Section 3-2

3-1. Continuous
3-2. Discrete
3-3. Continuous
3-4. Discrete
3-5. Discrete
3-6. Continuous
3-7. Discrete
3-8. Continuous
3-9. Continuous

Section 3-3

3-10. a) Engineers with at least 36 months of full-time employment.
b) Samples of cement blocks with compressive strength of at least 7000 kg per square centimeter.
c) Measurements of the diameter of forged pistons that conform to engineering specifications.
d) Cholesterol levels at most 180 or at least 220.

3-11. The intersection of A and B is empty, therefore $P(X \in A \cap B) = 0$.
a) Yes, since $P(X \in A \cap B) = 0$.
b) $P(X \in A') = 1 - P(X \in A) = 1 - 0.4 = 0.6$
c) $P(X \in B') = 1 - P(X \in B) = 1 - 0.6 = 0.4$
d) $P(X \in A \cup B) = P(X \in A) + P(X \in B) - P(X \in A \cap B) = 0.4 + 0.6 - 0 = 1$

3-12. a) $P(X \in A') = 1 - P(X \in A) = 1 - 0.3 = 0.7$
b) $P(X \in B') = 1 - P(X \in B) = 1 - 0.25 = 0.75$
c) $P(X \in C') = 1 - P(X \in C) = 1 - 0.6 = 0.4$
d) A and B are mutually exclusive if $P(X \in A \cap B) = 0$. To determine if A and B are mutually exclusive, solve the following for $P(X \in A \cap B)$:
$P(X \in A \cup B) = P(X \in A) + P(X \in B) - P(X \in A \cap B)$
$0.55 = 0.3 + 0.25 - P(X \in A \cap B)$
$0.55 = 0.55 - P(X \in A \cap B)$ and $P(X \in A \cap B) = 0$.
Therefore, A and B are mutually exclusive.
e) B and C are mutually exclusive if $P(X \in B \cap C) = 0$. To determine if B and C are mutually exclusive, solve the following for $P(X \in B \cap C)$:
$P(X \in B \cup C) = P(X \in B) + P(X \in C) - P(X \in B \cap C)$
$0.70 = 0.25 + 0.60 - P(X \in B \cap C)$
$0.70 = 0.85 - P(X \in B \cap C)$ and $P(X \in B \cap C) = 0.15$.
Therefore, B and C are not mutually exclusive.

3-13. a) $P(X > 15) = 1 - P(X \le 15) = 1 - 0.3 = 0.7$
b) $P(X \le 24) = P(X \le 15) + P(15 < X \le 24) = 0.3 + 0.6 = 0.9$
c) $P(15 < X \le 20) = P(X \le 20) - P(X \le 15) = 0.5 - 0.3 = 0.2$
d) $P(X \le 18) = P(15 < X \le 18) + P(X \le 15)$
where $P(15 < X \le 18) = P(15 < X \le 24) - P(18 < X \le 24) = 0.6 - 0.4 = 0.2$
Therefore, $P(X \le 18) = P(15 < X \le 18) + P(X \le 15) = 0.2 + 0.3 = 0.5$
Alternatively, $P(X \le 18) = P(X \le 24) - P(18 < X \le 24) = 0.9 - 0.4 = 0.5$.

3-14. A - Overfilled, B - Medium filled, C - Underfilled
a) $P(X \in C') = 1 - P(X \in C) = 1 - 0.20 = 0.8$
b) $P(X \in A \cup C) = P(X \in A) + P(X \in C) - P(X \in A \cap C) = 0.45 + 0.20 - 0 = 0.65$ ($P(X \in A \cap C) = 0$ since A and C are mutually exclusive)

3-15. a) $P(X \le 8000) = 1 - P(X > 8000) = 1 - 0.45 = 0.55$
b) $P(X > 6000) = 1 - P(X \le 6000) = 1 - 0.05 = 0.95$
c) $P(6000 < X \le 8000) = P(X \le 8000) - P(X \le 6000) = 0.55 - 0.05 = 0.50$

3-16. a) Probability that a component does not fail: $P(E_1') = 1 - P(E_1) = 1 - 0.15 = 0.85$
b) $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) = 0.15 + 0.30 = 0.45$

c) $P(E_1 \text{ or } E_2)' = 1 - 0.45 = 0.55$

3-17. a) $P(X = 2 \text{ or } X = 3) = 0.25 + 0.25 = 0.50$
b) $P(X < 2) = P(X = 0) + P(X = 1) = 0.10 + 0.15 = 0.25$
c) $P(X > 3) = P(X = 4) + P(X = 5) = 0.15 + 0.10 = 0.25$
d) $P(X > 0) = 1 - P(X = 0) = 1 - 0.1 = 0.9$

3-18. a) $P(X < 1) = P(X = 0) = 0.7$
b) $P(X > 3) = 1 - P(X \le 3) = 1 - 1 = 0$
c) $P(X > 0) = 1 - P(X = 0) = 1 - 0.7 = 0.3$
d) $P(X = 0) = 0.7$

3-19. a) $P(X > 50) = 1 - P(X \le 50)$. Because $P(0 \le X \le 49) = 1$, then $P(X \le 50) = 1$ and $P(X > 50) = 1 - 1 = 0$
b) $P(10 \le X \le 19) = P(X \le 19) - P(X < 10) = P(X \le 19) - P(X \le 9) = 0.7 - 0.4 = 0.3$
c) $P(20 \le X \le 29) = P(X \le 29) - P(X < 20) = P(X \le 29) - P(X \le 19) = 0.8 - 0.7 = 0.1$
d) P(More than 39 unique visitors) $= P(X > 39) = 1 - P(X \le 39) = 1 - 0.9 = 0.1$

3-20. a) $A \cap B$ = visit to hospital 4 that result in LWBS = 242.Then $P(A \cap B) = 242/22{,}252 = 0.011$
b) A' = visit to hospital 1, 2, or 3 = 5,292 + 6,991 + 5,640 = 17,926. Then P(A') = 17,926 /22,252 = 0.805
c) $A \cup B$ = visit to hospital 4 or a visit that results in LWBS, or both = 195 + 270 + 246 + 4329 = 5,040.
Then $P(A \cup B) = 5{,}040/22{,}252 = 0.227$
d) $A \cup B'$ = visit to hospital 4 or a visit that does not result in LWBS, or both = (5,292 – 195) + (6,991 – 270) + (5,640 – 246) + 4,329 = 21,541. Then $P(A \cup B') = 21{,}541/22{,}252 = 0.968$.
e) $A' \cap B'$= visit to hospital 1, 2, or 3 and does not result in LWBS = (5,292 – 195) + (6,991 – 270) + (5,640 – 246) = 17,212. Then $P(A' \cap B') = 17{,}212/22{,}252 = 0.774$. Another approach is to write $P(A' \cap B') = P(A \cup B)' = 1 - P(A \cup B) = 1 - 0.227 = 0.773$.

