A.2.1. Table 2.3.

Current Density

j, A/m^2 or Amps/meter^2

$j_i = \square_{ij} E_j$

Volt = Ohm. Ampere

Electrical Conductivity

□ , s/m or Siemens/meter

$$\frac{A}{m^2} = \frac{1}{\square m} \cdot \frac{V}{m}$$

$1/\square \cdot m$ or $\square^{\square 1} \cdot m^{\square 1}$

□ is ohms

Electric Field

E V/m or volt/meter

N/C or Newton/Coulomb

Flux of Atoms

$$\frac{\text{Atoms}}{\text{cm}^2 \cdot \text{sec}} = \frac{m^2}{\text{sec}} \frac{\text{atoms}}{m^3 \cdot s} \frac{1}{m}$$

J = D dC/dx

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dU = □dx

m = _ m (dimensionless)

F = □ A

N = $\frac{N}{m^2} \frac{1}{m^2}$

A.2.2.

With only this couple it will rotationally accelerate.

$\square_{32} = \square_{23}$ is required to avoid rotation.

A.2.3.

Because strain results from heating or cooling (changes in dimensions) and these changes may vary with orientation, thermal expansion is a second rank tensor as is strain.

A.2.4.

$P = d\sigma \Rightarrow P_i = d_{ijk}\sigma_{jk}$

d is a third rank tensor.

A.2.5.

$P = d\sigma$

$$\frac{\text{change}}{m^2} = \frac{\text{Coul.}}{N}\frac{N}{m^2}$$

$\varepsilon = dE$

$$= \frac{\text{Coul.}}{N}\cdot\frac{\text{Volt}}{m}$$

where is electric field

$$Volt = \frac{Joule}{Coulomb} = \frac{Newton\cdot Meter}{Coulomb}$$

so

$$\frac{Coulomb}{Newton}\frac{Newton\cdot Meter}{Coulomb}\frac{1}{Meter} = \text{dimensionless!}$$

A.2.6.

$$A = \frac{C_{44}}{\frac{1}{2}(C_{11} - C_{12})}$$

Since the anisotropy factor is only for cubic materials it makes sense to not include the hexagonal ones.

**Note to Instructors: You may want to only assign part of the list! Since I wrote this initially the list got longer. Also, there is some inconsistency between the c and s values – in part due to rounding effects.*

	C11	C12	C44	A(C)	S11	S12	S44	A(S)
Aluminum	108	61	28	1.19	15.7	-5.7	35.1	1.22
Copper	168	121	75.4	3.21	15	-6.3	13.3	3.20
Gold	186	157	42	2.90	23.3	-10.7	24	2.83
Lead	47	40	14.4	4.11	93	-42.4	69	3.92
Nickel	247	147	125	2.50	7.3	-2.7	8	2.50
Silver	124	93.4	46	3.01	22.9	-9.8	22	2.97
Iron	237	141	116	2.42	8	-2.8	8.6	2.51

Molybdenum	460	176	110	0.77	2.8	-0.78	9.1	0.79
Niobium	246	134	28.7	0.51	6.6	-2.3	34.8	0.51
Tantalum	267	161	82.5	1.56	6.9	-2.6	12.2	1.56
Tungsten	**520**	200	**160**	**1.00**	2.6	-0.7	6.6	1.00
C (diamond)	1076	125	566	1.19	1.1	-0.22	2.1	1.26
Germanium	129	48.3	67.1	1.66	9.8	-2.7	15	1.67
Silicon	166	63.9	79.6	1.56	7.7	-2.1	12.6	1.56
Potassium	4.6	3.7	2.6	5.78	820	-370	380	6.26
Zinc Sulfide	108	72	41	2.28	20	-8	24	2.33
Magnesium Oxide	296	95	156	1.55	4	-1	6.5	1.54
Sodium Chloride	49	12	13	0.70	23	-4.7	79	0.70
Lithium Fluoride	111	42	62.8	1.82	11.1	-3.1	15.9	1.79
Titanium Carbide	513	106	178	0.87	2.1	-0.36	5.61	0.88

Numbers in **bold** were incorrect in the first edition.

