

# Chapter 2

## Problem Solutions

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### Problem 2.1

The antenna beamwidth is  $2\tan^{-1}\left(\frac{0.25 \text{ km}}{2 \text{ km}}\right) = 14.3^\circ$ . Equation (2.5) gives

$$\text{Beamwidth} = 14.3^\circ = 66^\circ \frac{\lambda}{1 \text{ m}},$$

so  $\lambda = 0.216 \text{ m}$ . Then Equation (2.1) gives  $f = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.316} = 1.39 \text{ GHz}$ .

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### Problem 2.2

The received power is given by Equation (2.12). Substituting Equation (2.6) for the gain of the receiving antenna gives

$$P_r = \frac{P_t G_t \eta A_{er}}{4\pi d^2},$$

where  $P_t = P_0$  and  $G_t = 1$ . For the first antenna we have

$$P_r = \frac{P_0 \times 0.9 \times (0.06 \text{ m}^2)}{4\pi (2 \times 10^3 \text{ m})^2}.$$

For the second antenna we have

$$P_r = \frac{P_0 \times 0.8 \times (1.0 \text{ m}^2)}{4\pi d^2}.$$

Equating and solving for  $d$  gives  $d = 7.70 \text{ km}$ .

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### Problem 2.3

Wavelength is given by Equation (2.1) and gain is given by Equation (2.6). Substituting the given efficiencies, effective apertures, and frequencies gives

*Antenna Gains in dBi*

	$f = 800 \text{ MHz}$	$f = 1.9 \text{ GHz}$
Antenna 1	6.84	14.3
Antenna 2	18.5	26.1

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**Problem 2.4**

Taking  $A_{er} = \frac{\pi L^2}{4}$  gives diameters for the two antennas of  $L = 0.276 \text{ m}$  and  $L = 1.13 \text{ m}$  respectively. Using Equation (2.5), the beamwidth of each antenna at each frequency is given by

*Antenna Beamwidths*

	$f = 800 \text{ MHz}$	$f = 1.9 \text{ GHz}$
Antenna 1	69.2°	29.1°
Antenna 2	16.9°	7.1°

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**Problem 2.5**

The wavelength is given by Equation (2.1),  $\lambda = \frac{3 \times 10^8}{890 \times 10^6} = 0.337 \text{ m}$ .

Then the path loss can be found from Equation (2.21). That is,

$$L_{path} \Big|_{dB} = 10 \log \left( \frac{4\pi 5 \times 10^3}{0.337} \right)^2 = 105 \text{ dB}.$$

The gain of the receiving antenna

is 2 dBd, which is 4.15 dBi. We then have

$$-87 \text{ dBm} = EIRP \Big|_{dB} + 4.15 \text{ dB} - 105 \text{ dB} - 2.8 \text{ dB}.$$

Solving gives  $EIRP \Big|_{dB} = 17.1 \text{ dBm}$ .

Now  $17.1 \text{ dBm} = P_t \Big|_{dB} + 7.5 \text{ dB}$ , so  $P_t \Big|_{dB} = 9.56 \text{ dBm}$ . In watts this is  $P_t = 9.03 \text{ mW}$ .

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**Problem 2.6**

The received signal power is given by

$$\begin{aligned}
 P_r|_{dB} &= EIRP|_{dB} + G_r|_{dB} - L_{sys}|_{dB} - L_{path}|_{dB} \\
 &= 30 \text{ dBm} + 5 \text{ dB} - 2 \text{ dB} - L_{path}|_{dB} \\
 &= 33 \text{ dBm} - L_{path}|_{dB},
 \end{aligned}$$

with  $L_{path}|_{dB} = 10 \log \left( \frac{4\pi 10^3}{\lambda} \right)^2$  for a distance of 1 km.

- A. At a frequency of 900 MHz,  $\lambda = 0.333 \text{ m}$ ,  $L_{path}|_{dB} = 91.5 \text{ dB}$ , and  $P_r|_{dB} = -58.5 \text{ dBm}$ .
- B. At a frequency of 1800 MHz,  $\lambda = 0.167 \text{ m}$ ,  $L_{path}|_{dB} = 97.5 \text{ dB}$ , and  $P_r|_{dB} = -64.5 \text{ dBm}$ .
- C. The received signal level drops 6 dB when the frequency doubles. In this problem the receiving antenna gain is held constant. This means that the antenna aperture must decrease as the frequency increases. The smaller receiving antenna aperture in part B captures correspondingly less power than the larger aperture in part A.

