Solutions Manual

Solutions to Chapter 1 Problems

S.1.1

The principal stresses are given directly by Eqs (1.11) and (1.12) in which $\sigma_x = 80\text{N/mm}^2$, $\sigma_y = 0$ (or vice versa) and $\tau_{xy} = 45\text{N/mm}^2$. Thus, from Eq. (1.11)

$$\sigma_{\text{I}} = \frac{80}{2} + \frac{1}{2}\sqrt{80^2 + 4 \times 45^2}$$

i.e.

$$\sigma_{\text{I}} = 100.2 \text{ N/mm}^2$$

From Eq. (1.12)

$$\sigma_{\text{II}} = \frac{80}{2} - \frac{1}{2}\sqrt{80^2 + 4 \times 45^2}$$

i.e.

$$\sigma_{\text{II}} = -20.2 \text{ N/mm}^2$$

The directions of the principal stresses are defined by the angle θ in Fig. 1.8(b) in which θ is given by Eq. (1.10). Hence

$$\tan 2\theta = \frac{2 \times 45}{80 - 0} = 1.125$$

which gives

$$\theta = 24°11' \quad \text{and} \quad \theta = 114°11'$$

It is clear from the derivation of Eqs (1.11) and (1.12) that the first value of θ corresponds to σ_{I} while the second value corresponds to σ_{II}.

Finally, the maximum shear stress is obtained from either of Eqs (1.14) or (1.15). Hence from Eq. (1.15)

$$\tau_{\max} = \frac{100.2 - (-20.2)}{2} = 60.2\text{N}/\text{mm}^2$$

and will act on planes at 45° to the principal planes.

S.1.2

The principal stresses are given directly by Eqs (1.11) and (1.12) in which σ_x = 50N/mm^2, σ_y = –35 N/mm^2 and τ_{xy} = 40 N/mm^2. Thus, from Eq. (1.11)

$$\sigma_{\text{I}} = \frac{50-35}{2} + \frac{1}{2}\sqrt{(50+35)^2 + 4\times 40^2}$$

i.e.

$$\sigma_{\text{I}} = 65.9 \text{ N/mm}^2$$

and from Eq. (1.12)

$$\sigma_{\text{II}} = \frac{50-35}{2} - \frac{1}{2}\sqrt{(50+35)^2 + 4\times 40^2}$$

i.e.

$$\sigma_{\text{II}} = -50.9 \text{ N/mm}^2$$

From Fig. 1.8(b) and Eq. (1.10)

$$\tan 2\theta = \frac{2\times 40}{50+35} = 0.941$$

which gives

$$\theta = 21°38'(\sigma_{\text{I}}) \quad \text{and} \quad \theta = 111°38'(\sigma_{\text{II}})$$

The planes on which there is no direct stress may be found by considering the triangular element of unit thickness shown in Fig. S.1.2 where the plane AC represents the plane on which there is no direct stress. For equilibrium of the element in a direction perpendicular to AC

$$0 = 50\text{AB}\cos\alpha - 35\text{BC}\sin\alpha + 40\text{AB}\sin\alpha + 40\text{BC}\cos\alpha \qquad \text{(i)}$$

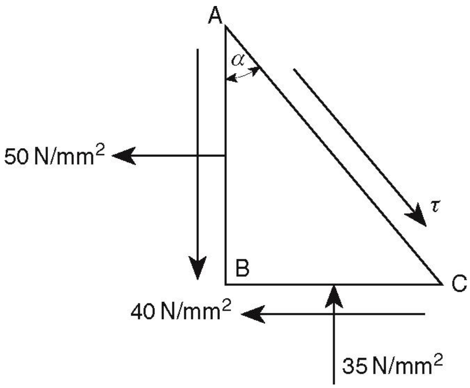

Fig. S.1.2

Dividing through Eq. (i) by AB

$$0 = 50\cos\alpha - 35\tan\alpha\sin\alpha + 40\sin\alpha + 40\tan\alpha\cos\alpha$$

which, dividing through by cos α, simplifies to

$$0 = 50 - 35\tan^2\alpha + 80\tan\alpha$$

from which

$$\tan\alpha = 2.797 \quad \text{or} \quad -0.511$$

Hence

$$\alpha = 70°21' \quad \text{or} \quad -27°5'$$

S.1.3

The construction of Mohr's circle for each stress combination follows the procedure described in Section 1.8 and is shown in Figs S.1.3(a)–(d).