A.2.7.

$$T_{14} = T_{1123} \qquad\qquad T_{34} = T_{3323}$$

$$C_{14} = C_{1123} \quad S_{14} = 2S_{1123} \qquad\qquad C_{3323} \quad 2S_{3323}$$

$$T_{54} = T_{1323} \qquad\qquad T_{63} = T_{1233}$$

$$C_{1323} \quad 4S_{1323} \qquad\qquad C_{1233} \quad 2S_{1233}$$

A.2.8.

Eq. 2.1.8 becomes

Tensor Notation	11	22	33	23	32	13	31	12	21
Matrix Notation	1	2	3	4	7	5	8	6	9

$$\square = \begin{vmatrix} \square_1 & \square_6 & \square_5 \\ \square_9 & \square_2 & \square_4 \\ \square_8 & \square_7 & \square_3 \end{vmatrix}$$

A.2.9.

$$\square_{11} = C_{1111}\square_{11} + C_{1122}\square_{22} + C_{1133}\square_{33}$$

or

$$\square_1 = C_{11}\square_1 + C_{12}\square_2 + C_{13}\square_3$$
$$= (2C_{44} + C_{12})\square_1 + C_{12}\square_2 + C_{13}\square_3 \text{ (using Eq. 2.26)}$$
$$= (2\square + \square)\square_1 + \square(\square_2 + \square_3)$$
$$= 2\square\square_1 + \square(\square_1 + \square_2 + \square_3)$$

so

$$\square_1 = 2\square\square_1 + \square(\square_1 + \square_2 + \square_3)$$
$$\square_2 = 2\square\square_2 + \square(\square_1 + \square_2 + \square_3)$$
$$\square_3 = 2\square\square_3 + \square(\square_1 + \square_2 + \square_3)$$

A.2.10.

Hexagonal materials are transversely isotropic about the basal normal. That means that all properties in the basal plane must be isotropic -- so an arbitrary rotation should return the same properties. Using Eq. (2.2.4) with A=1 we get

$$S_{44} = 2(S_{11} \square S_{12})$$

because S_{66} is the inverse shear modulus in the basal plane we can write

$$S_{66} = 2(S_{11} \square S_{12})$$
$$[S_{1212} = S_{66}]!$$

A.2.11.

$$C_{66} = C_{1212}$$

e)
$$\square_{12`} = 2C_{1212}\square_{12}$$
$$\square_6 = C_{1212}e_6$$
$$\square_6 = C_{66}e_6$$

A.2.12.

(a) No – cubic or isotropic
(b) Yes
(c) No – cubic or isotropic
(d) No – cubic or isotropic
(e) No – cubic or isotropic

A.2.13.

(a), (b), (c), (e), (f), (g)

Not d - $C_{1313} = C_{55}$
$C_{1133} = C_{3311} = C_{13}$

Not h - $S_{11} \neq S_{22}$ for orthotropic

A.2.14.

$$C_{11} = C_{22} = C_{33}$$
$$C_{12} = C_{23} = C_{13}$$
$$C_{44} = C_{55} = C_{66}$$

A.2.15.

(i) $\varepsilon_3 = s_{32}\sigma_z$
where σ_z = -10 MPa

A.2.16.

(a) (i) Four-fold symmetry about z
(b) $C_{1123} = 0$

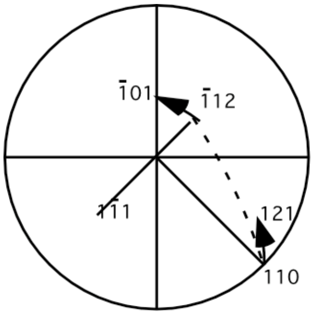
B.2.1.

