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Chapter 2

1. A tailless aircraft of 9072kg mass has a delta wing with aspect ratio 1 and area 37m^2. Show that the aerodynamic mean chord,

$$\bar{\bar{c}} = \frac{\int_0^{\frac{b}{2}} c^2 dy}{\int_0^{\frac{b}{2}} c dy} \tag{S2.1}$$

of a delta wing is two thirds of its root chord and that for this wing it is 8.11m.

Answer

Considering the half-span of a delta wing, shown in Figure S2.1, the relationship between local chord length *c* and spanwise coordinate *y* is,

$$y = -\frac{b}{2c_r}c + \frac{b}{2} \tag{S2.2}$$

where c_r denotes the root chord. Equation (S2.2) can be rearranged to give,

$$c = c_r - \frac{2c_r y}{b} \tag{S2.3}$$

Equation (S2.3) can then be inserted into (S2.1),

$$\bar{\bar{c}} = \frac{\int_0^{\frac{b}{2}}\left(c_r^2 - \frac{4c_r^2 y}{b} + \frac{4c_r^2 y^2}{b^2}\right)dy}{\int_0^{\frac{b}{2}}\left(c_r - \frac{2c_r y}{b}\right)dy} = \frac{\left[c_r^2 y - \frac{2c_r^2 y^2}{b} + \frac{4c_r^2 y^3}{3b^2}\right]_0^{\frac{b}{2}}}{\left[c_r y - \frac{c_r y^2}{b}\right]_0^{\frac{b}{2}}} = \frac{\frac{c_r^2 b}{2} - \frac{c_r^2 b}{2} + \frac{c_r^2 b}{6}}{\frac{c_r b}{2} - \frac{c_r b}{4}} = \frac{2}{3}c_r$$

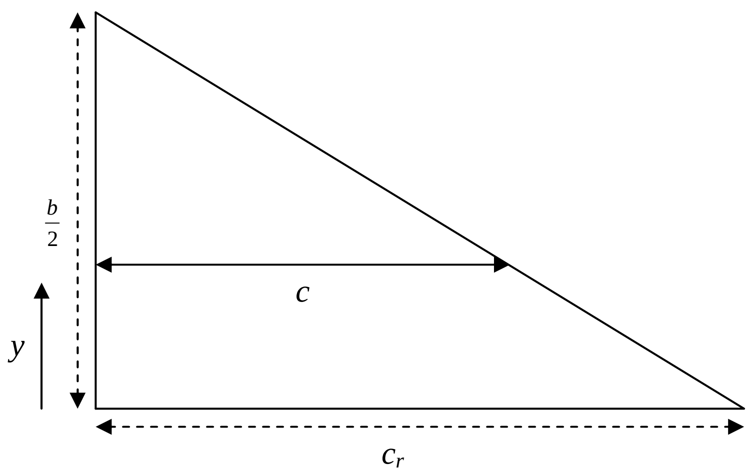

Figure S2.1 – Diagram of half-span of a delta wing

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Using the given numerical values for the wing, the root chord is calculated to be 12.165m and, hence, the aerodynamic mean chord is 8.11m.

2. With the aid of a diagram describe the axes systems used in aircraft stability and control analysis. State the conditions when the use of each axis system might be preferred.

Answer

Earth Axes: A right-handed orthogonal axes system ($o_E x_E y_E z_E$) which is fixed with respect to the earth. For aircraft stability and control analysis, it is usually adequate to assume flight over a flat, stationary earth. This is because only short term motion is of interest. The $o_E z_E$ axis points vertically down and is tied to the gravity vector, while the $o_E x_E$ axis points along the aircraft's direction of flight. The origin o_E is placed in the atmosphere at the most convenient location, which is frequently coincident with the origin of the aircraft body axes. This axes system is used to provide an inertial reference frame for short term aircraft motion.