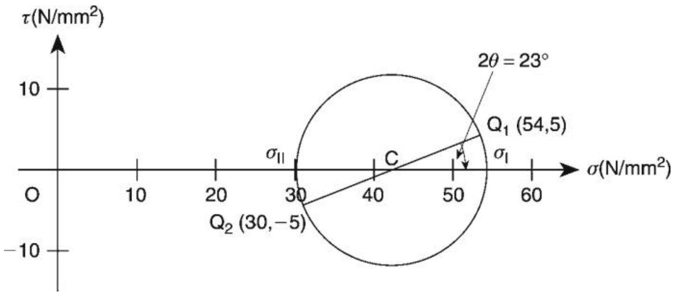

Fig. S.1.3(a)

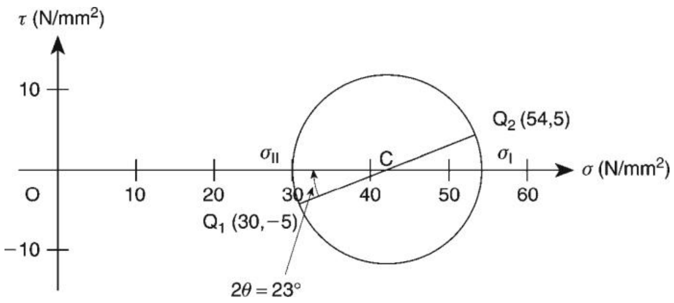

Fig. S.1.3(b)

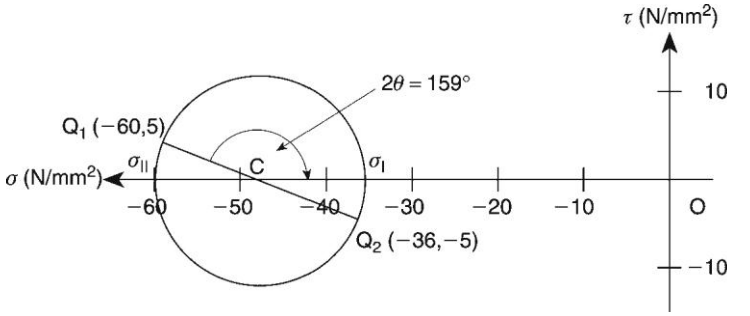

Fig. S.1.3(c)

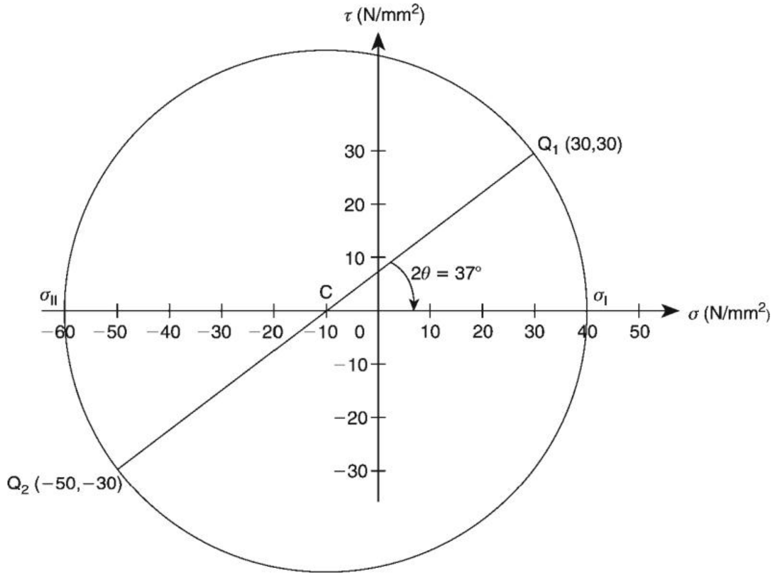

Fig. S.1.3(d)