Section 3-4

3-21. a) $\int_0^4 kx^2 dx = k\frac{x^3}{3}\Big|_0^4 = k\frac{64}{3}$. Therefore, k = 3/64.

$$E(X) = \int_0^4 \frac{3}{64}x^3 dx = \frac{3}{64}\frac{x^4}{4}\Big|_0^4 = 3$$

$$V(X) = \int_0^4 \frac{3}{64}x^2(x-3)^2 dx = \frac{3}{64}\int_0^4 (x^4 - 6x^3 + 9x^2)dx$$

$$= \frac{3}{64}\left(\frac{x^5}{5} - \frac{6x^4}{4} + \frac{9x^3}{3}\right)\Big|_0^4 = 0.6$$

b) $\int_0^2 k(1+2x)dx = k(x+x^2)\Big|_0^2 = k6$. Therefore, k = 1/6.

$$E(X) = \int_0^2 \frac{1}{6}x(1+2x)dx = \frac{1}{6}\int_0^2 (x+2x^2)dx = \frac{1}{6}\left(\frac{x^2}{2} + \frac{2x^3}{3}\right)\Big|_0^2 = 11/9$$

$$V(X) = \int_0^2 \tfrac{1}{6}(1+2x)(x-\tfrac{11}{9})^2 dx$$

$$= \tfrac{1}{6}\int_0^2 (x^2 - \tfrac{22}{9}x + \tfrac{121}{81} + 2x^3 - \tfrac{44}{9}x^2 + \tfrac{242}{81}x)dx$$

$$= \tfrac{1}{6}\left(\tfrac{x^3}{3} - \tfrac{22}{9}\tfrac{x^2}{2} + \tfrac{121}{81}x + 2\tfrac{x^4}{4} - \tfrac{44}{9}\tfrac{x^3}{3} + \tfrac{242}{81}\tfrac{x^2}{2}\right)\Big|_0^2 = \frac{1.704}{6} = 0.284$$

c) $\int_0^\infty ke^{-x}dx = k(-e^{-x})\Big|_0^\infty = k$. Therefore, k = 1.

$E(X) = \int_0^\infty xe^{-x}dx$, using integration by parts $E(X) = -xe^{-x}\Big|_0^\infty + \int_0^\infty e^{-x}dx = -e^{-x}\Big|_0^\infty = 1$

$V(X) = \int_0^\infty (x-1)^2 e^{-x}dx = \int_0^\infty (x^2e^{-x} - 2xe^{-x} + e^{-x})dx$. Now, using integration by parts

$\int_0^\infty x^2e^{-x}dx = 2\int_0^\infty xe^{-x}$. Therefore, $V(X) = \int_0^\infty e^{-x}dx = 1$, because e^{-x} is a probability density function.

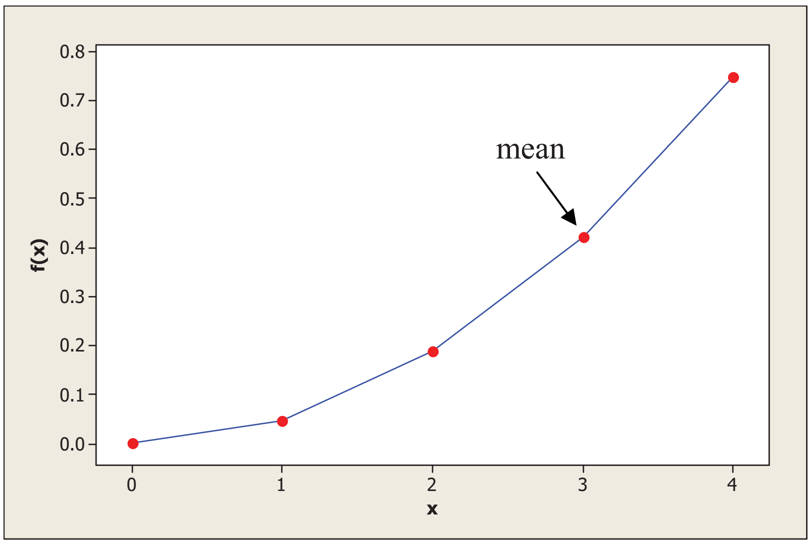
d) $\int_{100}^{100+k} kdx = kx\Big|_{100}^{100+k} = k^2$, $k^2 = 1$, so k =1 (where k > 0)

$E(X) = \int_{100}^{101} 1xdx = 100.5$

$V(X) = \int_{100}^{101} (x-100.5)^2 1dx = 0.08333$

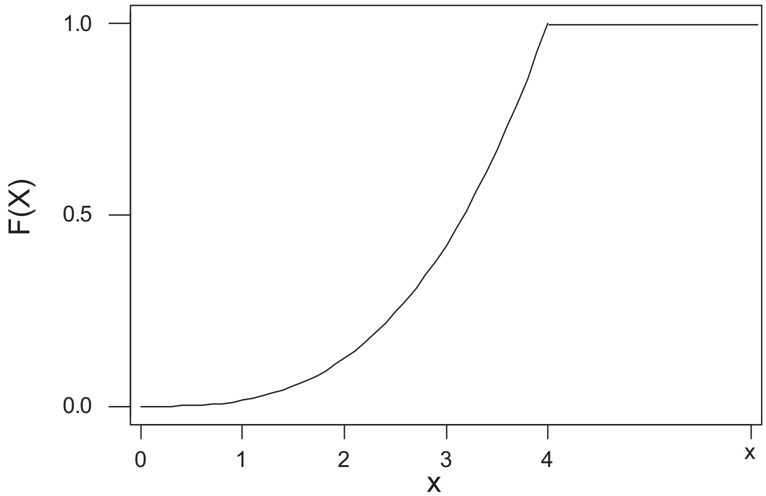
3-22. For 3-21a:

a) $f(x) = \dfrac{3x^2}{64}, \quad 0 < x < 4$



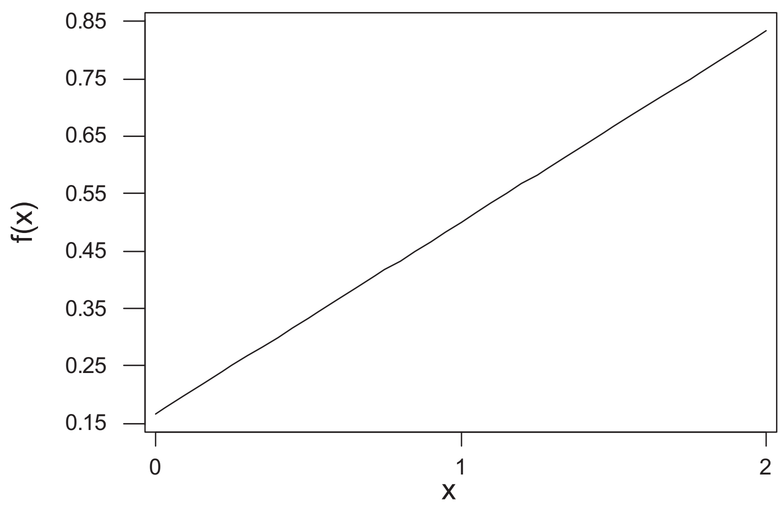
b) $F(X) = \int_0^x f(t)dt = \int_0^x \dfrac{3t^2}{64}dt = \dfrac{x^3}{64}$, 0 < x < 4

c)