$a_{\square_2} \cdot a_{\text{example 2.}}$

$$\begin{vmatrix} \frac{\sqrt{3}}{4} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{\sqrt{3}}{4} & 0 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{-1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{vmatrix}$$

$$= \begin{vmatrix} 0.408 & 0.816 & 0.408 \\ -.707 & 0 & .707 \\ 0.577 & -0.577 & 0.577 \end{vmatrix}$$

$$\square\ [121] = x'$$
$$[\bar{1}01] = y'$$
$$[1\bar{1}1] = z'$$



B.2.2. (Mathcad Example – ORIGIN=1 for subscripts in matrices)

There are a number of ways to approach this problem: but one that shows what is going on simply uses the values from taken directly from the rotation matrix and plots the points versus the angle of rotation. This problem was picked so that it seems easiest to start by defining an angle of rotation that involves rotation around the y-axis - we will put in the Euler angles a bit later.

$$a(\theta) := \begin{pmatrix} \cos(\theta) & 0 & \sin(\theta) \\ 0 & 1 & 0 \\ -\sin(\theta) & 0 & \cos(\theta) \end{pmatrix}$$

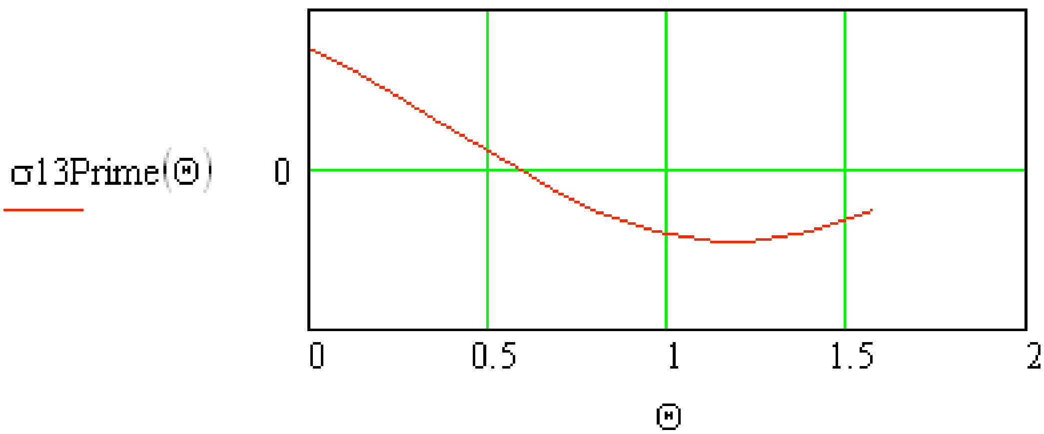
Example :

$$a\left(\frac{\pi}{6}\right)_{1,1} = 0.866$$

Write an equation that finds how □13' varieswith this rotation applying the second rank tensor transformation equation.

$$\begin{aligned} \sigma 13\mathrm{Prime}(\theta) := {} & a(\theta)_{(1,1)} \cdot a(\theta)_{(3,1)} \cdot (3) + a(\theta)_{(1,1)} \cdot a(\theta)_{(3,3)} \cdot (3) \ldots \\ & + a(\theta)_{(1,3)} \cdot a(\theta)_{(3,1)} \cdot (1) + a(\theta)_{(1,3)} \cdot a(\theta)_{(3,3)} \cdot (-1) \end{aligned}$$

$$\Theta := 0, \frac{\pi}{36} .. \frac{\pi}{2}$$



$x := 0.5$

$\mathrm{root}(\sigma13Prime(x), x) = 0.183\pi$

$\mathrm{root}(\sigma13Prime(x), x) = 32.8505\,\mathrm{deg}$

Then this gives our rotation matrix to rotate to the principal stresses as:

$$a(32.8505\cdot deg) = \begin{pmatrix} 0.84 & 0 & 0.542 \\ 0 & 1 & 0 \\ -0.542 & 0 & 0.84 \end{pmatrix}$$

The Euler angles that get you to this point are found by finding first the values of the first two Euler angles that get the new z axis to be the bottom row. Once there the value of 2 can get the other axes to where they need to be.