S.1.4

The principal stresses at the point are determined, as indicated in the question, by transforming each state of stress into a σ_x, σ_y, τ_{xy} stress system. Clearly, in the first case $\sigma_x = 0$, $\sigma_y = 10$ N/mm^2, $\tau_{xy} = 0$ (Fig. S.1.4(a)). The two remaining cases are transformed by considering the equilibrium of the triangular element ABC in Figs S.1.4(b), (c), (e) and (f). Thus, using the method described in Section 1.6 and the principle of superposition (see Section 5.9), the second stress system of Figs S.1.4(b) and (c) becomes the σ_x, σ_y, τ_{xy} system shown in Fig. S.1.4(d) while

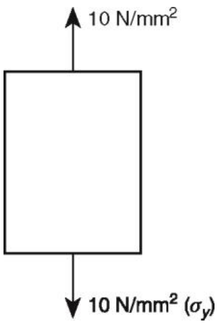

Fig. S.1.4(a)

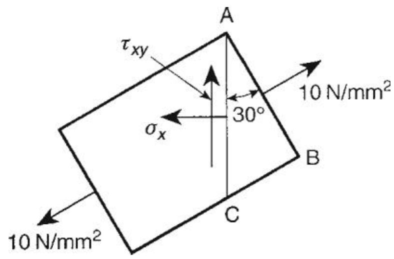

Fig. S.1.4(b)

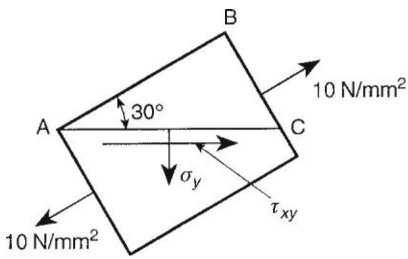

Fig. S.1.4(c)

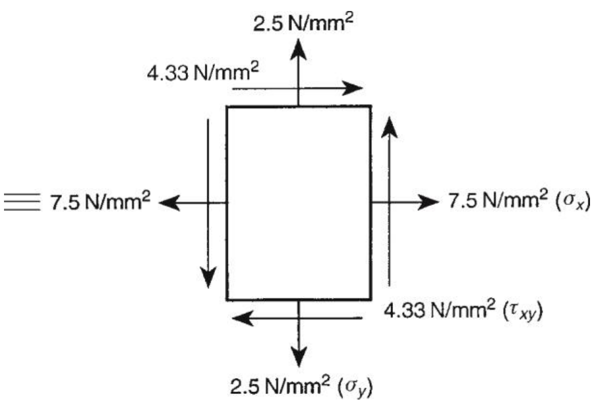

Fig. S.1.4(d)

the third stress system of Figs S.1.4(e) and (f) transforms into the σ_x, σ_y, τ_{xy} system of Fig. S.1.4(g).

Finally, the states of stress shown in Figs S.1.4(a), (d) and (g) are superimposed to give the state of stress shown in Fig. S.1.4(h) from which it can be seen that $\sigma_{\mathrm{I}} = \sigma_{\mathrm{II}} = 15\,\mathrm{N/mm}^2$ and that the *x* and *y* planes are principal planes.

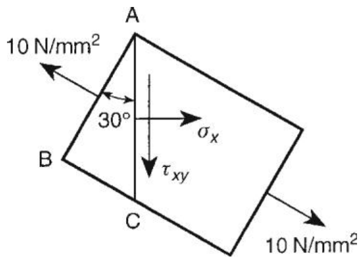

Fig. S.1.4(e)

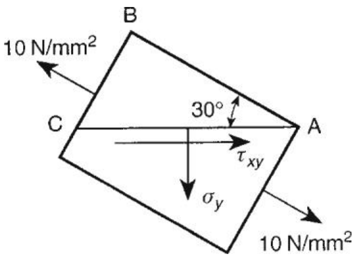

Fig. S.1.4(f)