Aircraft Body Axes: A right-handed orthogonal axes system ($ox_b y_b z_b$) which is fixed with respect to the aircraft and constrained to move with it. The ox_b axis is usually aligned with the horizontal fuselage datum, the oy_b axis points to starboard and the oz_b axis is directed 'downwards'. The origin o is fixed at a convenient reference point in the airframe, such as the centre of gravity. When the aircraft experiences a disturbance from trim, the motion is quantified by perturbation variables defined in body axes. A generalised body axes system will be used when deriving the equations of motion from first principles. It may also be preferred when gathering experimental data from the aircraft, which will subsequently used in the equations of motion.

Wind Axes: Also termed *aerodynamic* or *stability* axes, this is a right-handed orthogonal axes system ($ox_w y_w z_w$) which is fixed with respect to the aircraft. In steady symmetric flight, wind axes are a particular version of the body axes, which have been rotated about the oy_b axis through the trim angle of attach α_e. This results in the ox_w axis being aligned with the freestream velocity vector V_0. As there is a unique value of α_e for every flight condition, the orientation of the wind axes within the airframe varies with flight condition. However, this orientation is fixed at the outset and constrained to move with the aircraft during disturbed flight. Wind axes are preferred for aerodynamic measurements and computations, which are usually referenced to the freestream velocity vector.

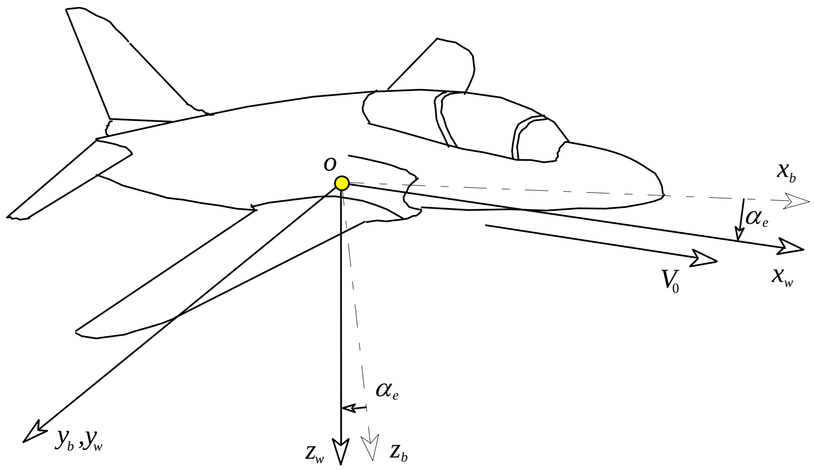

Figure S2.2 – Aircraft body and wind axes systems

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3. Show that in a longitudinal symmetric small perturbation the components of aircraft weight resolved into the *ox* and *oz* axes are given by,

$$\begin{aligned} X_g &= -mg\theta\cos\theta_e - mg\sin\theta_e \\ Z_g &= mg\cos\theta_e - mg\theta\sin\theta_e \end{aligned} \qquad \text{(S2.4)}$$

where θ is the perturbation in pitch attitude and θ_e is the equilibrium pitch attitude.

Answer

If it can be assumed that the aircraft is flying wings level in the initial symmetric flight condition, the components of its weight only appear in the longitudinal plane of symmetry. Thus, with reference to Figure S2.3, in the steady state the components of the weight resolved into aircraft axes are

$$\begin{bmatrix} X_{g_e} \\ Y_{g_e} \\ Z_{g_e} \end{bmatrix} = \begin{bmatrix} -mg\sin\theta_e \\ 0 \\ mg\cos\theta_e \end{bmatrix} \qquad \text{(S2.5)}$$

During the disturbance, it is assumed that the aircraft experiences an attitude perturbation in pitch only, given by θ. Perturbations in roll and yaw, ϕ and ψ respectively, are assumed to be zero. In this instance, the direction cosine matrix is given by,