We need to get x' to a position in the first rotation that is simultaneously 90 from the final z and the initial z - that would place the x' axis at the direction of the original y [010], so 1=90, then all we have to do is rotate z by the angle between z and the final z, which is a clockwise (or negative rotation) about the new x'.

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(-32.9\cdot deg) & \sin(-32.9\cdot deg) \\ 0 & -\sin(-32.9\cdot deg) & \cos(-32.9\cdot deg) \end{pmatrix} \cdot \begin{pmatrix} \cos(90\cdot deg) & \sin(90\cdot deg) & 0 \\ -\sin(90\cdot deg) & \cos(90\cdot deg) & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ -0.84 & 0 & -0.543 \\ -0.543 & 0 & 0.84 \end{pmatrix}$$

that cut-off result was:

$$\begin{pmatrix} 0 & 1 & 0 \\ -0.84 & 0 & -0.543 \\ -0.543 & 0 & 0.84 \end{pmatrix}$$

$$\begin{pmatrix} \cos(270\cdot deg) & \sin(270\cdot deg) & 0 \\ -\sin(270\cdot deg) & \cos(270\cdot deg) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 & 1 & 0 \\ -0.84 & 0 & -0.543 \\ -0.543 & 0 & 0.84 \end{pmatrix} = \begin{pmatrix} 0.84 & 0 & 0.543 \\ 0 & 1 & 0 \\ -0.543 & 0 & 0.84 \end{pmatrix}$$

so our Euler angles are (90, -32.86, 270) or something similar!

B.2.4.

$\nu = 0.4$ or $\nu = 0.6$

Note: When $\nu > 0.5$ there is an anomalous volume change – often this is not consistent with conventional elasticity (may be volume change from phase transformation or structural rearrangement).

$\varepsilon_x = 0.01$

$\nu = 0.4 \qquad \Delta = 0.01 - 0.004 - 0.004 = 0.002$

$\nu = 0.6 \qquad \Delta = 0.01 - 0.006 - 0.006 = -0.002$

Calculate from displacements (for these strain levels there is less than 1% difference in the engineering and true strain results).

$\nu = 0.4$	1.01 x 0.996 x 0.996 = 1.0019
	Δ = 0.0019
$\nu = 0.6$	1.01 x 0.994 x 0.994 = 0.9979
	Δ = -0.0021

B.2.5.

E = 2 GPa

For the axial dir.

$$\square = 0.005 = \frac{\square}{2000\text{MPa}}$$

$$\square = 10\ \text{MPa}$$

For the transverse dir.

$$\square_y = \frac{0 \square 0.4(\square_x)}{2\text{GPa}}$$

$$\frac{2000\text{MPa} \cdot 0.005}{\square 0.4} = \square_x$$

$$\square_x = \square 25\ \text{MPa}$$

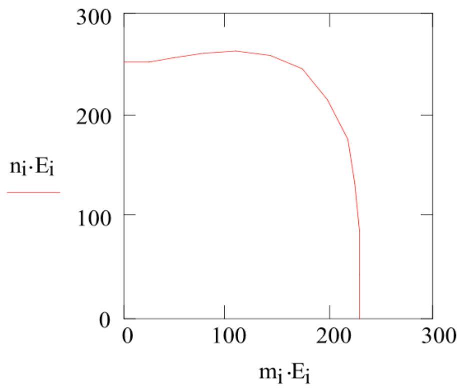
B.2.6 (Mathcad Example – ORIGIN=1 for subscripts in matrices)

