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SOLUTIONS TO PROBLEMS 2

2.1 From (2.8),

$$x^2 - \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)x + \left(\frac{1}{\alpha\beta}\right) = 0\,,\ \text{i.e.}\ \ \alpha\beta x^2 - (\alpha+\beta)x + 1 = 0\,.$$

But $\alpha + \beta = 2$ and $\alpha\beta = -3$, so the required equation is $3x^2 + 2x - 1 = 0$.

2.2 Rearranging the expression for p, we have the equation

$$5x^2 - 3px + (p+10) = 0\,.$$

Then using the general formula for the solution of a quadratic equation, (2.6b), real roots are only possible if $9p^2 \geq 20(p+10)$.

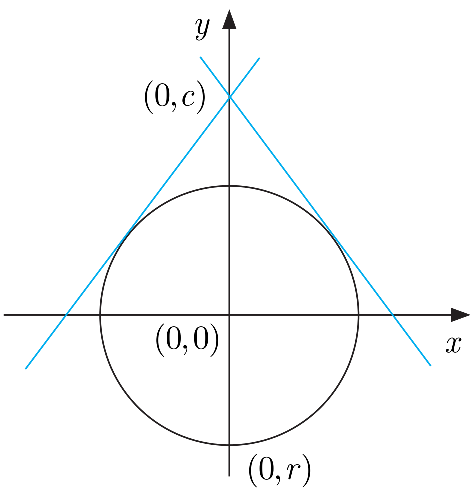
2.3 We need to solve the equation $y = mx + c$ simultaneously with the equation for the circle. Substituting for y gives, after rearrangement,

$$(1+m^2)x^2 + 2mcx + (c^2 - r^2) = 0\,,$$

which is a quadratic of the form $\alpha x^2 + \beta x + \gamma = 0$. This will have a single real root if $\beta^2 = 4\alpha\gamma$, i.e.

$$4m^2c^2 = 4(1+m^2)(c^2 - r^2), \quad \text{or} \quad m = \pm\left(\frac{c^2 - r^2}{r^2}\right)^{1/2}.$$

Thus there are two tangents as shown in the figure below.



2.4 The equations of the two circles are

$$(x-1)^2 + (y+1)^2 = 4, \quad \text{i.e.} \quad x^2 + y^2 - 2x + 2y - 2 = 0\,, \tag{1}$$

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and

$$x^2 + y^2 = 4\,, \tag{2}$$

respectively. Subtracting (2) from (1) gives

$$-2x + 2y = -2, \quad \text{i.e. } y = x - 1 \tag{3}$$

and substituting in (2) gives $2x^2 - 2x - 3 = 0$, with solutions

$$x = (1+\sqrt{7})/2 \quad \text{and} \quad x = (1-\sqrt{7})/2$$

respectively. From (3) the corresponding values of y are

$$y = (-1+\sqrt{7})/2 \quad \text{and} \quad y = (-1-\sqrt{7})/2\,.$$

Hence the co-ordinates of the points of intersection are

$$(x,y) = \left(\tfrac{1}{2} + \tfrac{\sqrt{7}}{2}, -\tfrac{1}{2} + \tfrac{\sqrt{7}}{2}\right) \text{ and } \left(\tfrac{1}{2} - \tfrac{\sqrt{7}}{2}, -\tfrac{1}{2} - \tfrac{\sqrt{7}}{2}\right)\,.$$

The length of the chord is $\sqrt{14}$ and the radius of the circle is 2 in each case. Hence the cosine rule (2.40a), with $a = \sqrt{14}, b = c = 2$, gives $\cos A = -6/8$ and $A = 2.42\,\text{rad} = 139^\circ$.