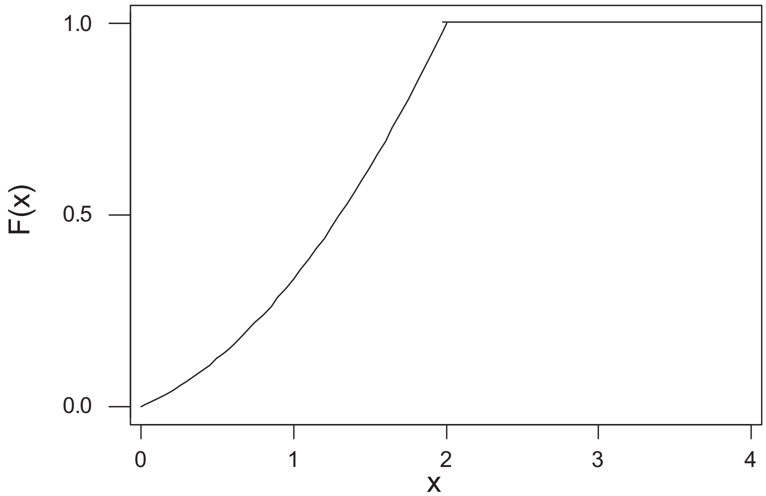
3-21b:

a) $f(x) = \frac{(1+2x)}{6}, \qquad 0 < x < 2$



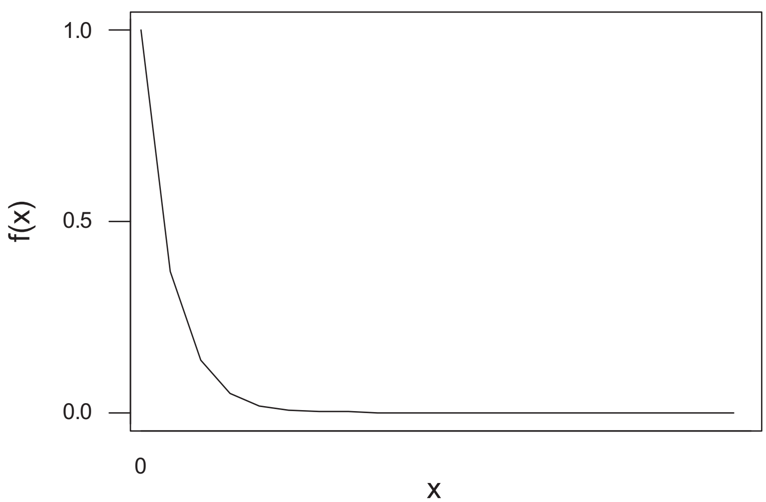
b) $F(X) = \int_0^x \frac{(1+2t)}{6} dt = \frac{1}{6}\left(x + x^2\right), \qquad 0 < x < 2$

c)



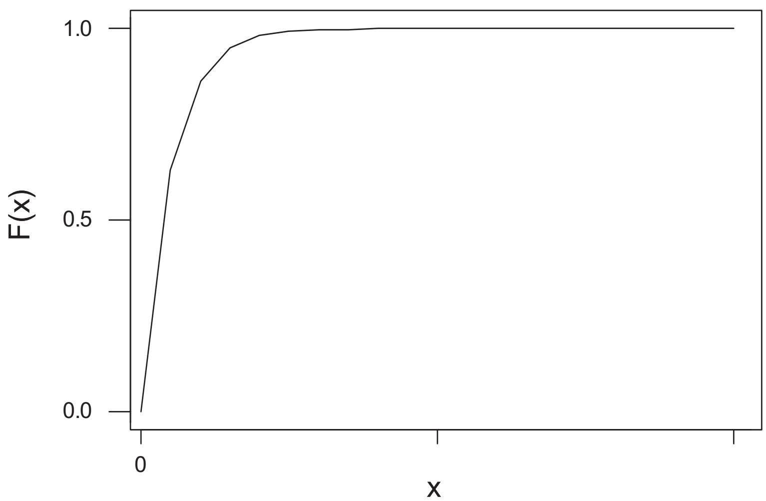
3-21c: $f(x) = e^{-x}$, x >0

a)



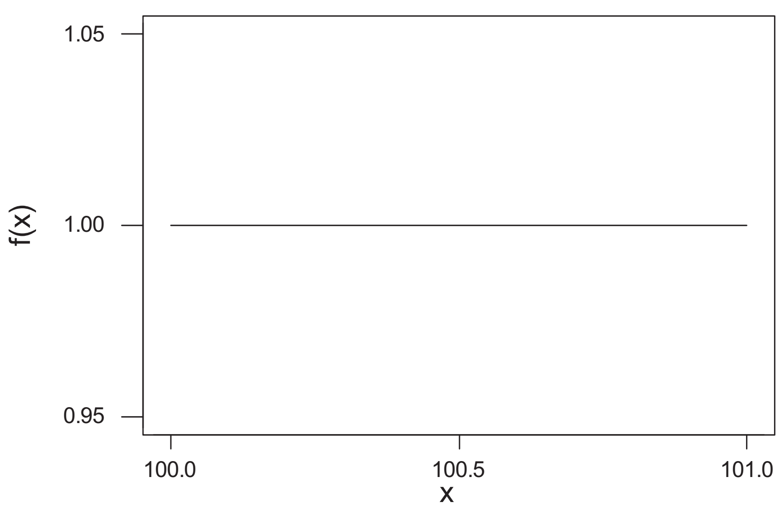
b) F(x) = 1 – e^{-x}, x > 0

c)



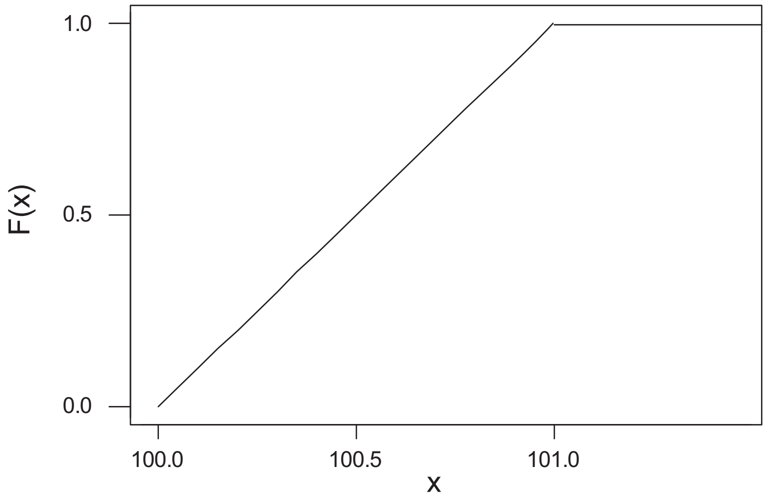
3-21 d: $f(x) = 1$, 100 < x < 101

a)



b) F(X) = x – 100, 100 < x < 101

c)



3-23. a) $P(X > 6) = \int_6^\infty e^{-(x-6)}dx = 1$

b) $P(6 \le X < 8) = \int_6^8 e^{-(x-6)}dx = e^0 - 0.1353 = 0.8647$

c) $P(X < 8) = \int_6^8 e^{-(x-6)}dx = 0.8647$

d) $P(X > 8) = 1 - P(X \le 8) = 1 - P(X < 8) = 1 - 0.8647 = 0.1353$

e) $P(X < x) = \int_6^x e^{-(x-6)}dx = 0.95$

$$\int_6^x e^{-(x-6)}dx = 1 - e^{-(x-6)}$$

$$1 - e^{-(x-6)} = 0.95$$

$$x = -\ln(0.05) + 6$$

$$x = 9$$

3-24. a) $P(0 < X) = 0.5$, by symmetry.