The (110) plane - same as above but start with z pointing in the [110] direction

$$a(\phi 1) := \begin{pmatrix} \cos(\phi 1) & \sin(\phi 1) & 0 \\ -\sin(\phi 1) & \cos(\phi 1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix} \qquad i := 1..13 \qquad \eta_i := \frac{\pi}{24}\cdot i - \frac{\pi}{24}$$

$$l_i := a(\eta_i)_{1,1} \qquad m_i := a(\eta_i)_{1,2} \qquad n_i := a(\eta_i)_{1,3}$$

$$E_i := E_{hkl}(l_i, m_i, n_i)$$

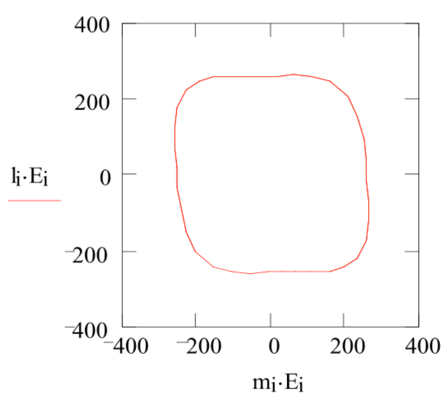


(123)plane with a little different plotting approach

$$a(\phi 1) := \begin{pmatrix} \cos(\phi 1) & \sin(\phi 1) & 0 \\ -\sin(\phi 1) & \cos(\phi 1) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{5}{\sqrt{42}} & \frac{-4}{\sqrt{42}} & \frac{1}{\sqrt{42}} \\ \frac{1}{\sqrt{14}} & \frac{2}{\sqrt{14}} & \frac{3}{\sqrt{14}} \end{pmatrix} \qquad i := 1..65 \qquad \eta_i := \frac{\pi}{18}\cdot i - \frac{\pi}{18}$$

$$l_i := a(\eta_i)_{1,1} \qquad m_i := a(\eta_i)_{1,2} \qquad n_i := a(\eta_i)_{1,3}$$

$$E_i := E_{hkl}(l_i, m_i, n_i)$$



B.2.7.

$$C'_{1133} = a_{1m}a_{1n}a_{3o}a_{3p}C_{mnop}$$

$$a_{11}a_{11}\cancel{a}^{o}_{31}\cancel{a}^{o}_{31}C_{1111} + \cancel{a}^{o}_{12}\cancel{a}^{o}_{12}a_{32}a_{32}C_{2222} + \cancel{a}^{o}_{13}\cancel{a}^{o}_{13}a_{33}a_{33}C_{3333}$$

$$\boxed{+a_{11}a_{11}a_{32}a_{32}C_{1122}} + \cancel{a}^{o}_{12}\cancel{a}^{o}_{12}\cancel{a}^{o}_{31}\cancel{a}^{o}_{31}C_{2211}$$

$$+\cancel{a}^{o}_{12}\cancel{a}^{o}_{12}a_{33}a_{33} + \cancel{a}^{o}_{13}\cancel{a}^{o}_{13}a_{32}a_{32}C_{3322}$$

$$\boxed{+a_{11}a_{11}a_{33}a_{33}C_{1133}} + \cancel{a}^{o}_{13}\cancel{a}^{o}_{13}a_{31}a_{31}C_{3311}$$

$$+\cancel{a}^{o}_{12}\cancel{a}^{o}_{12}a_{32}a_{32}C_{2323} + \cancel{a}^{o}_{12}\cancel{a}^{o}_{13}a_{33}a_{32}C_{2332} + \cancel{a}^{o}_{13}\cancel{a}^{o}_{12}a_{33}a_{32}C_{3232} + \cancel{a}^{o}_{13}\cancel{a}^{o}_{12}a_{32}a_{33}C_{3223}$$
$$+a_{11}\cancel{a}^{o}_{13}\cancel{a}^{o}_{31}a_{33}C_{1313} + a_{11}\cancel{a}^{o}_{13}a_{33}\cancel{a}^{o}_{31}C_{1331} + \cancel{a}^{o}_{13}a_{11}a_{33}\cancel{a}^{o}_{31}C_{3131} + \cancel{a}^{o}_{13}a_{11}\cancel{a}^{o}_{31}a_{33}C_{3113}$$
$$+a_{11}\cancel{a}^{o}_{12}\cancel{a}^{o}_{31}a_{32}C_{1212} + a_{11}\cancel{a}^{o}_{12}\cancel{a}^{o}_{32}\cancel{a}^{o}_{31}C_{1221} + \cancel{a}^{o}_{12}\cancel{a}^{o}_{11}\cancel{a}^{o}_{32}a_{31}C_{2121} + \cancel{a}^{o}_{12}a_{11}\cancel{a}^{o}_{31}a_{32}C_{2112}$$