2.5 (a) We have

$$(x^3 + x^2 - x - 4) = (x-1)(ax^2 + bx + c) + R(x)\,.$$

Setting $x = 1$ gives $R = -3$; and multiplying out the bracket and equating powers of x gives $a = 1, b = 2, \; c = 1$, so that

$$(x^3 + x^2 - x - 4) = (x-1)(x^2 + 2x + 1) - 3\,.$$

(b) By long division,

$$\begin{array}{r|l}
 & 3x^2 + 5x + 5 \\
(x^2 - 2x + 3) & 3x^4 - x^3 + 4x^2 + 5x + 15 \\
 & 3x^4 - 6x^3 + 9x^2 \\
 & \quad 5x^3 - 5x^2 + 5x + 15 \\
 & \quad 5x^3 - 10x^2 + 15x \\
 & \qquad 5x^2 - 10x + 15 \\
 & \qquad 5x^2 - 10x + 15 \\
 & \qquad\qquad 0
\end{array}$$

so that

$$3x^4 - x^3 + 4x^2 + 5x + 15 = (x^2 - 2x + 3)(3x^2 + 5x + 5)\,,$$

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with the remainder $R(x)=0$. Since both (x^2-2x+3) and $(3x^2+5x+5)$ are of the form ax^2+bx+c with $b^2<4ac$, both of them, and hence the quartic $f(x)$ itself, have no real roots.

2.6 By inspection, one finds that $x=1$ and $x=2$ are roots. Hence, by the factor theorem,

$$x^4-2x^3-2x^2+5x-2=(x-1)(x-2)(ax^2+bx+c).$$

By comparing powers of x^4 on both sides one finds $a=1$, and by comparing the constant term, $c=-1$. Hence

$$x^4-2x^3-2x^2+5x-2=(x-1)(x-2)(x^2+bx-1).$$

Also, comparing powers of x^3 gives $b=1$ and hence

$$x^4-2x^3-2x^2+5x-2=(x-1)(x-2)(x^2+x-1).$$

The roots of (x^2+x-1) are $(-1\pm\sqrt{5})/2$, and so finally the four roots are

$$x=1,\ 2,\ (-1+\sqrt{5})/2,\ (-1-\sqrt{5})/2\ .$$

2.7 Using the bisection method gives,

n	$\boldsymbol{x_n}$		$\boldsymbol{f(x_n)}$
1	$x_1=$	1.500000000	−0.12500000
2	$x_2=$	1.600000000	0.37600000
3	$x_3=\frac{1}{2}(x_1+x_2)=$	1.550000000	0.11888000
4	$x_4=\frac{1}{2}(x_1+x_3)=$	1.525000000	−0.00467200
5	$x_5=\frac{1}{2}(x_3+x_4)=$	1.537500000	0.05669000
6	$x_6=\frac{1}{2}(x_4+x_5)=$	1.531250000	0.02591100
7	$x_7=\frac{1}{2}(x_4+x_6)=$	1.528125000	0.01059000
8	$x_8=\frac{1}{2}(x_4+x_7)=$	1.526562500	0.00295500
9	$x_9=\frac{1}{2}(x_4+x_8)=$	1.525781250	−0.00086020
10	$x_{10}=\frac{1}{2}(x_8+x_9)=$	1.526171875	0.00104700
11	$x_{11}=\frac{1}{2}(x_9+x_{10})=$	1.525976563	0.00009315

Thus the root correct to three decimal places is 1.526.

2.8 (a) Setting

$$\frac{2(x^2-9x+11)}{(x-2)(x-3)(x+4)}=\frac{A}{(x-2)}+\frac{B}{(x-3)}+\frac{C}{(x+4)},$$

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gives

$$2(x^2-9x+11)=A(x-3)(x+4)+B(x-2)(x+4)+C(x-2)(x-3)$$

and using $x=2$, $x=3$ and $x=-4$ in turn, yields $A=1$, $B=-2$ and $C=3$, so that

$$\frac{2(x^2-9x+11)}{(x-2)(x-3)(x+4)}=\frac{1}{(x-2)}-\frac{2}{(x-3)}+\frac{3}{(x+4)}.$$

