Problem 1.1

The force, F, of the wind blowing against a building is given by $F = C_D \rho V^2 A/2$, where V is the wind speed, ρ the density of the air, A the cross-sectional area of the building, and C_D is a constant termed the drag coefficient. Determine the dimensions of the drag coefficient.

Solution 1.1

$$F = C_D \rho V^2 \frac{A}{2}$$

or

$$C_D = \frac{2F}{\rho V^2 A}, \text{ where}$$

$$F \doteq MLT^{-2}, \rho \doteq ML^{-3}, V \doteq LT^{-1}, A \doteq L^2$$

Thus,

$$C_D \doteq \frac{\left(MLT^{-2}\right)}{\left[\left(ML^{-2}\right)\left(LT^{-1}\right)^2\left(L^2\right)\right]} = M^0L^0T^0$$

Hence, C_D is dimensionless.

Problem 1.2

The Mach number is a dimensionless ratio of the velocity of an object in a fluid to the speed of sound in the fluid. For an airplane flying at velocity V in air at absolute temperature T, the Mach number Ma is,

$$\mathrm{Ma} = \frac{V}{\sqrt{kRT}},$$

where k is a dimensionless constant and R is the specific gas constant for air. Show that Ma is dimensionless.

Solution 1.2

We denote the dimension of temperature by θ and use Newton's second law to get $F = \dfrac{ML}{T^2}$.

Then

$$[M] = \frac{\left(\frac{L}{T}\right)}{\sqrt{(1)\left(\frac{FL}{M\theta}\right)\theta\left(\frac{ML}{T^2F}\right)}} = \frac{\left(\frac{L}{T}\right)}{\sqrt{\frac{L^2}{T^2}}}$$

or

$$[M] = [1].$$

Problem 1.3

Verify the dimensions, in both the FLT and MLT systems, of the following quantities which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Volume, **(b)** acceleration, **(c)** mass, **(d)** moment of inertia (area), and **(e)** work.

Solution 1.3

a) $\text{volume} \doteq \underline{\underline{L^3}}$

b) $\text{acceleration} = \text{time rate of change of velocity} \doteq \dfrac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$

c) $\text{mass} \doteq \underline{\underline{M}}$

or with $F \doteq MLT^{-2}$

$\text{mass} \doteq \underline{\underline{FL^{-1}T^2}}$

d) $\text{moment of inertia}\,(\text{area}) = \text{second moment of area} \doteq \left(L^2\right)\left(L^2\right) \doteq \underline{\underline{L^4}}$

e) $\text{work} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}}$

or with $F \doteq MLT^{-2}$

$\text{work} \doteq \underline{\underline{ML^2T^{-2}}}$

Problem 1.4

Verify the dimensions, in both the FLT and MLT systems, of the following quantities which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Angular velocity, **(b)** energy, **(c)** moment of inertia (area), **(d)** power, and **(e)** pressure.

Solution 1.4

a) $\text{angular velocity} = \dfrac{\text{angular displacement}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

b) energy $\sim$ capacity of body to do work

$\text{single work} = \text{force} \times \text{distance} \rightarrow \text{energy} \doteq \underline{\underline{FL}}$

or with $F \doteq MLT^{-2} \rightarrow \text{energy} \doteq \left(MLT^{-2}\right)(L) \doteq \underline{\underline{ML^2T^{-2}}}$

c) $\text{moment of inertia (area)} = \text{second moment of area} \doteq \left(L^2\right)\left(L^2\right) \doteq \underline{\underline{L^4}}$

d) $\text{power} = \text{rate of doing work} \doteq \dfrac{FL}{T} \doteq \underline{\underline{FLT^{-1}}} \doteq \left(MLT^{-2}\right)(L)\left(T^{-1}\right) \doteq \underline{\underline{ML^2T^{-3}}}$

e) $\text{pressure} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}} \doteq \left(MLT^{-2}\right)\left(L^{-2}\right) \doteq \underline{\underline{ML^{-1}T^{-2}}}$

Problem 1.5

Verify the dimensions, in both the *FLT* system and the *MLT* system, of the following quantities which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Frequency, **(b)** stress, **(c)** strain, **(d)** torque, and **(e)** work.