$$C'_{1133} = a_{11}a_{11}a_{32}a_{32}C_{1122} + a_{11}a_{11}a_{33}a_{33}C_{1133}$$
$$= 1.1\cdot(-\sin\theta)^2 C_{12} + 1.1\cdot\cos^2\theta\, C_{13}$$
$$= \cos^2\theta\, C_{13} + \sin^2\theta\, C_{12} \qquad \text{orthorhombic}$$
$$= (\cos^2\theta + \sin^2\theta)C_{12} \qquad \text{cubic}$$

B.2.8

Note: Probably should be C problem.

$$a = \begin{vmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} \\ \frac{1}{\sqrt{1}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{-1}{\sqrt{6}} & \frac{+2}{\sqrt{6}} \end{vmatrix}$$

$$[110]x[1\bar{1}2] = 2\bar{2}\bar{2} \;\square\; 1\bar{1}\bar{1}$$

on 'back' of projection

Can get there by

$\phi_1 = 45°$

$\Phi = 35.2°$

$\phi_2 = -90°$

$$\square = C'_{44} = C'_{2323} = \text{(a's implied)}$$

Here the a's are implied and the first and second subscripts are written offet – which when you do it by hand makes it easier to check if everything is correct.

$$2_1 3_1 2_1 3_1 C_{1111} + 2_2 3_2 2_2 3_2 C_{2222} + \cancel{2}_3 3_3 \cancel{2}_3 3_3 C_{3333}$$

$$\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}} \qquad \frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}}\frac{-1}{\sqrt{2}}\frac{-1}{\sqrt{6}} \qquad 0\frac{+2}{\sqrt{6}}0\frac{-2}{\sqrt{6}}$$

$$2_1 3_1 2_2 3_2 C_{1122} + 2_2 3_2 2_1 3_1 C_{2211} + \cancel{2}_1 \cancel{3}_1 \cancel{2}_3 \cancel{3}_3 \cancel{C}_{1133} +$$

$$\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}} \qquad \frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{+1}{\sqrt{6}} \qquad 0$$

$$\cancel{2}_3 \cancel{3}_3 \cancel{2}_1 \cancel{3}_1 \cancel{C}_{3311} + \cancel{2}_2 \cancel{3}_2 \cancel{2}_2 \cancel{3}_2 \cancel{C}_{2233} + \cancel{2}_3 \cancel{3}_3 \cancel{2}_2 3_2 \cancel{C}_{3322}$$

$$0 \qquad\qquad 0 \qquad\qquad 0$$

$$2_2 3_3 2_2 3_3 C_{2323} + \cancel{2}_2 \cancel{3}_3 \cancel{2}_3 \cancel{3}_2 \cancel{C}_{2332} + \cancel{2}_3 3_2 \cancel{2}_3 3_2 C_{3232} + \cancel{2}_3 \cancel{3}_2 \cancel{2}_2 \cancel{3}_3 \cancel{C}_{3223} +$$

$$\frac{1}{\sqrt{2}}\frac{2}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{2}{\sqrt{6}} \qquad \frac{1}{\sqrt{2}}\frac{2}{\sqrt{2}}0 \qquad 0\frac{+2}{\sqrt{6}}0\frac{-2}{\sqrt{6}} \qquad 0$$