(b) Setting

$$\frac{7x^2+6x-13}{(2x+1)(x^2+2x-4)}=\frac{Ax+B}{x^2+2x-4}+\frac{C}{2x+1},$$

gives

$$7x^2+6x-13=(Ax+B)(2x+1)+C(x^2+2x-4)$$

and equating coefficients of powers of x yields $A=2$, $B=-1$ and $C=3$, so that

$$\frac{7x^2+6x-13}{(2x+1)(x^2+2x-4)}=\frac{2x-1}{x^2+2x-4}+\frac{3}{2x+1}.$$

(c) Setting

$$\frac{2(3x^2+4x+2)}{(x-1)(2x+1)^2}=\frac{A}{x-1}+\frac{B}{2x+1}+\frac{C}{(2x+1)^2},$$

gives

$$2(3x^2+4x+2)=A(2x+1)^2+B(2x+1)(x-1)+C(x-1)$$

and equating coefficients of powers of x yields $A=2$, $B=-1$ and $C=-1$, so that

$$\frac{2(3x^2+4x+2)}{(x-1)(2x+1)^2}=\frac{2}{x-1}-\frac{1}{2x+1}-\frac{1}{(2x+1)^2}.$$

2.9 (a) Performing a long division gives

$$\frac{x^3-2x^2+10}{(x-1)(x+2)}=(x-3)+\frac{5x+4}{(x-1)(x+2)}.$$

Then, setting

$$\frac{5x+4}{(x-1)(x+2)}=\frac{A}{x-1}+\frac{B}{x+2}$$

gives

$$A(x+2)+B(x-1)=5x+4\Rightarrow A=3,\ B=2$$

and so

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$$\frac{x^3-2x^2+10}{(x-1)(x+2)}=(x-3)+\frac{3}{(x-1)}+\frac{2}{(x+2)}.$$

(b) Setting

$$\frac{3x^2-5x-4}{(x+2)(3x^2+x-1)}=\frac{A}{(x+2)}+\frac{Bx+C}{(3x^2+x-1)},$$

gives

$$3x^2-5x-4=A(3x^2+x-1)+(x+2)(Bx+C),$$

and choosing $x=-2$, yields $A=2$. Then

$$3x^2-5x-4=(6+B)x^2+(2B+C+2)x+(2C-2),$$

and equating coefficients of powers of x, gives $B=-3$ and $C=-1$. So finally,

$$\frac{3x^2-5x-4}{(x+2)(3x^2+x-1)}=\frac{2}{(x+2)}-\frac{3x+1}{(3x^2+x-1)}.$$

(c) Setting

$$\frac{3x^2-x+2}{(x-1)(x-3)^3}=\frac{A}{(x-1)}+\frac{B}{(x-3)}+\frac{C}{(x-3)^2}+\frac{D}{(x-3)^3},$$

gives

$$3x^2-x+2=A(x-3)^3+B(x-1)(x-3)^2+C(x-1)(x-3)+D(x-1).$$

Letting $x=1$ yields $A=-1/2$, and letting $x=3$ yields $D=13$. Then equating the coefficients of x^3 gives $B=1/2$ and equating the constants on both sides gives $C=2$. So, finally

$$\frac{3x^2-x+2}{(x-1)(x-3)^3}=-\frac{1}{2(x-1)}+\frac{1}{2(x-3)}+\frac{2}{(x-3)^2}+\frac{13}{(x-3)^3}.$$

2.10 (a) Using the double-angle formulas,

$$\cos 4\theta\equiv 2\cos^2 2\theta-1 \quad\text{and}\quad \cos 2\theta\equiv 2\cos^2\theta-1,$$

so that

$$\cos 4\theta\equiv 2(2\cos^2\theta-1)^2\equiv 8\cos^4\theta-8\cos^2\theta+1.$$

(b) Using the identities (2.36c) and (2.36d).

$$\sin(n\theta)+\sin[(n+4)\theta]\equiv 2\sin[(n+2)\theta]\cos(2\theta)$$

and

$$\cos(n\theta)+\cos[(n+4)\theta]\equiv 2\cos[(n+2)\theta]\cos(2\theta),$$

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in the left-hand side of the identity gives

$$\frac{\sin[(n+2)\theta][1+2\cos(2\theta)]}{\cos[(n+2)\theta][1+2\cos(2\theta)]} \equiv \tan[(n+2)\theta].$$