Solution 1.5

a) $\text{frequency}=\dfrac{\text{cycles}}{\text{time}} \doteq \underline{\underline{T^{-1}}}$

b) $\text{stress}=\dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$$\text{stress} \doteq \frac{MLT^{-2}}{L^2} \doteq \underline{\underline{ML^{-1}T^{-2}}}$$

c) $\text{strain}=\dfrac{\text{change in length}}{\text{length}} \doteq \dfrac{L}{L} \doteq \underline{\underline{L^0}}\,(\text{dimensionless})$

d) $\text{torque}=\text{force}\times\text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)(L) \doteq \underline{\underline{ML^2T^{-2}}}$

e) $\text{work}=\text{force}\times\text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)(L) \doteq \underline{\underline{ML^2T^{-2}}}$

Problem 1.6

If u is a velocity, x a length, and t a time, what are the dimensions (in the MLT system) of **(a)** $\partial u / \partial t$, **(b)** $\partial^2 u / \partial x \partial t$, and **(c)** $\int (\partial u / \partial t) dx$?

Solution 1.6

a) $\dfrac{\partial u}{\partial t} \doteq \dfrac{LT^{-1}}{T} \doteq \underline{\underline{LT^{-2}}}$

b) $\dfrac{\partial^2 u}{\partial x \partial t} \doteq \dfrac{LT^{-1}}{(L)(T)} \doteq \underline{\underline{T^{-2}}}$

c) $\int \dfrac{\partial u}{\partial t} \partial x \doteq \dfrac{\left(LT^{-1}\right)}{T}(\mathrm{L}) \doteq \underline{\underline{L^2 T^{-2}}}$

Problem 1.7

Verify the dimensions, in both the FLT system and the MLT system, of the following quantities which appear in Table B.1 Physical Properties of Water (BG/EE Units).

(a) Acceleration, **(b)** stress, **(c)** moment of a force, **(d)** volume, and **(e)** work.

Solution 1.7

a) $\text{acceleration} = \dfrac{\text{velocity}}{\text{time}} \doteq \dfrac{L}{T^2} \doteq \underline{\underline{LT^{-2}}}$

b) $\text{stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$\text{stress} \doteq \dfrac{MLT^{-2}}{L^2} \doteq \underline{\underline{ML^{-1}T^{-2}}}$

c) $\text{moment of a force} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)L \doteq \underline{\underline{ML^2T^{-2}}}$

d) $\text{volume} = (\text{length})^3 \doteq \underline{\underline{L^3}}$

e) $\text{work} = \text{force} \times \text{distance} \doteq \underline{\underline{FL}} \doteq \left(MLT^{-2}\right)L \doteq \underline{\underline{ML^2T^{-2}}}$

Problem 1.8

If p is a pressure, V a velocity, and ρ a fluid density, what are the dimensions (in the MLT system) of **(a)** p/ρ, **(b)** $pV\rho$, and **(c)** $p/\rho V^2$?

Solution 1.8

a) $\frac{p}{\rho} \doteq \frac{FL^{-2}}{ML^{-3}} = \frac{MLT^{-2}L^{-2}}{ML^{-3}} = \frac{ML^{-1}T^{-2}}{ML^{-3}} \doteq \underline{\underline{L^2T^{-2}}}$

b) $pV\rho \doteq \left(ML^{-1}T^{-2}\right)\left(LT^{-1}\right)\left(ML^{-3}\right) \doteq \underline{\underline{M^2L^{-3}T^{-3}}}$

c) $\frac{p}{\rho V^2} \doteq \frac{ML^{-1}T^{-2}}{\left(ML^{-3}\right)\left(LT^{-1}\right)^2} \doteq M^0L^0T^0 \left(\underline{\underline{\text{dimensionless}}}\right)$

Problem 1.9

If P is a force and x a length, what are the dimensions (in the FLT system) of **(a)** dP/dx, **(b)** d^3P/dx^3, and **(c)** $\int P\,dx$?

Solution 1.9

a) $\dfrac{dP}{dx} \doteq \dfrac{F}{L} \doteq \underline{\underline{FL^{-2}}}$

b) $\dfrac{d^3P}{dx^3} \doteq \dfrac{F}{L^3} \doteq \underline{\underline{FL^{-3}}}$

c) $\int P dx \doteq \underline{\underline{FL}}$

Problem 1.10

If V is a velocity, ℓ a length, and ν a fluid property (the kinematic viscosity) having dimensions of L^2T^{-1}, which of the following combinations are dimensionless: **(a)** $V\ell\nu$, **(b)** $V\ell/\nu$, **(c)** $V^2\nu$, **(d)** $V/\ell\nu$?