b) $P(0.5 < X) = \int_{0.5}^1 1.5x^2dx = 0.5x^3\Big|_{0.5}^1 = 0.5 - 0.0625 = 0.4375$

c) $P(-0.5 \le X \le 0.5) = \int_{-0.5}^{0.5} 1.5x^2dx = 0.5x^3\Big|_{-0.5}^{0.5} = 0.125$

d) P(X < –2) = 0

e) P(X < 0 or X > –0.5) = 1

f) $P(x < X) = \int_x^1 1.5x^2 dx = 0.5x^3\Big|_x^1 = 0.5 - 0.5x^3 = 0.05$

Then, x = 0.9655

3-25. a) $P(X > 1000) = \int_{1000}^{\infty} \frac{e^{-\frac{x}{4000}}}{4000} dx = 0.7788$

b) $P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{-\frac{x}{4000}}}{4000} dx = 0.1723$

c) $P(X < 4000) = \int_{0}^{4000} \frac{e^{-\frac{x}{4000}}}{4000} dx = 0.6321$

d) $P(X < x) = \int_{0}^{x} \frac{e^{-\frac{x}{4000}}}{4000} dx = 0.10$

$\int_{0}^{x} \frac{e^{-\frac{x}{4000}}}{4000} dx = -e^{-\frac{x}{4000}}\Big|_0^x = 1 - e^{-\frac{x}{4000}} = 0.1$

$e^{-\frac{x}{4000}} = 0.9$

x = 421.6

e) $E(X) = \int_{0}^{\infty} x \frac{e^{-\frac{x}{4000}}}{4000} dx = 4000$

$V(X) = (4000)^2 = 16000000$

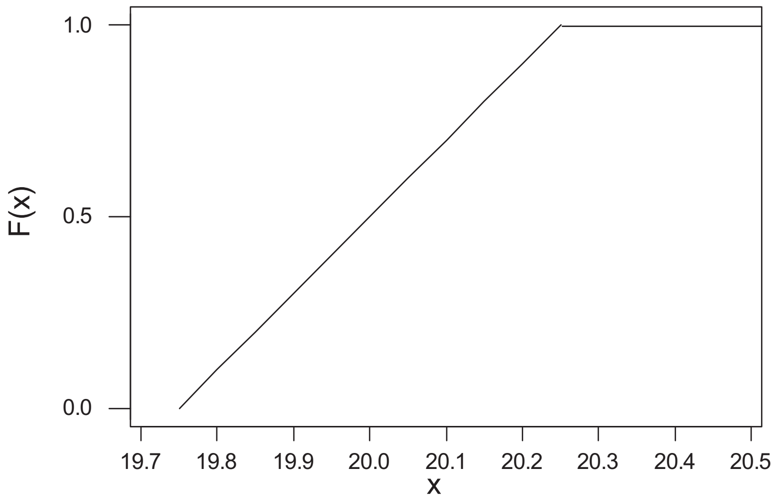
3-26. a) $P(19.75 < X < 20) = \int_{19.75}^{20} 2.0 dx = 0.5$

b) $P(19.8 < X < 20.0) = \int_{19.8}^{20.0} 2.0 dx = 0.4$

c) $E(X) = \int_{19.75}^{20.25} 2.0 x dx = 20$

$V(X) = \int_{19.75}^{20.25} 2.0 x^2 dx - [E(X)]^2 = 400.02 - (20)^2 = 0.02$

d) $F(x) = \int_{19.75}^{x} 2.0 dx = 2x - 39.5$, 19.75 < x < 20.25



3-27. a) P(X < 905) = F(905) = 0.1(905) – 90 = 0.5
b) P(900 < X < 905) = F(905) – F(900) = 0.5 – 0 = 0.5
c) P(X > 908) = 1 – P(X ≤ 908) = 1 – [0.1(908) – 90] = 0.2
d) In specification = P(802 < X < 808) =F(808) – F(802) = 0.8 – 0.2 = 0.6.
Therefore, out of specification is 1 – 0.6 = 0.4.

3-28. a) $P(X \le 2.0080) = F(2.0080) = 200(2.0080) - 401 = 0.6$
b) $P(X > 2.0055) = 1 - P(X \le 2.0055) = 1 - F(2.0055) = 1 - 0.1 = 0.9$
c) $P(2.0080 < X < 2.0090) = FF(2.0090) - (2.0080) = 0.8 - 0.6 = 0.2$

3-29. a) Show that $\int_1^\infty 2x^{-3}dx = 1$:

$$\int_1^\infty 2x^{-3}dx = -x^{-2}\Big|_1^\infty = 0 - (-1^{-2}) = 1$$

b) $F(X) = \int_1^x 2x^{-3}dx = -x^{-2}\Big|_1^x = 1 - x^{-2}$

c) $E(X) = \int_1^\infty 2x^{-2}dx = 2.0$

d) $P(X < 5) = \int_1^5 2x^{-3}dx = 0.96$

e) $P(X > 7) = \int_7^\infty 2x^{-3}dx = 0.0204$

3-30. a) $E(X) = \int_5^\infty x10e^{-10(x-5)}dx$.

Using integration by parts with u = x and $dv = 10e^{-10(x-5)}dx$, we obtain

$$E(X) = -xe^{-10(x-5)}\Big|_5^\infty + \int_5^\infty e^{-10(x-5)}dx = 5 - \frac{e^{-10(x-5)}}{10}\Big|_5^\infty = 5.1$$

Now, $V(X) = \int_5^\infty (x-5.1)^2 10e^{-10(x-5)}dx$. Using the integration by parts with $u = (x-5.1)^2$ and

$dv = 10e^{-10(x-5)}$, we obtain $V(X) = -(x-5.1)^2 e^{-10(x-5)}\Big|_5^\infty + 2\int_5^\infty (x-5.1)e^{-10(x-5)}dx$.

From the definition of E(X), the integral above is recognized to equal 0. Therefore, $V(X) = (5-5.1)^2 = 0.01$.

b) $P(X > 5.1) = \int_{5.1}^\infty 10e^{-10(x-5)}dx = -e^{-10(x-5)}\Big|_{5.1}^\infty = e^{-10(5.1-5)} = 0.3679$

3-31. a) P(X < 1408) = F(1408) = 0.1(1408) – 140 = 0.8

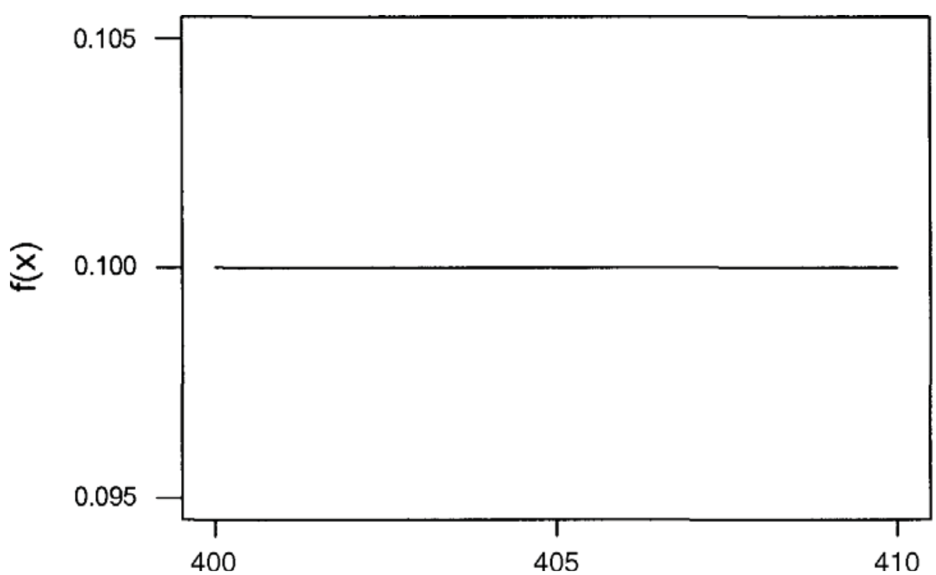
b) P(1395 < X < 1405) = F(1405) – F(1395) = [0.1(1405)-140] – 0 = 0.5