$$2_1 3_3 2_1 3_3 C_{1313} + \cancel{2}_1 \cancel{3}_3 \cancel{2}_3 \cancel{3}_1 \cancel{C}_{1331} + \cancel{2}_3 \cancel{3}_1 \cancel{2}_3 \cancel{3}_1 \cancel{C}_{3131} + \cancel{2}_3 \cancel{3}_1 \cancel{2}_1 \cancel{3}_3 \cancel{C}_{3113} +$$

$$\frac{1}{\sqrt{2}}\frac{2}{\sqrt{6}}\frac{2}{\sqrt{2}}\frac{2}{\sqrt{6}} \qquad 0 \qquad 0 \qquad 0$$

$$2_1 3_2 2_1 3_2 C_{1212} + 2_1 3_2 2_2 3_1 C_{1221} + 2_2 3_1 2_2 3_1 C_{2121} + 2_2 3_1 2_1 3_2 C_{2112}$$

$$\frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}} \qquad \frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}} \qquad \frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}} \qquad \frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}}$$

$$\left(\overbrace{\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}}\right)}^{\frac{1}{12}} + \overbrace{\left(\frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}}\frac{+1}{\sqrt{2}}\frac{=1}{\sqrt{6}}\right)}^{\frac{1}{12}}\right) C_{11} +$$

$$\left(\overbrace{\left(\frac{1}{\sqrt{2}}\frac{1}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}}\right)}^{-\frac{1}{12}} + \overbrace{\left(\frac{1}{\sqrt{2}}\frac{-1}{\sqrt{6}}\frac{1}{\sqrt{2}}\frac{+1}{\sqrt{6}}\right)}^{-\frac{1}{12}}\right) C_{12} +$$

$$\overset{\frac{4}{12}}{\left(\frac{1}{\sqrt2}\frac{2}{\sqrt6}\frac{1}{\sqrt2}\frac{2}{\sqrt6}\right)}+\overset{\frac{4}{12}}{\left(\frac{1}{\sqrt2}\frac{2}{\sqrt6}\frac{1}{\sqrt2}\frac{2}{\sqrt6}\right)}+\overset{\frac{1}{12}}{\left(\frac{1}{\sqrt2}\frac{-1}{\sqrt6}\frac{1}{\sqrt2}\frac{-1}{\sqrt6}\right)}+$$

$$\overset{\frac{-1}{12}}{\left(\frac{1}{\sqrt2}\frac{-1}{\sqrt6}\frac{1}{\sqrt2}\frac{2}{\sqrt6}\right)}+\overset{\frac{1}{12}}{\left(\frac{1}{\sqrt2}\frac{1}{\sqrt6}\frac{1}{\sqrt2}\frac{1}{\sqrt6}\right)}+\overset{\frac{-1}{12}}{\left(\frac{1}{\sqrt2}\frac{-1}{\sqrt6}\frac{1}{\sqrt2}\frac{-1}{\sqrt6}\right)}C_{44}$$

$$\left(\frac{1}{6}\right)C_{11}+\left(\frac{-1}{6}\right)C_{12}+\frac{2}{3}C_{44}$$

$$\frac{1}{6}(108)+\frac{-1}{6}(61)+\frac{2}{3}(18)=26.5\text{ GPa for Al}$$

$$\frac{1}{6}(237)+\frac{-1}{6}(141)+\frac{2}{3}(116)=93\text{ GPa for Fe}$$

B.2.9

$$a=\begin{vmatrix}\frac{1}{\sqrt2} & \frac{1}{\sqrt2} & 0\\ \frac{-1}{\sqrt2} & \frac{1}{\sqrt2} & 0\\ 0 & 0 & 1\end{vmatrix}$$

$$S^1_{1111}=a_{1m}a_{1n}a_{1o}a_{1p}S_{mnop}$$

(a's implied)

$$\underset{\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}}{11\ 11\ 11\ 11}\ S_{1111}+\underset{\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}}{12\ 12\ 12\ 12}\ S_{2222}+\cancel{13}\ \cancel{13}\ \cancel{13}\ \cancel{13}\ S_{3333}+$$

