and the definition of the reaction force R. Let P be the point on the axle. Note that $y_P = 0$. The coordinates of the mass center of the rod are $x_G = x_P - (L/2)\sin\theta$ and $y_G = -(L/2)\cos\theta$. Thus

$$\ddot{x}_G = \ddot{x}_P - \frac{L}{2}\ddot{\theta}\cos\theta + \frac{L}{2}\dot{\theta}^2\sin\theta$$

$$\ddot{y}_G = \frac{L}{2}\ddot{\theta}\sin\theta + \frac{L}{2}\dot{\theta}^2\cos\theta$$

Let m be the mass of the rod. Summing forces in the x direction:

$$m\ddot{x}_G = f \qquad \text{or} \qquad m\left(\ddot{x}_P - \frac{L}{2}\ddot{\theta}\cos\theta + \frac{L}{2}\dot{\theta}^2\sin\theta\right) = f \qquad (1)$$

Summing forces in the y direction:

$$m\ddot{y}_G = R - mg \qquad \text{or} \qquad m\left(\frac{L}{2}\ddot{\theta}\sin\theta + \frac{L}{2}\dot{\theta}^2\cos\theta\right) = R - mg \qquad (2)$$

Summing moments about the mass center: $I_G\ddot{\theta} = (fL/2)\cos\theta - (RL/2)\sin\theta$. Substituting for R from (2) and using the fact that $I_G = mL^2/12$, we obtain

$$\frac{1}{12}mL^2\ddot{\theta} = \frac{fL}{2}\cos\theta - \frac{mgL}{2}\sin\theta - \frac{mL^2}{4}\sin^2\theta\,\ddot{\theta} - \frac{mL^2}{4}\dot{\theta}^2\sin\theta\cos\theta \qquad (3)$$

The model consists of (1) and (3) with $m = 20$ kg and $L = 1.4$ m.

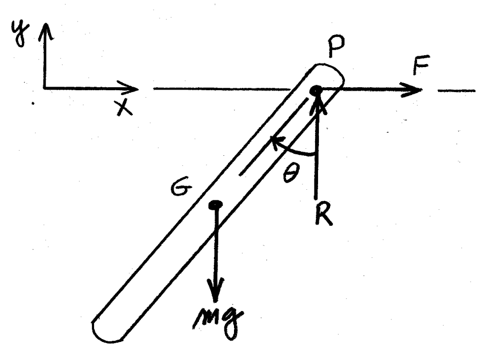

Figure : for Problem 2.11

Problem 2.12 a) $2kx = mg$, so $x = mg/2k$.

b)

$$T_0 = 0 \qquad V_0 = \frac{1}{2}k(0.06)^2$$

$$T_1 = \frac{1}{2}mv^2 \qquad V_1 = \frac{1}{2}k[0.06 + 2(0.09)]^2 - mg(0.09)$$

From conservation of energy,

$$T_0 + V_0 = T_1 + V_1$$

Thus

$$\frac{1}{2}mv^2 = T_0 + V_0 - V_1 = \frac{1}{2}k(0.06)^2 - \frac{1}{2}k[0.06 + 2(0.09)]^2 + mg(0.09)$$

Solve for v using $m = 30$ and $k = 600$ to obtain $v = 0.828$ m/s.

Problem 2.13 a) From conservation of energy,

$$T_0 + V_0 - W_{01} = T_1 + V_1$$

Thus

$$0 + mgd \sin\theta - \mu mgd \cos\theta = \frac{1}{2}mv^2 + 0$$

Thus

$$v = \sqrt{2gd(\sin\theta - \mu \cos\theta)} = 2.04 \ \text{m/s}$$

b) From conservation of energy,

$$T_1 + V_1 - W_{12} = T_2 + V_2$$

Thus

$$\frac{1}{2}mv_1^2 + mgx \sin\theta - \mu mgx \cos\theta = 0 + \frac{1}{2}kx^2$$

This gives

$$6.2421 + 20.807x - 8.3228x = 500x^2$$

which has the roots $x = 0.1249$ and $x = -0.0999$. Choosing the positive root, we see that the spring is compressed by 0.1249 m.

Problem 2.14

$$W = fx - \frac{1}{2}k\left[(x + x_0)^2 - x_0^2\right] - mgx\sin\theta$$

or

$$W = 400(2) - \frac{1}{2}44\left[(2.5)^2 - (0.5)^2\right] - 11(9.81)2(0.5) = 560.09 \quad \text{N} \cdot \text{m}$$

Problem 2.15 Let x_1 be the initial stretch in the spring from its free length. Then

$$(L + x_1)^2 = (D_1 - r)^2 + D_2^2$$

where r is the radius of the cylinder. This gives $x_1 = 1.828$ m.

From conservation of energy,

$$T_1 + V_1 = T_2 + V_2$$

or

$$0 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + \frac{1}{2}kx_2^2$$

where $v = R\omega$ and $x_2 = D_1 - r - L = 1$ m. This gives

$$0 + 41.77 = \frac{1}{2}10(0.25\omega)^2 + \frac{1}{2}0.4\omega^2 + 12.5$$

or $\omega = 7.56$ rad/s.