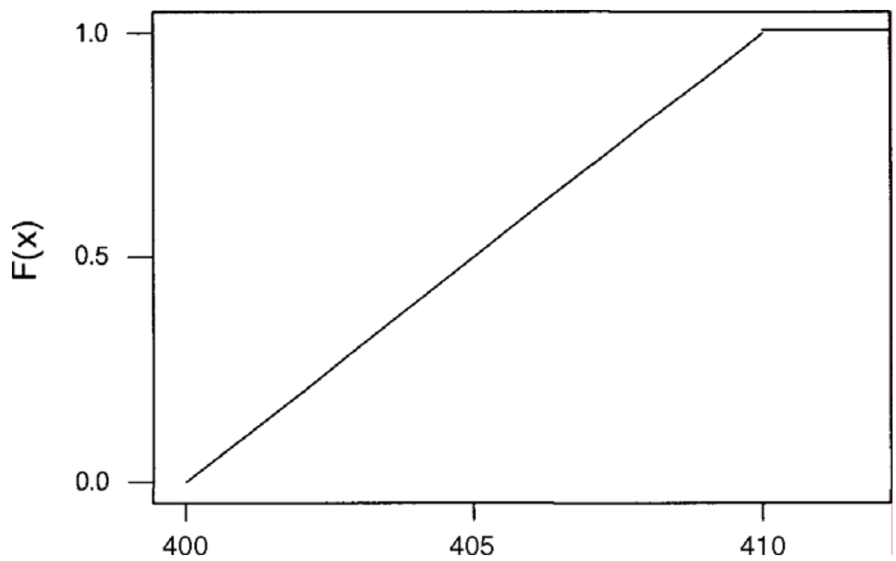
3-32. a) $E(X) = \int_{100}^{120} x\frac{600}{x^2}dx = 600\ln x\,|_{100}^{120} = 600(\ln 120 - \ln 100) = 109.39\mu m$

$V(X) = \int_{100}^{120} x^2\frac{600}{x^2}dx - E(x)^2 = 600x\,|_{100}^{120} - 109.39^2 = 33.19\mu m^2$

b) Average Cost = Coating Cost × Mean of the Coating Thickness
= 0.50 × 109.39 = \$54.70

3-33. a) P(X < 409) = 0.1(409) – 40 = 0.9
b) P(400 < X < 408) = F(408) – F(400) = 0.8 – 0 = 0.8
c) P(X > 409) = 1 – F(409) = 1 – 0.9 = 0.1
d) f(x) = 0.1 for 400 ≤ x ≤ 410
e)





f) E(X) = 205, V(X) = 8.3333

3-34. Now, $f(x) = \frac{e^{-x/10}}{10}$ for 0 < x and

$$F_x(x) = \frac{1}{10}\int_0^x e^{-u/10}du = -e^{-u/10}\Big|_0^x = 1 - e^{-x/10} \quad \text{for } 0 < x.$$

Then, $F_x(x) = \begin{cases} 0, x \le 0 \\ 1 - e^{-x/10}, x > 0 \end{cases}$

a) P(X<60) = F(60) =1 - e^{-6} = 1 - 0.002479 = 0.9975

b) $\frac{1}{10}\int_{15}^{30} e^{-x/10}dx = e^{-1.5} - e^{-3} = 0.173343$

c) $\frac{1}{10}\int_0^x e^{-u/10}du = 0.5 \rightarrow x \approx 6.93147$

d) P(15<X<30) = F(30)-F(15) = $e^{-1.5}$- e^{-3}=0.173343

e) $\mu = E(X) = \int_0^\infty xf(x)dx = \frac{1}{10}\int_0^\infty xe^{-x/10}dx = e^{-x/10}(-x-10)\,|_0^\infty = 0 + 10 = 10$

$$\sigma^2 = V(X) = \int_0^\infty x^2 f(x)dx - \mu^2 = \frac{1}{10}\int_0^\infty x^2 e^{-x/10}dx = e^{-x/10}((-x-20)x-200)\,|_0^\infty - (10)^2$$
$$= 0 + 200 - 100 = 100$$

$$\sigma = \sqrt{\sigma^2} = \sqrt{100} = 10$$

3-35. a) $P(1 < X < 5) = \int_1^5 f(x)dx = -\frac{32}{3|_x}\Bigg|^5 = 0.826$

b) $E(X) = \int_1^{32} xf(x)dx = \int_1^{32}\frac{32}{31x}dx = \frac{32}{31}\ln x\,|_1^{32} = 3.578$

$E(X^2) = \int_1^{32} x^2 f(x)dx = \int_1^{32}\frac{32}{31}dx = 32$

Var(X) = $E(X^2) - [E(X)]^2 = 32 - 12.8021 = 19.1979$

c) 3*3.578 = 10.73

3-36. a) P(X < 1) = F(1) = 1 – exp(–1 /2) = 1 – 0.607 = 0.393
b) P(X > 2) = 1 – F(2) = 1 – (1 – exp(–2/2)) = 1– (1 – 0.368) = 0.368
c) P(1 ≤ X < 2) = F(2) – F(1) = 0.632 – 0.393 = 0.239
d) P(X < x) = 0.95 = 1 – exp(-x/2). Then, exp(-x/2) = 0.05 and x = 6
e) f(x) = $0.5e^{-x/2}$ for 0 ≤ x

3-37. $F_x(x) = \int_0^x f(u)du = (\frac{1}{18}u^2)|_0^x = \frac{1}{18}x^2$ for 0 < x < 3

$$F_x(x) = \int_0^3 f(u)du + \int_3^x f(u)du = 0.5 + (\frac{2}{3}u - \frac{1}{18}u^2)|_3^x$$

$= \frac{2}{3}x - \frac{1}{18}x^2 - 1$, for 3 < x < 6.

Then, $F_x(x) = \begin{cases} \frac{1}{18}x^2, & 0 < x < 3 \\ \frac{2}{3}x - \frac{1}{18}x^2 - 1, & 3 < x < 6 \end{cases}$

a) P(X < 4) = F(4) = $\frac{2}{3}(4) - \frac{1}{18}(4)^2 - 1 = \frac{7}{9}$

b) P(X > 5) = 1 – F(5) = $1 - [\frac{2}{3}(5) - \frac{1}{18}(5)^2 - 1] = 1 - \frac{17}{18} = 1 - 0.944 = 0.056$

c) P(X ≤ 0.5) = F(0.5) = $\frac{1}{18}(0.5^2) = 0.014$

d) P(X > x) = 1 – F(x) = 0.1. Therefore F(x) = 0.9.
From part (b), F(5) = 0.944, the range of 3 < x < 6 are selected.
F(x) = 0.9 = $\frac{2}{3}(x) - \frac{1}{18}(x)^2 - 1$. Then, $\frac{2}{3}(x) - \frac{1}{18}(x)^2 = 1.9$.
Solving the equation above to obtain x = 4.658

e) $E(X) = \int_0^3 xf(x)dx + \int_3^6 xf(x)dx = \int_0^3 x\frac{1}{9}xdx + \int_3^6 x(\frac{2}{3} - \frac{1}{9}x)dx = (\frac{1}{27}x^3|_0^3) + (\frac{1}{3}x^2 - \frac{1}{27}x^3|_3^6) = 1 + (4-2) = 3$