### Problem 2.7

- A. At 1.9 GHz the wavelength is given by  $\lambda = \frac{3 \times 10^8}{1.9 \times 10^9} = 0.158 \text{ m}$ . The path loss is then given by

$$L_{path}|_{dB} = 10 \log \left( \frac{4\pi d}{\lambda} \right)^2 = 10 \log \left( \frac{4\pi 10 \times 10^3}{0.158} \right)^2 = 118 \text{ dB.}$$

- B. Antenna gains are given in terms of the effective apertures by Equation (2.6). For the transmitter we have

$$G_t|_{dB} = 10 \log \left( \frac{4\pi A_{et}}{\lambda^2} \right) = 10 \log \left( \frac{4\pi 0.9}{(0.158)^2} \right) = 26.6 \text{ dBi,}$$

and for the receiver

$$G_r|_{dB} = 10 \log \left( \frac{4\pi 100 \times 10^{-4}}{(0.158)^2} \right) = 7.02 \text{ dBi.}$$

- C. The transmit power is 5 W, which is  $P_t|_{dB} = 37 \text{ dBm}$ . Then  $EIRP|_{dB} = 37 \text{ dBm} + 26.6 \text{ dB} = 63.6 \text{ dBm}$ .
- D. The power at the output of the receive antenna is

$$\begin{aligned}
 P_r|_{dB} &= EIRP|_{dB} + G_r|_{dB} - L_{path}|_{dB} - L_{sys}|_{dB} \\
 &= 63.6 \text{ dBm} + 7.02 \text{ dB} - 118 \text{ dB} - 2 \text{ dB} \\
 &= -49.4 \text{ dBm}.
 \end{aligned}$$

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### Problem 2.8

Begin with some preliminaries:

The transmitted power is  $P_t|_{dB} = 10 \log\left(\frac{5 \text{ W}}{0.001 \text{ W}}\right) = 37.0 \text{ dBm}$ .

The effective aperture of the transmit antenna is

$$A_{et} = \pi \frac{(0.3)^2}{4} = 0.0707 \text{ m}^2.$$

The wavelength is  $\lambda = \frac{3 \times 10^8}{1800 \times 10^6} = 0.167 \text{ m}$ .

Then the gain of the transmit antenna is  $G_t|_{dB} = 10 \log\left(\frac{4\pi A_{et}}{\lambda^2}\right) = 15.0 \text{ dB}$ .

The gain of the receive antenna is  $G_r|_{dB} = 2.15 \text{ dB}$  for a dipole.

A. Since a received signal level of  $-80 \text{ dBm}$  is needed, we have

$$-80 \text{ dBm} = 37.0 \text{ dBm} + 15.0 \text{ dB} + 2.15 \text{ dB} - L_{path}|_{dB}.$$

This gives  $L_{path}|_{dB} = 134 \text{ dB}$ . Then

$$L_{path}|_{dB} = 20 \log\left(\frac{4\pi d}{\lambda}\right),$$

from which  $d = 68.1 \text{ km}$ .

B. At the 3 dB point of the beam we must have 3 dB less path loss to maintain the received signal level. Using  $L_{path}|_{dB} = 131 \text{ dB}$  gives  $d = 47.2 \text{ km}$ .

C. The beamwidth of the transmit antenna is  $55^\circ \frac{0.167 \text{ m}}{0.3 \text{ m}} = 30.6^\circ$ . Then if  $y$  is the separation between antennas,

$$\sin\left(\frac{30.6^\circ}{2}\right) = \frac{y/2}{47.2 \text{ km}}.$$

Solving gives  $y = 24.9 \text{ km}$ .

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### Problem 2.9

Combining the resistors in parallel and series gives an equivalent resistance of  $R_{eq} = 889 \text{ k}\Omega$ . Equation (2.39) gives

$$P = 4kTRB = 4(1.38 \times 10^{-23})(290)(889 \times 10^3)(100 \times 10^3) \\ = 1.49 \times 10^{-9} \text{ V}^2.$$

The RMS voltage is

$$V_{RMS} = \sqrt{1.49 \times 10^{-9}} = 37.7 \text{ }\mu\text{V}.$$

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### Problem 2.10

Equation (2.77) gives the noise bandwidth of a lowpass Butterworth filter in terms of the 3 dB bandwidth as

$$B = \frac{\frac{\pi}{2} f_{3dB}}{n \sin\left(\frac{\pi}{2n}\right)}.$$

If  $B = 1.01f_{3dB}$ , we can solve for  $n$ . The result is  $n = 6.4$ , which must be rounded up to the integer  $n = 7$ .

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### Problem 2.11

The power spectrum of the noise at the amplifier output is given by

$$S_o(f) = |H(f)|^2 S_i(f) = \frac{404 \times 10^{24} \frac{N_0}{2}}{f^4 - 200 \times 10^{12} f^2 + 1.04 \times 10^{28}}.$$

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### Problem 2.12

For the given amplifier,

$$P_o = \int_{-\infty}^{\infty} S_o(f) df \\ = \int_{-\infty}^{\infty} \frac{404 \times 10^{24} \frac{N_0}{2}}{f^4 - 200 \times 10^{12} f^2 + 1.04 \times 10^{28}} df \\ = 3.13 \times 10^6 N_0.$$