Solution 1.10

a) $V\ell\nu \doteq \left(LT^{-1}\right)(L)\left(L^2T^{-1}\right) \doteq L^4T^{-2}$ $\left(\underline{\underline{\text{not dimensionless}}}\right)$

b) $\dfrac{V\ell}{\nu} \doteq \dfrac{\left(LT^{-1}\right)(L)}{\left(L^2T^{-1}\right)} \doteq L^0T^0$ $\left(\underline{\underline{\text{dimensionless}}}\right)$

c) $V^2\nu \doteq \left(LT^{-1}\right)^2\left(L^2T^{-1}\right) \doteq L^4T^{-3}$ $\left(\underline{\underline{\text{not dimensionless}}}\right)$

d) $\dfrac{V}{\ell\nu} \doteq \dfrac{\left(LT^{-1}\right)}{(L)\left(L^2T^{-1}\right)} \doteq L^{-2}$ $\left(\underline{\underline{\text{not dimensionless}}}\right)$

Problem 1.11

The momentum flux is given by the product $\dot{m}V$, where $\dot{m}$ is mass flow rate and V is velocity. If mass flow rate is given in units of mass per unit time, show that the momentum flux can be expressed in units of force.

Solution 1.11

$$[\dot{m}V] = \left(\frac{M}{T}\right)\left(\frac{L}{T}\right) = M\frac{L}{T^2} = \left(\frac{FT^2}{L}\right)\frac{L}{T^2} = \underline{\underline{F}}$$

Problem 1.12

An equation for the frictional pressure loss Δp (inches H_2O) in a circular duct of inside diameter $d\,(\text{in.})$ and length $L\,(\text{ft})$ for air flowing with velocity $V\,(\text{ft/min})$ is

$$\Delta p = 0.027\left(\frac{L}{d^{1.22}}\right)\left(\frac{V}{V_o}\right)^{1.82},$$

where V_0 is a reference velocity equal to $1000\,\text{ft/min}$. Find the units of the "constant" 0.027.

Solution 1.12

Solving for the constant gives

$$0.027 = \frac{\Delta p_L}{\left(\dfrac{L}{D^{1.22}}\right)\left(\dfrac{V}{V_o}\right)^{1.82}}\ .$$

The units give

$$[0.027] = \frac{(\text{in. H}_2\text{O})}{\left(\dfrac{\text{ft}}{\text{in.}^{1.22}}\right)\left(\dfrac{\frac{\text{ft}}{\text{min}}}{\frac{\text{ft}}{\text{min}}}\right)^{1.82}}$$

$$\boxed{[0.027] = \frac{\text{in. H}_2\text{O}\cdot\text{in.}^{1.22}}{\text{ft}}}$$

Problem 1.13

The volume rate of flow, Q , through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8 \mu \ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ a fluid property called viscosity $\left(FL^{-2}T\right)$, and ℓ the length of pipe. What are the dimensions of the constant $\pi/8$? Would you classify this equation as a general homogeneous equation? Explain.

Solution 1.13

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\frac{\left[L^4\right]\left[FL^2\right]}{\left[FL^{-2}T\right]\left[L\right]}$$

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\left[L^3T^{-1}\right]$$

The constant is $\frac{\pi}{8}$ is dimensionless.

Yes. This is a general homogeneous equation
because it is valid in any consistent units system.

Problem 1.14

Show that each term in the following equation has units of lb/ft^3. Consider u a velocity, y a length, x a length, p a pressure, and μ an absolute viscosity.

$$0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2}.$$

Solution 1.14

$$\left[\frac{\partial p}{\partial x}\right] = \frac{\left[\frac{\text{lb}}{\text{ft}^2}\right]}{[\text{ft}]} \quad \text{or} \quad \boxed{\left[\frac{\partial p}{\partial x}\right] = \left[\frac{\text{lb}}{\text{ft}^3}\right]},$$

and

$$\left[\mu \frac{\partial^2 u}{\partial y^2}\right] = \left[\frac{\text{lb}\cdot\text{sec}}{\text{ft}^2}\right] \frac{\left[\frac{\text{ft}}{\text{sec}}\right]}{\left[\text{ft}^2\right]} \quad \text{or} \quad \boxed{\left[\mu \frac{\partial^2 u}{\partial y^2}\right] = \left[\frac{\text{lb}}{\text{ft}^3}\right]}.$$

Problem 1.15

The pressure difference, Δp , across a partial blockage in an artery (called a stenosis) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1 \right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity $(FL^{-2}T)$, ρ the blood density $\left(ML^{-3}\right)$, D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the stenosis. Determine the dimensions of the constants K_υ and K_u . Would this equation be valid in any system of units?