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & -\theta \\ 0 & 1 & 0 \\ \theta & 0 & 1 \end{bmatrix} \qquad \text{(S2.6)}$$

where $\cos\theta \approx 1$ and $\sin\theta \approx \theta$. Combining equations (S2.5) and (S2.6) gives,

$$\begin{bmatrix} X_g \\ Y_g \\ Z_g \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\theta \\ 0 & 1 & 0 \\ \theta & 0 & 1 \end{bmatrix} \begin{bmatrix} X_{g_e} \\ Y_{g_e} \\ Z_{g_e} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\theta \\ 0 & 1 & 0 \\ \theta & 0 & 1 \end{bmatrix} \begin{bmatrix} -mg\sin\theta_e \\ 0 \\ mg\cos\theta_e \end{bmatrix} \qquad \text{(S2.7)}$$

which can be multiplied out to obtain the solution

$$\begin{aligned} X_g &= -mg\theta\cos\theta_e - mg\sin\theta_e \\ Z_g &= mg\cos\theta_e - mg\theta\sin\theta_e \end{aligned}$$

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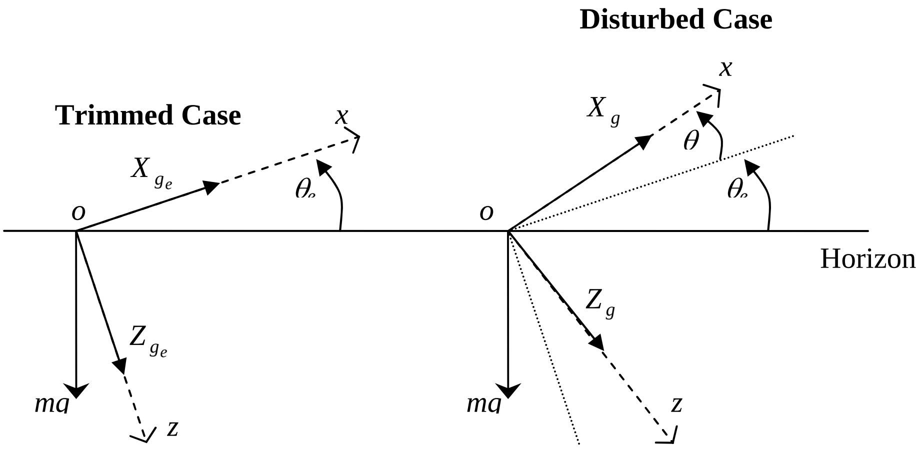

Figure S2.3 – Aircraft weight components in the longitudinal plane

4. With the aid of a diagram showing a generalised set of aircraft body axes, define the parameter notation used in the mathematical modelling of aircraft motion.

Answer

X	Axial 'drag' force
Y	Side force
Z	Normal 'lift' force
L	Rolling moment
M	Pitching moment
N	Yawing moment
p	Roll rate
q	Pitch rate
r	Yaw rate
U	Axial velocity
V	Lateral velocity
W	Normal velocity
ϕ	Roll attitude
θ	Pitch attitude
ψ	Yaw attitude

Table S2.1 – Parameter notation used in the mathematical modelling of aircraft motion

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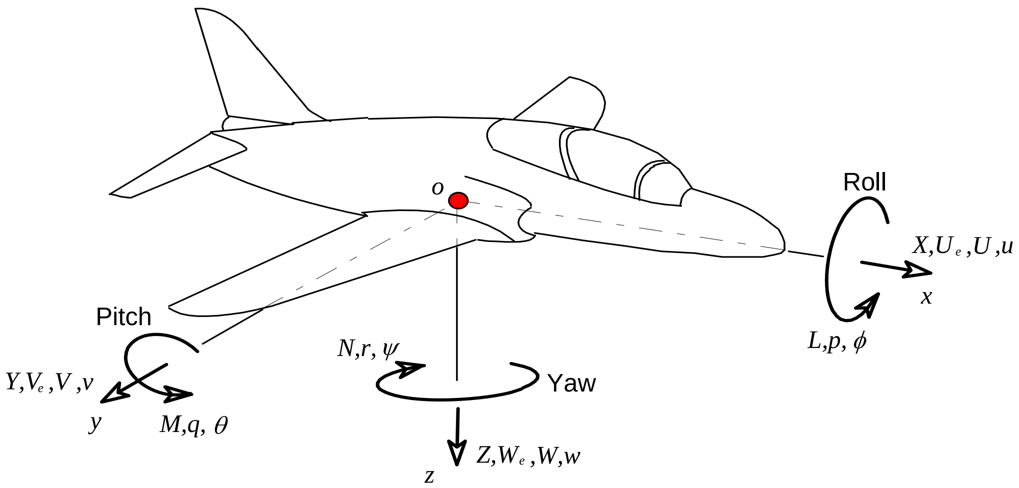