(c)

$$\left(\frac{\sin 5\theta}{\sin\theta}\right)^2 - \left(\frac{\cos 5\theta}{\cos\theta}\right)^2 = \left(\frac{\sin 5\theta}{\sin\theta} - \frac{\cos 5\theta}{\cos\theta}\right)\left(\frac{\sin 5\theta}{\sin\theta} + \frac{\cos 5\theta}{\cos\theta}\right)$$
$$= \left(\frac{\sin 5\theta\cos\theta - \cos 5\theta\sin\theta}{\sin\theta\cos\theta}\right)\left(\frac{\sin 5\theta\cos\theta + \cos 5\theta\sin\theta}{\sin\theta\cos\theta}\right)$$
$$= \frac{4\sin 4\theta\sin 6\theta}{\sin 2\theta\sin 2\theta}.$$

Then from (2.36c), $\sin 6\theta$ may be written

$$\sin 6\theta = \sin 4\theta\cos 2\theta + \cos 4\theta\sin 2\theta\,,$$

and using the double-angle formulas (2.37a) on the right-hand side,

$$\sin 6\theta = 3\sin 2\theta - 4\sin^3 2\theta\,,$$

so that

$$\frac{4\sin 4\theta\sin 6\theta}{\sin 2\theta\sin 2\theta} = \frac{8\sin 2\theta\cos 2\theta(3\sin 2\theta - 4\sin^3 2\theta)}{\sin^2 2\theta},$$

and finally

$$\left(\frac{\sin 5\theta}{\sin\theta}\right)^2 - \left(\frac{\cos 5\theta}{\cos\theta}\right)^2 = 8\cos 2\theta(3-4\sin^2 2\theta) = 8\cos 2\theta(4\cos^2 2\theta - 1).$$

2.11 (a) Using the double-angle formula (2.37a), we have

$$2\cos\theta\cos 2\theta + 2\sin\theta\cos\theta = 2\cos\theta(3\cos^2\theta - 1)\,,$$

so the first solution is $\cos\theta = 0$, i.e. $\theta = \frac{\pi}{2},\ \frac{3\pi}{2}$. If $\cos\theta \neq 0$, we can divide by $2\cos\theta$ to give

$$\cos 2\theta + \sin\theta = 3\cos^2\theta - 1\,,$$

which again using the double-angle formula and writing everything in terms of $\sin\theta$, reduces to the quadratic $\sin^2\theta + \sin\theta - 1 = 0$ which leads directly to solutions $\theta = 0.666$ and 2.475 radians.

(b) Combining the first and third terms using (2.36) gives

$$2\sin 2\theta\cos\theta - 2\sin\theta\cos\theta = 0\,,$$

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so the first solution is

$$\cos\theta = 0 \Rightarrow \theta = \tfrac{\pi}{2},\ \tfrac{3\pi}{2}\,.$$

If $\cos\theta \neq 0$, then dividing gives

$$\sin\theta(2\cos\theta - 1) = 0\,.$$

The two possibilities are

$$\sin\theta = 0 \Rightarrow \theta = \pi\,,\ \text{or}\ \cos\theta = \tfrac{1}{2} \Rightarrow \theta = \tfrac{\pi}{3},\ \tfrac{5\pi}{3}\,.$$

So finally

$$\theta = \tfrac{\pi}{3},\ \tfrac{\pi}{2},\ \pi,\ \tfrac{3\pi}{2}\ \text{and}\ \tfrac{5\pi}{3}\,.$$

2.12 We have

$$\sin k\theta - \sin\theta \equiv 2\sin[\tfrac{1}{2}(k-1)\theta]\cos[\tfrac{1}{2}(k+1)\theta]\,.$$

Thus one of the two brackets must be zero. The first bracket is zero if

$$\tfrac{1}{2}(k-1)\theta = \pm n\pi,\ \text{i.e.}\ k\theta = \pm n\pi + \theta \Rightarrow \theta = \pm 2n\pi/(k-1)\,,$$

for all integer n and $k \neq 1$. The second bracket is zero if

$$\tfrac{1}{2}(k+1)\theta = \pm\tfrac{1}{2}(2n+1)\pi,\ \text{i.e.}\ k\theta = \pm(2n+1)\pi - \theta \Rightarrow \theta = \pm(2n+1)\pi/(k+1)\,,$$

for all integer n and $k \neq -1$.