Section 3-5

3-38. a) P(-1 < Z < 1) = P(Z < 1) – P(Z < -1) = 0.841345 – 0.158655 = 0.68269
b) P(-2 < Z < 2) = P(Z < 2) – P(Z < -2) = 0.97725 – 0.02275 = 0.9545
c) P(-3 < Z < 3) = P(Z < 3) – P(Z < -3) = 0.998650 – 0.00135 = 0.997300
d) P(Z < -3) = 0.00135
e) P(0 < Z ≤ 3) = P(Z < 3) – P(Z < 0) = 0.998650 – 0. 5 = 0.498650

3-39. a) P(Z < z) = 0.500000
z = 0
b) P(Z < z) = 0.001001
z = -3.09
c) P(Z > z) = 0.881000
Can be rewritten as: 1 – P(Z < z) = 0.881000
P(Z < z) = 0.119000
z = -1.18
d) P(Z > z) = 0.866500
Can be rewritten as: 1 – P(Z < z) = 0.866500
P(Z < z) = 0.133500
z = -1.11

e) P(-1.3 < Z < z) = 0.863140

P(Z < z) – P(Z < -1.3) = 0.863140
P(Z < z) – 0.096801 = 0.863140
P(Z < z) = 0.959941, Z = 1.75

3-40. a) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.90
So P(Z<-z) = 0.5(1 – 0.90) = 0.05
z = 1.65
b) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.97
So P(Z<-z) = 0.5(1 – 0.97) = 0.015
z = 2.17
c) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.61
So P(Z<-z) = 0.5(1 – 0.61) = 0.195
z = 0.86
d) P(-z < Z < z) = P(Z < z) – P(Z < -z) = 1 – 2P(Z<-z) = 0.99
So P(Z<-z) = 0.5(1 – 0.99) = 0.005
z = 2.58

3-41. a) P(X < 34) = P(Z < 2) = 0.97725
b) P(X > 28) = P(Z > -1) = 0.841345
c) P(28 < X < 32) = P(-1 < Z < 1) = P(Z < 1) – P(Z < -1) = 0.841345 – 0.158655 = 0.68269
d) P(24 < X < 36) = P(-3 < Z < 3) = 0.99865 – 0.00135 = 0.9973
e) P(26 < X < 30) = P(-2 < Z < 0) = P(Z < 0) – P(Z < -2) = 0.5 – 0.02275 = 0.47725
f) P(30 < X < 36) = P(0 < Z < 3) = P(Z < 3) – P(Z < 0) = 0.99865 – 0.50 = 0.49865

3-42. a) P(X > x) = P(Z > z) = 0.5 where z = $\frac{x-20}{2}$ = 0. Thus x =20
b) P(X > x) = P(Z > z) = 0.95 where z = $\frac{x-20}{2}$ = -1.645. Thus x =16.71
c) P(x < X < 20) = P(Z < 0) – P(Z < z)= 0.5 – P(Z < z) = 0.2
So P(Z < z) = 0.3 where z = $\frac{x-20}{2}$ = -0.53. Thus x =18.94

3-43. a) P(X < 21) = P(Z < -3) = 0.001350
b) P(X > 20) = P(Z > -3.5) = 0.999767
c) P(23 < X < 27) = P(-2 < Z < 0) = P(Z < 0) – P(Z < -2) = 0.5 – 0.02275 = 0.47725
d) P(22 < X < 29) = P(-2.5 < Z < 1) = 0.841345 – 0.00621 = 0.835135
e) P(20 < X < 28) = P(-3.5 < Z < 0.5) = 0.691462 – 0.000233 = 0.691229

3-44. a) P(X > x) = 1 – P(X < x)
$= 1 - P\left(Z < \frac{x-6}{3}\right) = 0.50.$
And, $P\left(Z < \frac{x-6}{3}\right) = 0.50$
Therefore, $\frac{x-6}{3} = 0$ and x = 6.
b) P(X > x) = 1 – P(X < x)
$= 1 - P\left(Z < \frac{x-6}{3}\right) = 0.95.$
And, $P\left(Z < \frac{x-6}{3}\right) = 0.05$
Therefore, $\frac{x-6}{3} = -1.645$ and x = 1.065.

c) P(x < X < 9) = 0.20

$$P\left(\frac{x-6}{3} < Z < \frac{9-6}{3}\right) = P\left(\frac{x-6}{3} < Z < 1\right)$$

$$P(Z<1) - P\left(Z < \frac{x-6}{3}\right) = 0.20$$

$$0.841345 - P\left(Z < \frac{x-6}{3}\right) = 0.20$$

$$P\left(Z < \frac{x-6}{3}\right) = 0.841345 - 0.20$$

$$P\left(Z < \frac{x-6}{3}\right) = 0.641345$$

Therefore, $\frac{x-6}{3}$ =0.36, and x = 7.08.

d) P(3 < X < x) = 0.80

$$P\left(\frac{3-6}{3} < Z < \frac{x-6}{3}\right) = P\left(-1 < Z < \frac{x-6}{3}\right)$$

$$P\left(Z < \frac{x-6}{3}\right) - P(Z < -1) = 0.80$$

$$P\left(Z < \frac{x-6}{3}\right) - 0.158655 = 0.80$$

$$P\left(Z < \frac{x-6}{3}\right) = 0.958655$$

Therefore, $\frac{x-6}{3} \cong 1.74$, and x = 11.22.

3-45. a) $P(X < 5250) = P\left(Z < \frac{5250-5000}{100}\right)$

= P(Z < 2.5)
= 0.9938

b) P(4800 < X < 4900) = P(–2 < Z < –1)
= P(Z <– 1) – P(Z < –2)
= 0.1359

c) $P(X > x) = P\left(Z > \frac{x-5000}{100}\right) = 0.95.$

Therefore, $\frac{x-5000}{100}$ = –1.64 and x = 4835.51.

3-46. a) P(X < 269) = P(Z < 2) = 0.97725
b) P(X < 200) = P(Z < -3) = 0.00170

3-47. a) $P(X > 0.62) = P\left(Z > \frac{0.62-0.5}{0.05}\right)$

= P(Z > 2.4)
= 1 – P(Z < 2.4)
= 0.0082

b) P(0.47 < X < 0.63) = P(–0.6 < Z < 2.6)
= P(Z < 2.6) – P(Z < –0.6)
= 0.99534 – 0.27425

$= 0.7211$

c) $P(X < x) = P\left(Z < \frac{x - 0.5}{0.05}\right) = 0.90.$

Therefore, $\frac{x-0.5}{0.05} = 1.28$ and $x = 0.5641$.

3-48. a) $P(X < 355) = P(Z < \frac{355-367}{3}) = P(Z < -4) \cong 0$

b) $P(X < 358) = P\left(Z < \frac{358-367}{3}\right)$

$= P(Z < -3)$
$= 0.00135$

and

$P(X > 373) = P\left(Z > \frac{373-367}{3}\right)$

$= P(Z > 2)$
$= 0.02275$

Therefore, the proportion of cans scrapped is $0.00135 + 0.02275 = 0.0241$.

c) $P(367 - x < X < 367 + x) = 0.99$.

Therefore, $P\left(-\frac{x}{3} < Z < \frac{x}{3}\right) = 0.99$

Consequently, $P\left(Z < \frac{x}{3}\right) = 0.995$ and $x = 3(2.58) = 7.74$.

The specifications are $(359.26; 374.74)$.