Figure S2.4 – Aircraft parameter notation

5. In the context of aircraft motion, what are the Euler angles? If the standard right handed aircraft axis set is rotated through pitch θ and yaw ψ angles only, show that the initial vector quantity (x_0,y_0,z_0) is related to the transformed vector quantity (x,y,z) as follows,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\sin\psi & \cos\psi & 0 \\ \sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \qquad \text{(S2.8)}$$

Answer

In the context of aircraft motion, the Euler angles represent the aircraft's attitude, which is defined to be the angular orientation of the airframe fixed axes with respect to earth axes.

Transformation of a vector quantity in one axis set to another is achieved by rotating vector through roll, pitch and yaw angles in turn:

i Roll angle ϕ about ox is zero therefore,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} \qquad \text{(S2.9)}$$

ii Pitching about oy_2 through angle θ,

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \qquad \text{(S2.10)}$$

iii Yawing about oz_1 through angle ψ,

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$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} \qquad \text{(S2.11)}$$

By repeated substitution equations (S2.9), (S2.10) and (S2.11) can be combined to give the required transformation relationship,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta\cos\psi & \cos\theta\sin\psi & -\sin\theta \\ -\sin\psi & \cos\psi & 0 \\ \sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}$$

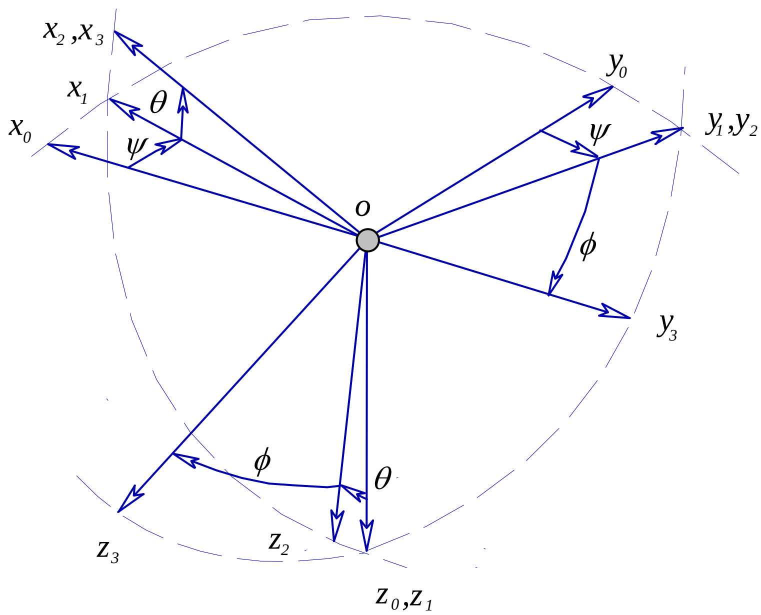

Figure S2.5 – Euler angles

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6. Define the span, gross area, aspect ratio and mean aerodynamic chord of an aircraft wing.