$$\underset{\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}}{11\ 11\ 12\ 12}\ S_{1122}+\underset{\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}}{12\ 12\ 11\ 11}\ S_{2211}+\cancel{11}\ \cancel{11}\ \cancel{13}\ \cancel{13}\ S_{1133}+$$

$$\underset{\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}}{13\ 13\ 11\ 11}\ S_{3311}+\underset{\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}\frac{1}{\sqrt2}}{12\ 12\ \cancel{13}\ \cancel{13}}\ S_{2233}+\overset{0}{\cancel{13}\ \cancel{13}\ 12\ 12}\ S_{3322}+$$

$$\underset{0}{13\ 13\ 11\ 11\ S_{3311}} + \underset{0}{12\ 12\ 13\ 13\ S_{2233}} + \underset{0}{13\ 13\ 12\ 12\ S_{3322}} +$$

$$\underset{0}{12\ 13\ 12\ 13\ S_{2323}} + \underset{0}{12\ 13\ 13\ 12\ S_{2332}} + \underset{0}{13\ 12\ 12\ 13\ S_{3223}} +$$

$$\underset{0}{11\ 13\ 11\ 13\ S_{1313}} + \underset{0}{11\ 13\ 13\ 11\ S_{1331}} + \underset{0}{13\ 11\ 13\ 11\ S_{3131}} + \underset{0}{13\ 11\ 11\ 13\ S_{3113}} +$$

$$\underset{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}{\overset{(\frac{1}{4}S_{44})}{11\ 12\ 11\ 12\ S_{1212}}} + \underset{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}{\overset{(\frac{1}{4}S_{44})}{11\ 12\ 12\ 11\ S_{1221}}} + \underset{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}{\overset{(\frac{1}{4}S_{44})}{12\ 11\ 12\ 11\ S_{2121}}} + \underset{\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}}{\overset{(\frac{1}{4}S_{44})}{12\ 11\ 11\ 12\ S_{2112}}}$$

$$\left(\frac{1}{4} + \frac{1}{4}\right)S_{11} + \left(\frac{1}{4} + \frac{1}{4}\right)S_{12} + \left(\frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}\right)\frac{S_{44}}{4}$$

$$\frac{1}{2}(0.98) + \frac{1}{2}(-0.27) + \frac{1.5}{4} = 0.73\text{x}10^{-11}\text{Pa}$$

$$E = 137\ \text{GPa}$$

B.2.10. (Mathcad Example – ORIGIN=1 for subscripts in matrices)

$i := 1..6$

$s11_i :=$	$s12_i :=$	$s44_i :=$
0.68	−.23	2.35
0.66	−.23	3.38
0.69	−.16	1.21
0.3	−.04	.99
0.28	−.08	.91
0.24	−.07	.62

$$E_{hkl}(s11, s12, s44, l, m, n) := \frac{1}{10^{-2}\cdot\left[s11 - 2\cdot\left(s11 - s12 - \frac{s44}{2}\right)\cdot\left(l^2\cdot m^2 + m^2\cdot n^2 + l^2\cdot n^2\right)\right]}$$

(a)

Just for comparison [100]

$metal_i :=$	$E_{hkl}\left(s11_i, s12_i, s44_i, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$	$E_{hkl}(s11_i, s12_i, s44_i, 1, 0, 0)$
V	116.7	147.059
Nb	83.8	151.515
Ta	189.9	144.928
Cr	247.9	333.333
Mo	291.3	357.143
W	416.7	416.667

(b) $\varepsilon_1 := S_{ij}\cdot\sigma_3$

Because Chromium is cubic - as long as we have not rotated the axes s12=s13!

$\varepsilon_1 := -0.04\cdot 10^{-11}\cdot Pa^{-1}\cdot -5\cdot 10^{6}\cdot Pa$

$\varepsilon_1 = 2 \times 10^{-6}$

(c) should stay square since the elastic constants are equal as shown in (b)