2.13 From the equation of the straight line $x = (p - y\cos\theta)/\sin\theta$ and substituting into the equation for the hyperbola gives

$$\frac{p^2 - 2py\cos\theta + y^2\cos^2\theta}{a^2\sin^2\theta} - \frac{y^2}{b^2} = 1\,,$$

which is a quadratic in y of the form $Ay^2 + By + C = 0$, where

$$A = \left[\frac{\cos^2\theta}{a^2\sin^2\theta} - \frac{1}{b^2}\right],\ B = -\left[\frac{2p\cos\theta}{a^2\sin^2\theta}\right]\ \text{and}\ C = \left[\frac{p^2}{a^2\sin^2\theta} - 1\right].$$

If the line is to be a tangent, then there can be only one solution of this quadratic, the condition for which is that $B^2 = 4AC$, i.e.

$$\left[\frac{2p\cos\theta}{a^2\sin^2\theta}\right]^2 = 4\left[\frac{\cos^2\theta}{a^2\sin^2\theta} - \frac{1}{b^2}\right]\left[\frac{p^2}{a^2\sin^2\theta} - 1\right],$$

which after simplifying gives $a^2\sin^2\theta - b^2\cos^2\theta = p^2$, as required.

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The y-co-ordinate of the point of intersection is $y = -B/2A$, i.e.

$$y = \frac{2p\cos\theta}{2a^2\sin^2\theta}\cdot\frac{a^2b^2\sin^2\theta}{b^2\cos^2\theta - a^2\sin^2\theta} = \frac{-b^2\cos\theta}{p}$$

and hence

$$x = \frac{p - y\cos\theta}{\sin\theta} = \frac{a^2\sin\theta}{p}.$$

2.14 Equation (2.37c) may be used on the left-hand side to give

$$\frac{1+\sin\theta+\cos\theta}{1+\sin\theta-\cos\theta} \equiv \frac{1+(2\sin\theta/2\cos\theta/2)+(2\cos^2\theta/2-1)}{1+(2\sin\theta/2\cos\theta/2)-(1-2\sin^2\theta/2)}$$
$$\equiv \frac{\cos\theta/2}{\sin\theta/2} \equiv \frac{2\cos^2\theta/2}{2\sin\theta/2\cos\theta/2} \equiv \frac{1+\cos\theta}{\sin\theta}.$$

2.15 From the sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \sin A = \frac{a\sin B}{b}.$$

So $A = \arcsin\left[5\sin(0.5)/4\right] = 0.643\text{ rad} = 36.84^0$ and hence $C = 1.999\,\text{rad} = 114.51^0$. Again using the sine rule, $c = b\sin C/\sin B = 7.59\,\text{cm}$.

2.16 The lengths of the sides are: $a = BC = \sqrt{(5-7)^2+(6-2)^2} = \sqrt{20}$ and likewise $b = AC = \sqrt{37}$ and $c = AB = \sqrt{25}$. Then using the cosine rule

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{21}{5\sqrt{37}},$$

giving $A = 0.809\,\text{rad} = 46.35^0$. In a similar way, $B = 1.391\text{ rad} = 79.70^0$ and $C = 0.942\text{ rad} = 53.97^0$.

2.17 For $n = 0$ the results are trivial. So we just need to show that if both are true for any $n \geq 0$ they are true for $m = n+1$. Suppose both results are true for n. Then

$$\begin{aligned}\sin[(2m+1)\theta] &= \sin[(2n+1)\theta + 2\theta] \\ &= \sin[(2n+1)\theta]\cos 2\theta + \cos[(2n+1)\theta]\sin 2\theta \\ &= \sin[(2n+1)\theta][1-2\sin^2\theta] + \left[\frac{\cos[(2n+1)\theta]}{\cos\theta}\right]2\sin\theta\cos^2\theta,\end{aligned}$$

which is a polynomial in $\sin\theta$ since both $\sin[(2n+1)\theta]$ and $\cos[(2n+1)\theta]/\cos\theta$ are polynomials and $\cos^2\theta = 1-\sin^2\theta$. Similarly,