Answer

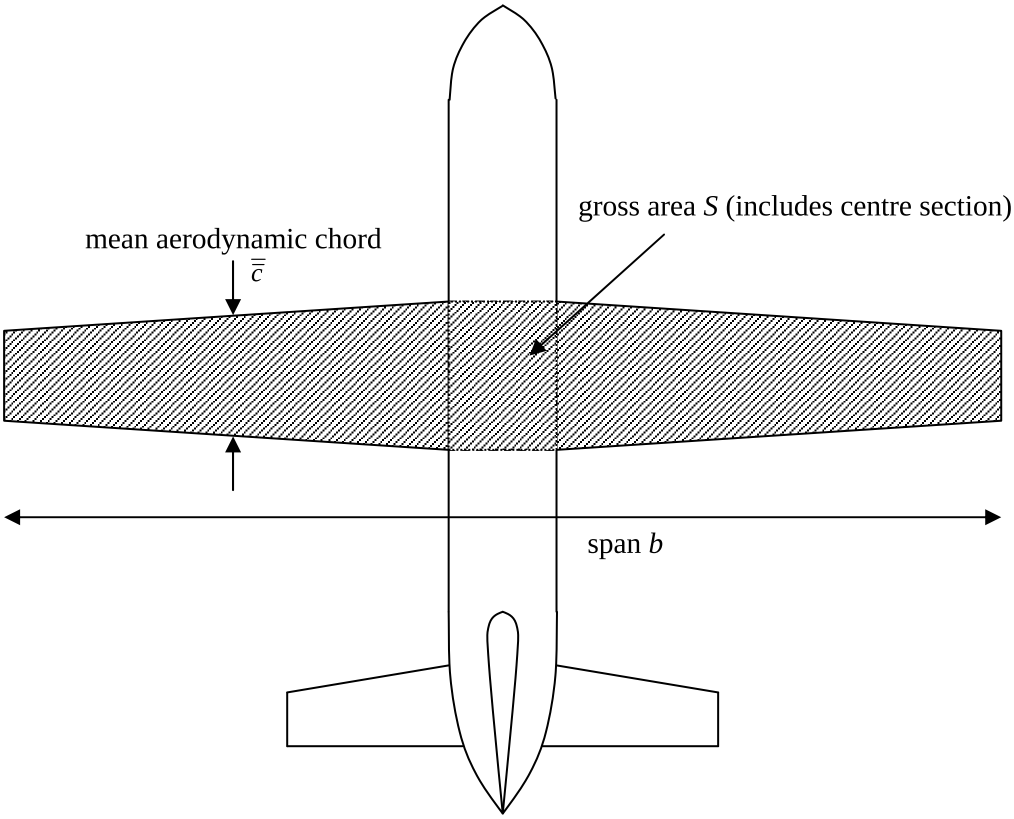

Figure S2.6 – Depiction of gross area, span and mean aerodynamic chord

Gross Area: see Figure S2.6

Span: see Figure S2.6

Aspect Ratio: Denoted by *A*, this is a measure of the spanwise slenderness of a wing and is given by the equation,

$$A = \frac{b^2}{S} \tag{S2.12}$$

Mean aerodynamic chord (mac): Denoted by $\bar{\bar{c}}$ and defined by the expression,

$$\bar{\bar{c}} = \frac{\int_0^{\frac{b}{2}} c^2 dy}{\int_0^{\frac{b}{2}} c dy} \tag{S2.13}$$

The *mac* represents the chord of an aerodynamically equivalent wing of rectangular planform and span *b*. The centre of pressure of the actual wing and equivalent

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rectangular wing are collocated on the *mac* and the lift, drag and pitching moment characteristics of both wings are the same. It provides a convenient aerodynamic reference geometry for a wing of any planform.

7. Distinguish between the centre of pressure and the aerodynamic centre of an aerofoil. Explain why the pitching moment about the quarter chord point of an aerofoil is nominally constant in subsonic flight.

Answer

Centre of pressure (cp): The point at which the resultant aerodynamic force (F) acts – this can be resolved into lift (L) and drag (D) components.

Aerodynamic centre (ac): The point at which the aerodynamic force due to angle of attack acts. This is usually at, or very near, the quarter chord position $\bar{\bar{c}}/4$ and ahead of *cp*.

The pitching moment about the quarter chord position is the moment of the aerodynamic force due to camber, acting at $\bar{\bar{c}}/2$. As the aerodynamic force due to camber is constant, the pitching moment about $\bar{\bar{c}}/4$ is also constant.