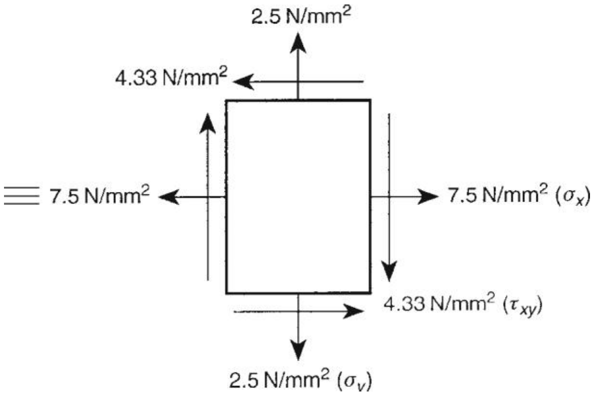

Fig. S.1.4(g)

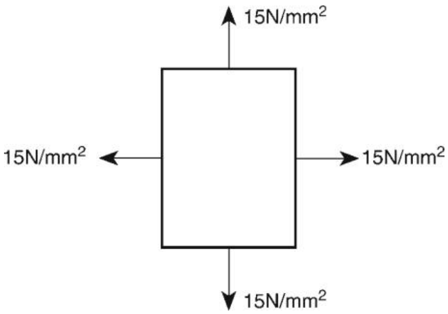

Fig. S.1.4(h)

S.1.5

The geometry of Mohr's circle of stress is shown in Fig. S.1.5 in which the circle is constructed using the method described in Section 1.8.

From Fig. S.1.5

$$\sigma_x = OP_1 = OB - BC + CP_1 \qquad \text{(i)}$$

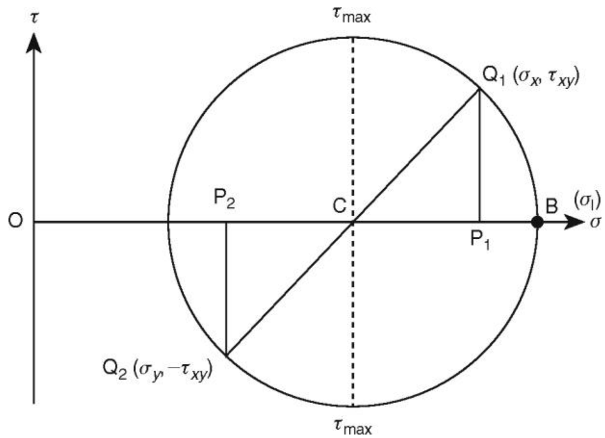

Fig. S.1.5

In Eq. (i) OB = σ_I, BC is the radius of the circle which is equal to τ_{max} and $CP_1 = \sqrt{CQ_1^2 - Q_1P_1^2} = \sqrt{\tau_{max}^2 - \tau_{xy}^2}$. Hence

$$\sigma_x = \sigma_1 - \tau_{max} + \sqrt{\tau_{max}^2 - \tau_{xy}^2}$$

Similarly

$$\sigma_y = OP_2 = OB - BC - CP_2 \text{ in which } CP_2 = CP_1$$

Thus

$$\sigma_y = \sigma_I - \tau_{max} - \sqrt{\tau_{max}^2 - \tau_{xy}^2}$$

S.1.6

From bending theory the direct stress due to bending on the upper surface of the shaft at a point in the vertical plane of symmetry is given by

$$\sigma_x = \frac{My}{I} = \frac{25 \times 10^6 \times 75}{\pi \times 150^4 / 64} = 75 \text{ N/mm}^2$$

From the theory of the torsion of circular section shafts the shear stress at the same point is

$$\tau_{xy} = \frac{Tr}{J} = \frac{50 \times 10^6 \times 75}{\pi \times 150^4 / 32} = 75\text{N} / \text{mm}^2$$

Substituting these values in Eqs (1.11) and (1.12) in turn and noting that $\sigma_y = 0$

$$\sigma_{\mathrm{I}} = \frac{75}{2} + \frac{1}{2}\sqrt{75^2 + 4 \times 75^2}$$

i.e.