Solution 1.15

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left[\frac{A_0}{A_1} - 1 \right]^2 \rho V^2$$

$$FL^{-2} \doteq [K_v] \frac{FT}{L^2} \frac{L}{T} \frac{1}{L} \quad + [K_u] \left(\frac{L^2}{L^2} - 1 \right)^2 \left(\frac{FT^2}{L} \frac{1}{L^3} \right) \left(\frac{L}{T} \right)^2$$

$$FL^{-2} \doteq [K_v] \left(FL^{-2} \right) \quad + [K_u] \left(FL^{-2} \right)$$

K_υ and K_u are <u>dimensionless</u> because each term in the equation must have the same dimensions,.

<u>Yes.</u> The equation would be valid in any consistent system of units.

Problem 1.16

Assume that the speed of sound, c, in a fluid depends on an elastic modulus, E_υ, with dimensions FL^{-2}, and the fluid density, ρ, in the form $c = (E_\upsilon)^a (\rho)^b$. If this is to be a dimensionally homogeneous equation, what are the values for a and b? Is your result consistent with the standard formula for the speed of sound? (See the equation $c = \sqrt{\dfrac{E_\upsilon}{\rho}}$.)

Solution 1.16

Substituting $[c] = LT^{-1}$ $[E_v] = FL^{-2}$ $[\rho] = FL^{-4}T^2$ into the equation provided yields:

$$\left[LT^{-1}\right] = \left[\left(FL^{-2}\right)^a\right]\left[\left(FL^{-4}T^2\right)^b\right] = F^{a+b}L^{-2a-4b}T^{2b}$$

Dimensional homogeneity requires that the exponent of each dimension on both sides of the equal sign be the same.

F: 0 =a+b
L: 1 =-2a-4b
T: -1 =2b

Therefore:

T: -1 =2b → b= -1/2
F: a =-b → a= 1/2
L: 1 =-2a-4b = -2(1/2) -4(-1/2)= 1✓

$$\boxed{a = \frac{1}{2};\quad b = -\frac{1}{2}}$$

Yes, this is consistent with the standard formula for the speed of sound.

Problem 1.17

A formula to estimate the volume rate of flow, Q, flowing over a dam of length, B, is given by the equation

$$Q = 3.09\,BH^{3/2}$$

where H is the depth of the water above the top of the dam (called the head). This formula gives Q in $\mathrm{ft^3/s}$ when B and H are in feet. Is the constant, 3.09, dimensionless? Would this equation be valid if units other than feet and seconds were used?

Solution 1.17

$$Q = 3.09\,BH^{\frac{3}{2}}$$

$$\left[L^3T^{-1}\right] \doteq [3.09][L][L]^{\frac{3}{2}}$$

$$\left[L^3T^{-1}\right] \doteq [3.09][L]^{\frac{5}{2}}$$

Since each term in the equation must have the same dimensions the constant 3.09 must have dimensions of $L^{\frac{1}{2}}T^{-1}$ and is therefore not dimensionless. No.

Since the constant has dimensions its value will change with a change in units.

No.

Problem 1.18

A commercial advertisement shows a pearl falling in a bottle of shampoo. If the diameter D of the pearl is quite small and the shampoo sufficiently viscous, the drag $\mathscr{D}$ on the pearl is given by Stokes's law,

$$\mathscr{D} = 3\pi\mu VD,$$

where V is the speed of the pearl and μ is the fluid viscosity. Show that the term on the right side of Stokes's law has units of force.