3-49. d) If $P(X > 355) = 0.999$, then $P\left(Z > \frac{355-\mu}{3}\right) = 0.999.$

Therefore, $\frac{355-\mu}{3} = -3.09$ and $\mu = 364.27$.

e) If $P(X > 355) = 0.999$, then $P\left(Z > \frac{355-\mu}{1.5}\right) = 0.999.$

Therefore, $\frac{355-\mu}{1.5} = -3.09$ and $\mu = 359.635$.

3-50. a) $P(X > 0.5) = P\left(Z > \frac{0.5-0.4}{0.05}\right)$

$= P(Z > 2)$
$= 1 - 0.97725$
$= 0.02275$

b) $P(0.4 < X < 0.5) = P(0 < Z < 2)$
$= P(Z < 2) - P(Z < 0)$
$= 0.47725$

c) $P(X > x) = 0.90$. Therefore, $\frac{x-0.4}{0.05} = -1.28$ and $x = 0.336$.

3-51. a) $P(90.3 < X) = P\left(\frac{90.3-90.2}{0.1} < Z\right)$

$= P(1 < Z)$
$= P(Z > 1)$

$= 1 - P(Z < 1)$
$=1 - 0.841345$
$= 0.158655$

$$P(X < 89.7) = P\left(Z < \frac{89.7 - 90.2}{0.1}\right)$$
$= P(Z < -5)$
$\cong 0$.

Therefore, the answer is 0.1587.

b) The process mean should be set at the center of the specifications; that is, at $\mu = 90.0$.

c) $P(89.7 < X < 90.3) = P\left(\frac{89.7 - 90}{0.1} < Z < \frac{90.3 - 90}{0.1}\right)$
$= P(-3 < Z < 3) = 0.9973$.

3-52. a) $P(X > 1140) = P(Z > \frac{1140 - 1000}{60}) = P(Z > 2.33) = P(Z < -2.33) = 0.009903$

b) $P(X < 900) = P(Z > \frac{900 - 1000}{60}) = P(Z > -0.17) = 0.432505$

3-53. a) $P(X > 9) = P(Z > 1.34) = 0.09012$
b) $P(5.5 < X < 8.5) = P(-0.35 < Z < 1.10) = 0.864334 - 0.363169 = 0.501165$
c) Threshold $= \mu + 3.75\sigma = 6.23 + 3.75(2.064) = 13.97$

3-54. a) $P(X < 5000) = P\left(Z < \frac{5000 - 7000}{600}\right)$
$= P(Z < -3.33)$
$= 0.0004$.

b) $P(X > x) = 0.95$. Therefore, $P\left(Z > \frac{x - 7000}{600}\right) = 0.95$ and $\frac{x - 7000}{600} = -1.64$. Consequently, $x = 6016$.

3-55. a) $P(X > 0.066) = P\left(Z > \frac{0.066 - 0.05}{0.01}\right)$
$= P(Z > 1.6)$
$= 0.0548$.

b) $P(0.036 < X < 0.066) = P(-1.4 < Z < 1.6)$
$= 0.8644$.

c) $P(0.036 < X < 0.066) = P\left(\frac{0.036 - 0.05}{\sigma} < Z < \frac{0.066 - 0.05}{\sigma}\right)$
$$= P\left(\frac{-0.014}{\sigma} < Z < \frac{0.016}{\sigma}\right) = 0.995$$

Therefore, $P\left(Z < \frac{0.016}{\sigma}\right) = 0.995$. Consequently, $\frac{0.016}{\sigma} = 2.81$ and $\sigma = 0.00569$.

3-56. a) $P(X > 62) = P(Z > 2) = 1 - P(Z < 2) = 1 - 0.977250 = 0.02275$

b) $P(X < 62) = 0.999$
$P\left(Z < \frac{62 - 59}{\hat{\sigma}}\right) = 0.999$
$\frac{62 - 59}{\hat{\sigma}} = 3.09$
$\hat{\sigma} = 0.97087$

c) P(X < 62) = P(Z < z) = 0.999 where $z = \frac{62-\hat{\mu}}{1.5} = 3.09$. Thus $\hat{\mu} = 57.365$

3-57. X is a lognormal distribution with θ=5 and ω^2=9

a) $P(X < 13300) = P(e^W < 13300) = P(W < \ln(13300)) = \Phi\left(\frac{\ln(13330)-5}{3}\right)$

$= \Phi(1.50) = 0.9332$

b) Find the value for which P(X≤x)=0.95

$$P(X \le x) = P(e^W \le x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x)-5}{3}\right) = 0.95$$

$\frac{\ln(x)-5}{3} = 1.65 \quad x = e^{1.65(3)+5} = 20952.2$

c) $\mu = E(X) = e^{\theta+\omega^2/2} = e^{5+9/2} = e^{9.5} = 13359.7$

$V(X) = e^{2\theta+\omega^2}(e^{\omega^2}-1) = e^{10+9}(e^9-1) = e^{19}(e^9-1) = 1.45x10^{12}$

3-58. a) X is a lognormal distribution with θ=2 and ω^2=4

$P(X < 500) = P(e^W < 500) = P(W < \ln(500)) = \Phi\left(\frac{\ln(500)-2}{2}\right)$

$= \Phi(2.11) = 0.9826$

b)

$P(500 < X < 1000) = P(500 < e^W < 1000) = P(\ln(500) < W < \ln(1000)) = \Phi\left(\frac{\ln(1000)-2}{2}\right) - \Phi\left(\frac{\ln(500)-2}{2}\right)$

$= \Phi(2.45) - \Phi(2.11) = 0.99286 - 0.9826$

$= 0.01026$

c)

$P(1500 < X < 2000) = P(1500 < e^W < 2000) = P(\ln(1500) < W < \ln(2000)) = \Phi\left(\frac{\ln(2000)-2}{2}\right) - \Phi\left(\frac{\ln(1500)-2}{2}\right)$

$= \Phi(2.80) - \Phi(2.66) = 0.99744 - 0.99609$

$= 0.00135$

d) The product has significantly degraded over the first 500 hours. The degradation is less significant after 500 hours.

3-59. X is a lognormal distribution with θ=0.5 and ω^2=1

a)

$$P(X > 10) = P(e^W > 10) = P(W > \ln(10)) = 1 - \Phi\left(\frac{\ln(10)-0.5}{1}\right)$$

$= 1 - \Phi(1.80) = 1 - 0.96407 = 0.03593$

b) $P(X \le x) = P(e^W \le x) = P(W < \ln(x)) = \Phi\left(\frac{\ln(x)-0.5}{1}\right) = 0.50$

$\frac{\ln(x)-0.5}{1} = 0 \quad x = e^{0(1)+0.5} = 1.65$ seconds

c) $\mu = E(X) = e^{\theta+\omega^2/2} = e^{0.5+1/2} = e^1 = 2.7183$

$V(X) = e^{2\theta+\omega^2}(e^{\omega^2}-1) = e^{1+1}(e^1-1) = e^2(e^1-1) = 12.6965$

3-60. a) Find the values of θ and ω^2 given that E(X) = 10000 and σ= 20,000

$10000 = e^{\theta+\omega^2/2}$ $\qquad 20000^2 = e^{2\theta+\omega^2}(e^{\omega^2}-1)$

Let $x = e^{\theta}$ and $y = e^{\omega^2}$ then (1) $10000 = x\sqrt{y}$ and (2) $20000^2 = x^2 y(y-1) = x^2y^2 - x^2y$

Square (1): $10000^2 = x^2 y$ and substitute into (2)