$$\sigma_{\mathrm{I}} = 121.4\,\mathrm{N/mm^2}$$

$$\sigma_{\mathrm{II}} = \frac{75}{2} - \frac{1}{2}\sqrt{75^2 + 4 \times 75^2}$$

i.e.

$$\sigma_{\mathrm{II}} = -46.4\,\mathrm{N/mm^2}$$

The corresponding directions as defined by θ in Fig. 1.8(b) are given by Eq. (1.10) i.e.

$$\tan 2\theta = \frac{2 \times 75}{75 - 0} = 2$$

Hence

$$\theta = 31°43'(\sigma_{\mathrm{I}})$$

and

$$\theta = 121°43'(\sigma_{\mathrm{II}})$$

S.1.7

The direct strains are expressed in terms of the stresses using Eqs (1.42), i.e.

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \quad \text{(i)}$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \quad \text{(ii)}$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \quad \text{(iii)}$$

Then

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1}{E}[\sigma_x + \sigma_y + \sigma_z - 2\nu(\sigma_x + \sigma_y + \sigma_z)]$$

i.e.

$$e = \frac{(1-2\nu)}{E}(\sigma_x + \sigma_y + \sigma_z)$$

whence

$$\sigma_y + \sigma_z = \frac{Ee}{(1-2\nu)} - \sigma_x$$

Substituting in Eq. (i)

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\frac{Ee}{1-2\nu} - \sigma_x\right)\right]$$

so that

$$E\varepsilon_x = \sigma_x(1+\nu) - \frac{\nu Ee}{1-2\nu}$$

Thus

$$\sigma_x = \frac{\nu Ee}{(1-2\nu)(1+\nu)} + \frac{E}{(1+\nu)}\varepsilon_x$$

or, since $G = E/2(1+\nu)$ (see Section 1.15)

$$\sigma_x = \lambda e + 2G\varepsilon_x$$

Similarly

$$\sigma_y = \lambda e + 2G\varepsilon_y$$

and

$$\sigma_z = \lambda e + 2G\varepsilon_z$$

S.1.8

The implication in this problem is that the condition of plane strain also describes the condition of plane stress. Hence, from Eqs (1.52)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \qquad \text{(i)}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \qquad \text{(ii)}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2(1+\nu)}{E}\tau_{xy} \qquad \text{(see Section 1.15)} \qquad \text{(iii)}$$

The compatibility condition for plane strain is

$$\frac{\partial^2\gamma_{xy}}{\partial x\,\partial y} = \frac{\partial^2\varepsilon_y}{\partial x^2} + \frac{\partial^2\varepsilon_x}{\partial y^2} \qquad \text{(see Section 1.11)} \qquad \text{(iv)}$$

Substituting in Eq. (iv) for ε_x, ε_y and γ_{xy} from Eqs (i)–(iii), respectively, gives

$$2(1+\nu)\frac{\partial^2\tau_{xy}}{\partial x\,\partial y} = \frac{\partial^2}{\partial x^2}(\sigma_y - \nu\sigma_x) + \frac{\partial^2}{\partial y^2}(\sigma_x - \nu\sigma_y) \qquad \text{(v)}$$

Also, from Eqs (1.6) and assuming that the body forces X and Y are zero

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{zy}}{\partial y} = 0 \tag{vi}$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = 0 \tag{vii}$$

Differentiating Eq. (vi) with respect to x and Eq. (vii) with respect to y and adding gives

$$\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \tau_{xy}}{\partial y\,\partial x} + \frac{\partial^2 \sigma_y}{\partial y^2} + \frac{\partial^2 \tau_{xy}}{\partial x\,\partial y} = 0$$

or

$$2\frac{\partial^2 \tau_{xy}}{\partial x\,\partial y} = -\left(\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2}\right)$$