Problem 2.16 The velocity of the mass is zero initially and also when the maximum compression is attained. Therefore $\Delta T = 0$ and we have $\Delta T + \Delta V = \Delta T + \Delta V_s + \Delta V_g = 0$ or $\Delta V_s + \Delta V_g = 0$. That is, if the mass is dropped from a height h above the middle spring and if we choose the gravitational potential energy to be zero at that height, then the maximum spring compression x can be found by adding the change in the gravitational potential energy $0 - W(h + x) = -W(h + x)$ to the change in potential energy stored in the springs. Thus, letting $W = mg$,

$$\frac{1}{2}k_1(x^2 - 0) + [0 - W(h + x)] \qquad \text{if } x < d$$

where d is the difference in the spring lengths ($d = 0.1$ m). This gives the following quadratic equation to solve for x:

$$\frac{1}{2}k_1x^2 - Wx - Wh = 0 \qquad \text{if } x < d \qquad (1)$$

If $x \geq d$, $\Delta V_s + \Delta V_g = 0$ gives

$$\frac{1}{2}k_1(x^2 - 0) + \frac{1}{2}(2k_2)\left[(x - d)^2 - 0\right] + [0 - W(h + x)] = 0 \qquad \text{if } x \geq d$$

which gives the following quadratic equation to solve for x.

$$(k_1 + 2k_2)x^2 - (2W + 4k_2d)x + 2k_2d^2 - 2Wh = 0 \qquad \text{if } x \geq d \qquad (2)$$

For the given values, equation (1) becomes

$$10^4x^2 - 200(9.81)x - 200(9.81)(0.75) = 0 \qquad \text{if } x < 0.1$$

which has the roots $x = 0.494$, which is greater than 0.1, and $x = -0.2978$, which is not feasible. Thus, since there is no solution for which $x < 0.1$, the side springs will also be compressed. From equation (2)

$$2.6 \times 10^4x^2 - (1962 + 3200)x + 160 - 1471.5 = 0$$

which has the solutions: $x = 0.344$ and $x = -0.146$. We discard the second solution because it is negative. So the outer springs will be compressed by $0.344 - 0.1 = 0.244$ m and the middle spring will be compressed 0.344 m.

Problem 2.17 From conservation of angular momentum,

$$H_{O1} = H_{O2}$$

or

$$mv_1(0.5) = \left[I_G + m_2(1)^2 + m_1(0.5)^2\right]\omega$$

or

$$0.8(10)v_1(0.5) = \left[\frac{1}{12}(4)(1)^2 + 4 + 0.8(0.5)^2\right]\omega$$

Solve for ω:

$$\omega = 0.882 \quad \text{rad/s}$$

Problem 2.18 From the figure,

$$v_{B1} = 10 \cos 30^\circ$$

$$e = 1 = \frac{v_{A2} - v_{B2}}{v_{B1} - v_{A1}} = \frac{0.6\omega - v_{B2}}{v_{B1} - 0}$$

Thus

$$v_{B1} = 10 \cos 30^0 = 0.6\omega - v_{B2}$$

and

$$v_{B2} = 0.6\omega - 8.66$$

From conservation of momentum,

$$H_{O1} = H_{O2}$$

or

$$m_1 v_{B1}(0.6) = I_O\omega + m_1 v_{B2}(0.6)$$

where

$$I_O = I_G + m_2(0.6)^2 = \frac{1}{12}m_2(0.8)^2 + 4(0.36) = 1.653$$

Thus

$$0.8(8.66)(0.6) = 1.653\omega + 0.8(0.6)(0.6\omega - 8.66)$$

This gives

$$\omega = 4.28 \ \ \text{rad/s}$$

Problem 2.19 From conservation of angular momentum,

$$H_{O1} = H_{O2}$$

or

$$mv_1(2L) = \left[I_G + m_2L^2 + m_1(2L)^2\right]\omega$$

where

$$I_G = \frac{1}{12}m_2(2L)^2 = \frac{1}{12}4.5(2.4)^2 = 2.16$$

Thus

$$0.005(365)(2.4) = \left[2.16 + 4.5(1.2)^2 + 0.005(2.4)^2\right]\omega$$

Solve for ω:

$$\omega = 0.505 \ \ \text{rad/s}$$

Problem 2.20 Let f_n be the reaction force on the pendulum from the pivot in the normal direction. When $\theta = \pi/2$, f_n is vertical and is positive upward. Let f_t be the reaction force on the pendulum from the pivot in the tangential direction. When $\theta = \pi/2$, f_t is horizontal and is positive to the right.

Summing moments about the pivot at point O gives the equation of motion of the pendulum.

$$I_O\ddot{\theta} = mg\frac{L}{2}\cos\theta$$

Summing moments about the mass center gives

$$I_G\ddot{\theta} = f_t\frac{L}{2}$$

Comparing these two expressions, we find that

$$f_t = \frac{mgI_G}{I_O}\cos\theta$$

Thus the tangential reaction force is $f_t = 0$ when $\theta = \pi/2$.

To compute the normal reaction force, sum forces in the vertical direction to obtain

$$ma_n = f_n - mg$$

where the normal acceleration is given by the following expression for circular motion.

$$a_n = \frac{L}{2}\omega^2$$

Thus

$$f_n = mg + ma_n = mg + m\frac{L}{2}\omega^2$$

We can compute ω either from energy conservation or by integrating the equation of motion. With the energy method we use the fact that the kinetic energy at $\theta = 90°$ equals the original potential energy at $\theta = 0$. Thus

$$\frac{1}{2}I_O\omega^2 = mg\frac{L}{2}$$

Since $I_O = mL^2/3$ for a slender rod, this gives $\omega^2 = 3g/L$.

(Continued on the next page)