Solution 1.18

$$[\mathscr{D}] = [3\pi\mu VD] = \left(\frac{M}{LT}\right)\left(\frac{L}{T}\right)L = M\frac{L}{T^2} = \frac{FT^2}{L}\frac{L}{T^2} = \underline{\underline{F}}$$

Problem 1.20

Express the following quantities in SI units: **(a)** $10.2\ \text{in./min}$, **(b)** $4.81\ \text{slugs}$, **(c)** $3.02\ \text{lb}$, **(d)** $73.1\ \text{ft/s}^2$, **(e)** $0.0234\ \text{lb}\cdot\text{s/ft}^2$.

Solution 1.20

a) $$10.2\,\frac{\text{in.}}{\text{min}} = \left(10.2\,\frac{\text{in.}}{\text{min}}\right)\left(2.540\times10^{-2}\,\frac{\text{m}}{\text{in.}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 4.32\times10^{-3}\,\frac{\text{m}}{\text{s}} = \underline{\underline{4.32\,\frac{\text{mm}}{\text{s}}}}$$

b) $$4.81\,\text{slugs} = (4.81\,\text{slugs})\left(1.459\times10\,\frac{\text{kg}}{\text{slug}}\right) = \underline{\underline{70.2\,\text{kg}}}$$

c) $$3.02\,\text{lb} = (3.02\,\text{lb})\left(4.448\,\frac{\text{N}}{\text{lb}}\right) = \underline{\underline{13.4\,\text{N}}}$$

d) $$73.1\,\frac{\text{ft}}{\text{s}^2} = \left(73.1\,\frac{\text{ft}}{\text{s}^2}\right)\left(3.048\times10^{-1}\,\frac{\frac{\text{m}}{\text{s}^2}}{\frac{\text{ft}}{\text{s}^2}}\right) = \underline{\underline{22.3\,\frac{\text{m}}{\text{s}^2}}}$$

e) $$0.0234\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2} = \left(0.0234\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(4.788\times10\,\frac{\frac{\text{N}\cdot\text{s}}{\text{m}^2}}{\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}\right) = \underline{\underline{1.12\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}}$$

Problem 1.21

Express the following quantities in BG units: **(a)** $14.2\ \text{km}$, **(b)** $8.14\ \text{N/m}^3$, **(c)** $1.61\ \text{kg/m}^3$, **(d)** $0.0320\ \text{N}\cdot\text{m/s}$, **(e)** $5.67\ \text{mm/hr}$.

Solution 1.21

a) $$14.2\ \text{km} = \left(14.2\times10^3\ \text{m}\right)\left(3.281\ \frac{\text{ft}}{\text{m}}\right) = \underline{\underline{4.66\times10^4\ \text{ft}}}$$

b) $$8.14\ \frac{\text{N}}{\text{m}^3} = \left(8.14\ \frac{\text{N}}{\text{m}^3}\right)\left(6.366\times10^{-3}\ \frac{\frac{\text{lb}}{\text{ft}^3}}{\frac{\text{N}}{\text{m}^3}}\right) = \underline{\underline{5.18\times10^{-2}\ \frac{\text{lb}}{\text{ft}^3}}}$$

c) $$1.61\ \frac{\text{kg}}{\text{m}^3} = \left(1.61\ \frac{\text{kg}}{\text{m}^3}\right)\left(1.940\times10^{-3}\ \frac{\frac{\text{slugs}}{\text{ft}^3}}{\frac{\text{kg}}{\text{m}^3}}\right) = \underline{\underline{3.12\times10^{-3}\ \frac{\text{slugs}}{\text{ft}^3}}}$$

d) $$0.0320\ \frac{\text{N}\cdot\text{m}}{\text{s}} = \left(0.0320\ \frac{\text{N}\cdot\text{m}}{\text{s}}\right)\left(7.376\times10^{-1}\ \frac{\frac{\text{ft}\cdot\text{lb}}{\text{s}}}{\frac{\text{N}\cdot\text{m}}{\text{s}}}\right) = \underline{\underline{2.36\times10^{-2}\ \frac{\text{ft}\cdot\text{lb}}{\text{s}}}}$$

e) $$5.67\ \frac{\text{mm}}{\text{hr}} = \left(5.67\times10^{-3}\ \frac{\text{m}}{\text{hr}}\right)\left(3.281\ \frac{\text{ft}}{\text{m}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right) = \underline{\underline{5.17\times10^{-6}\ \frac{\text{ft}}{\text{s}}}}$$