Substituting in Eq. (v)

$$-(1+v)\left(\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2}\right) = \frac{\partial^2}{\partial x^2}(\sigma_y - v\sigma_x) + \frac{\partial^2}{\partial y^2}(\sigma_x - v\sigma_y)$$

so that

$$-(1+v)\left(\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2}\right) = \frac{\partial^2 \sigma_y}{\partial x^2} + \frac{\partial^2 \sigma_x}{\partial y^2} - v\left(\frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2}\right)$$

which simplifies to

$$\frac{\partial^2 \sigma_y}{\partial x^2} + \frac{\partial^2 \sigma_x}{\partial y^2} + \frac{\partial^2 \sigma_x}{\partial x^2} + \frac{\partial^2 \sigma_y}{\partial y^2} = 0$$

or

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)(\sigma_x + \sigma_y) = 0$$

S.1.9

Suppose that the load in the steel bar is P_{st} and that in the aluminium bar is P_{al}. Then, from equilibrium

$$P_{\text{st}} + P_{\text{al}} = P \tag{i}$$

From Eq. (1.40)

$$\varepsilon_{\text{st}} = \frac{P_{\text{st}}}{A_{\text{st}}E_{\text{st}}} \qquad \varepsilon_{\text{al}} = \frac{P_{\text{al}}}{A_{\text{al}}E_{\text{al}}}$$

Since the bars contract by the same amount

$$\frac{P_{st}}{A_{st}E_{st}} = \frac{P_{al}}{A_{al}E_{al}} \qquad \text{(ii)}$$

Solving Eqs (i) and (ii)

$$P_{st} = \frac{A_{st}E_{st}}{A_{st}E_{st} + A_{al}E_{al}}P \qquad P_{al} = \frac{A_{al}E_{al}}{A_{st}E_{st} + A_{al}E_{al}}P$$

from which the stresses are

$$\sigma_{st} = \frac{E_{st}}{A_{st}E_{st} + A_{al}E_{al}}P \qquad \sigma_{al} = \frac{E_{al}}{A_{st}E_{st} + A_{al}E_{al}}P \qquad \text{(iii)}$$

The areas of cross-section are

$$A_{st} = \frac{\pi \times 75^2}{4} = 4417.9\,\text{mm}^2 \quad A_{al} = \frac{\pi(100^2 - 75^2)}{4} = 3436.1\,\text{mm}^2$$

Substituting in Eq. (iii) we have

$$\sigma_{st} = \frac{10^6 \times 200\,000}{(4417.9 \times 200\,000 + 3436.1 \times 80\,000)} = 172.6\,\text{N/mm}^2 \text{ (compression)}$$

$$\sigma_{al} = \frac{10^6 \times 80\,000}{(4417.9 \times 200\,000 + 3436.1 \times 80\,000)} = 69.1\,\text{N/mm}^2 \text{ (compression)}$$

Due to the decrease in temperature in which no change in length is allowed the strain in the steel is $\alpha_{st}T$ and that in the aluminium is $\alpha_{al}T$. Therefore due to the decrease in temperature

$$\sigma_{st} = E_{st}\alpha_{st}T = 200\,000 \times 0.000012 \times 150 = 360.0\,\text{N/mm}^2 \text{ (tension)}$$

$$\sigma_{al} = E_{al}\alpha_{al}T = 80\,000 \times 0.000005 \times 150 = 60.0\,\text{N/mm}^2 \text{ (tension)}$$

The final stresses in the steel and aluminium are then

$$\sigma_{st}\text{(total)} = 360.0 - 172.6 = 187.4\,\text{N/mm}^2 \text{ (tension)}$$

$$\sigma_{al}\text{(total)} = 60.0 - 69.1 = -9.1\,\text{N/mm}^2 \text{ (compression)}$$

S.1.10

The principal strains are given directly by Eqs (1.69) and (1.70). Thus

$$\varepsilon_I = \frac{1}{2}(-0.002 + 0.002) + \frac{1}{\sqrt{2}}\sqrt{(-0.002 + 0.002)^2 + (+0.002 + 0.002)^2}$$

i.e.

$$\varepsilon_{\mathrm{I}} = +0.00283$$

Similarly

$$\varepsilon_{\mathrm{II}} = -0.00283$$

The principal directions are given by Eq. (1.71), i.e.