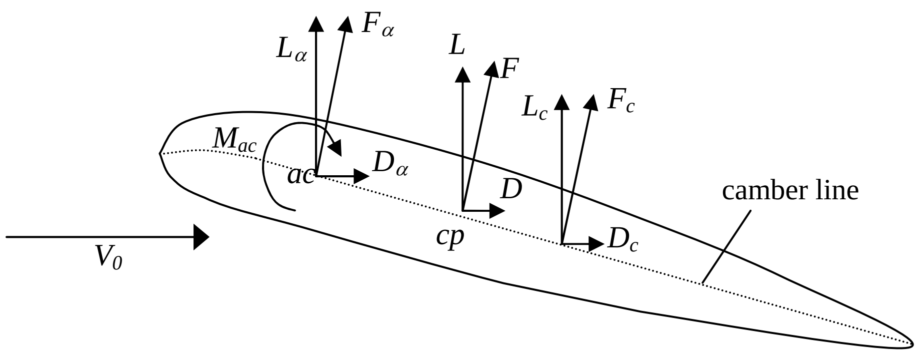

Figure S2.7 – Aerodynamic reference centres

8. What is the adverse effect of roll control by means of ailerons? Sketch the first few seconds of a typical time response in roll, roll rate, yaw and yaw rate to an aileron step input.

Answer

A positive stick displacement in roll is defined as a displacement to the right. This causes the right aileron to deflect up and the left aileron to deflect down and, in turn, leads to a reduction in lift on the starboard wing and increase in lift on the port wing. The differential lift then causes the aircraft to roll to the right as commanded.

However, as well as causing a differential lift effect, the deflection of the ailerons also leads to an induced drag differential – drag increases on the port wing and decreases on the starboard wing. This means that the aircraft will yaw to left. As this is not commanded by the pilot, it is an adverse effect.

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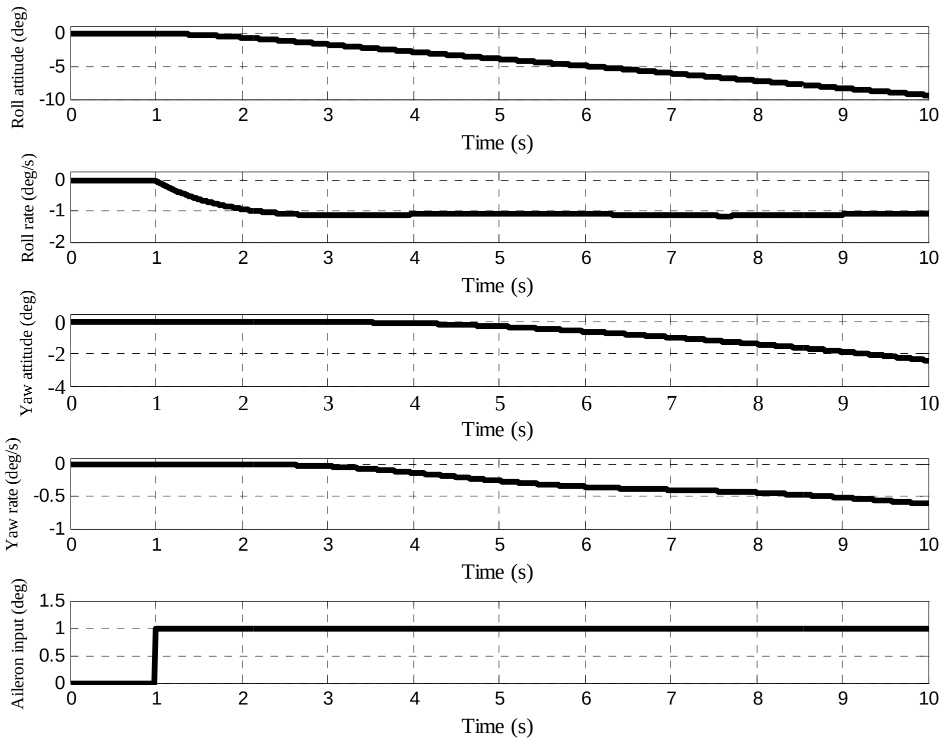

Figure S2.8 – Typical response to an aileron step input