$$\tan 2\theta = \frac{2(-0.002) + 0.002 - 0.002}{0.002 + 0.002} = -1$$

Hence

$$2\theta = -45^\circ \text{ or } +135^\circ$$

and

$$\theta = -22.5^\circ \text{ or } +67.5^\circ$$

S.1.11

The principal strains at the point **P** are determined using Eqs (1.69) and (1.70). Thus

$$\varepsilon_{\mathrm{I}} = \left[\frac{1}{2}(-222 + 45) + \frac{1}{\sqrt{2}}\sqrt{(-222 + 213)^2 + (-213 - 45)^2}\right] \times 10^{-6}$$

i.e.

$$\varepsilon_{\mathrm{I}} = 94.0 \times 10^{-6}$$

Similarly

$$\varepsilon_{\mathrm{II}} = -217.0 \times 10^{-6}$$

The principal stresses follow from Eqs (1.67) and (1.68). Hence

$$\sigma_{\mathrm{I}} = \frac{31000}{1 - (0.2)^2}(94.0 - 0.2 \times 271.0) \times 10^{-6}$$

i.e.

$$\sigma_{\mathrm{I}} = 1.29 \text{ N/mm}^2$$

Similarly

$$\sigma_{\mathrm{II}} = -814 \text{ N/mm}^2$$

Since **P** lies on the neutral axis of the beam the direct stress due to bending is zero. Therefore, at **P**, $\sigma_x = 7$ N/mm^2 and $\sigma_y = 0$. Now subtracting Eq. (1.12) from (1.11)

$$\sigma_{\mathrm{I}} - \sigma_{\mathrm{II}} = \sqrt{\sigma_x^2 + 4\tau_{xy}^2}$$

i.e.

$$1.29+8.14=\sqrt{7^2+4\tau_{xy}^2}$$

from which τ_{xy} = 3.17 N/mm^2.

The shear force at **P** is equal to Q so that the shear stress at **P** is given by

$$\tau_{xy}=3.17=\frac{3Q}{2\times 150\times 300}$$

from which

$$Q=95\,100\text{N}=95.1\text{kN}.$$

Solutions to Chapter 2 Problems

S.2.1

The stress system applied to the plate is shown in Fig. S.2.1. The origin, O, of the axes may be chosen at any point in the plate; let **P** be the point whose coordinates are (2, 3).

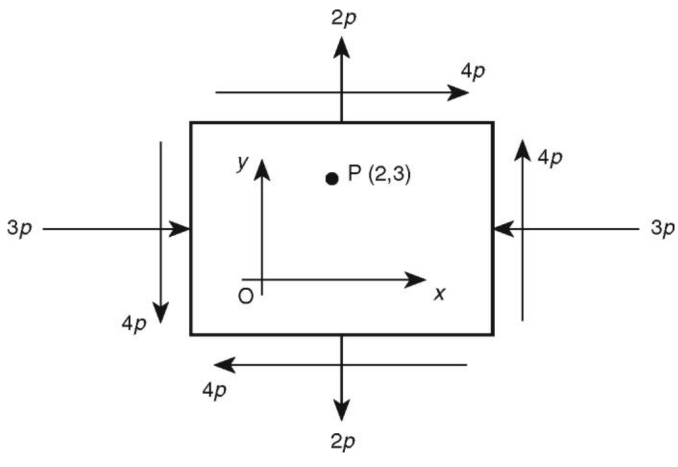

Fig. S.2.1

From Eqs (1.42) in which σ_z = 0

$$\varepsilon_x=-\frac{3p}{E}-\nu\frac{2p}{E}=-\frac{3.5p}{E} \qquad \text{(i)}$$

$$\varepsilon_y=-\frac{3p}{E}-\nu\frac{3p}{E}=-\frac{2.75p}{E} \qquad \text{(ii)}$$

Hence, from Eqs (1.27)

$$\frac{\partial u}{\partial x}=-\frac{3.5p}{E} \quad \text{so that} \quad u=-\frac{3.5p}{E}x+f_1(y) \qquad \text{(iii)}$$