$20000^2 = 10000^2(y-1)$

$y = 5$

Substitute y into (1) and solve for x: $x = \frac{10000}{\sqrt{5}} = 4472.1360$

$\theta = \ln(4472.1360) = 8.4056$ and $\omega^2 = \ln(5) = 1.6094$

b) $P(X > 10000) = P(e^W > 10000) = P(W > \ln(10000)) = 1 - \Phi\left(\frac{\ln(10000) - 8.4056}{1.2686}\right)$

$= 1 - \Phi(0.63) = 1 - 0.7357 = 0.2643$

c) $P(X > x) = P(e^W > x) = P(W > \ln(x)) = \Phi\left(\frac{\ln(x) - 8.4056}{1.2686}\right) = 0.1$

$\frac{\ln(x) - 8.4056}{1.2686} = -1.28 \qquad x = e^{-1.280(1.2686)+8.4056} = 881.65$ hours

3-61. β = 0.2 and δ = 100 hours

$E(X) = 100\Gamma(1 + \frac{1}{0.2}) = 100 \times 5! = 12{,}000$

$V(X) = 100^2\Gamma(1 + \frac{2}{0.2}) - 100^2[\Gamma(1 + \frac{1}{0.2})]^2 = 3.61 \times 10^{10}$

3-62. a) $P(X < 10000) = F_X(10000) = 1 - e^{-100^{0.2}} = 1 - e^{-2.512} = 0.9189$

b) $P(X > 5000) = 1 - F_X(5000) = e^{-50^{0.2}} = 0.1123$

3-63. Let X denote lifetime of a bearing. β=2 and δ=10000 hours

a) $P(X > 8000) = 1 - F_X(8000) = e^{-\left(\frac{8000}{10000}\right)^2} = e^{-0.8^2} = 0.5273$

b)

$E(X) = 10000\Gamma(1 + \frac{1}{2}) = 10000\Gamma(1.5)$

$= 10000(0.5)\Gamma(0.5) = 5000\sqrt{\pi} = 8862.3$

= 8862.3 hours

c) Let Y denote the number of bearings out of 10 that last at least 8000 hours. Then, Y is a binomial random variable with n = 10 and p = 0.5273.

$P(Y = 10) = \binom{10}{10} 0.5273^{10}(1 - 0.5273)^0 = 0.00166$.

3-64. a) $E(X) = \delta\Gamma(1 + \frac{1}{\beta}) = 900\Gamma(1 + 1/3) = 900\Gamma(4/3) = 900(0.89298). = 803.68$ hours

b)

$V(X) = \delta^2\Gamma(1 + \frac{2}{\beta}) - \delta^2\left[\Gamma(1 + \frac{2}{\beta})\right]^2 = 900^2\Gamma(1 + \frac{2}{3}) - 900^2\left[\Gamma(1 + \frac{1}{3})\right]^2$

$= 900^2(0.90274) - 900^2(0.89298)^2 = 85314.64 \text{ hours}^2$

c) $P(X < 500) = F_X(500) = 1 - e^{-\left(\frac{500}{900}\right)^3} = 0.1576$

3-65. $E(X) = \delta\Gamma\left(1 + \frac{1}{2}\right) = 2.5$

So $\delta = \dfrac{2.5}{\Gamma\left(1+\frac{1}{2}\right)} = \dfrac{5}{\sqrt{\pi}}$

$Var(X) = \delta^2\Gamma(2) - (E(X))^2 = \dfrac{25}{\pi} - 2.5^2 = 1.7077$

Stdev(X)= 1.3068

3-66. $\Gamma(r) = \int_0^\infty x^{r-1}e^{-x}dx$. Use integration by parts with $u = x^{r-1}$ and $dv = e^{-x}$. Then,

$\Gamma(r) = -x^{r-1}e^{-x}\Big|_0^\infty + (r-1)\int_0^\infty x^{r-2}e^{-x}dx = (r-1)\Gamma(r-1)$.

3-67. a) $\Gamma(6) = 5! = 120$

b) $\Gamma\left(\frac{5}{2}\right) = \frac{3}{2}\Gamma\left(\frac{3}{2}\right) = \frac{3}{2}\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{3}{4}\pi^{1/2} = 1.32934$

c) $\Gamma\left(\frac{9}{2}\right) = \frac{7}{2}\Gamma\left(\frac{7}{2}\right) = \frac{7}{2}\frac{5}{2}\frac{3}{2}\frac{1}{2}\Gamma\left(\frac{1}{2}\right) = \frac{105}{16}\pi^{1/2} = 11.6317$

3-68. E(X) = 6/3 = 2; V(X) = $6/3^2$ = 0.6667

3-69. E(X) = 3.2/2.5 = 1.28; V(X) = $3.2/(2.5)^2$ = 0.512

3-70. r/λ = 3, so r = 3λ; Also, $r/\lambda^2 = 1.5$; Using substitution, $3\lambda/\lambda^2 = 1.5$, and λ = 2;
Giving r = 6.

3-71. r/λ = 11.4, so r = 11.4λ; Also, $r/\lambda^2 = 40.32$; Using substitution, $\frac{11.4\lambda}{\lambda^2} = 40.32$, and λ = 0.28;
Giving r = 3.19.

3-72. r/λ = 4, so r = 4λ; Also, $r/\lambda^2 = 2$; Using substitution, $4\lambda/\lambda^2 = 2$, and λ = 2;
Giving r = 8.

Section 3-6

3-73. a) $P(X < 0.25) = \int_0^{0.25} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1} = \int_0^{0.25} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}$

$= \int_0^{0.25} \frac{\Gamma(3.5)}{\Gamma(2.5)\Gamma(1)})x^{1.5} = \frac{(2.5)(1.5)(0.5)\sqrt{\pi}}{(1.5)(0.5)\sqrt{\pi}} \frac{x^{2.5}}{2.5}\Big|_0^{0.25} = 0.25^{2.5} = 0.0313$

b) $P(0.25 < X < 0.75) = \int_{0.25}^{0.75} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1} = \int_{0.25}^{0.75} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}$

$= \int_{0.25}^{0.75} \frac{\Gamma(3.5)}{\Gamma(2.5)\Gamma(1)})x^{1.5} = \frac{(2.5)(1.5)(0.5)\sqrt{\pi}}{(1.5)(0.5)\sqrt{\pi}} \frac{x^{2.5}}{2.5}\Big|_{0.25}^{0.75} = 0.75^{2.5} - 0.25^{2.5} = 0.4559$

c) $\mu = E(X) = \frac{\alpha}{\alpha+\beta} = \frac{2.5}{2.5+1} = 0.7143$

$\sigma^2 = V(X) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} = \frac{2.5}{(3.5)^2(4.5)} = 0.0454$

3-74. a) $P(X<0.25)=\int_0^{0.25}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}=\int_0^{0.25}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}$

$$=\int_0^{0.25}\frac{\Gamma(5.2)}{\Gamma(1)\Gamma(4.2)})(1-x)^{3.2}=\frac{(4.2)(3.2)(2.2)(1.2)\Gamma(1.2)}{(3.2)(2.2)(1.2)\Gamma(1.2)}\frac{-(1-x)^{4.2}}{4.2}\Big|_0^{0.25}=-(0.75)^{4.2}+1=0.7013$$

b) $P(0.5<X)=\int_{0.5}^{1}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}=\int_{0.5}^{1}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}$

$$=\int_{0.5}^{1}\frac{\Gamma(5.2)}{\Gamma(1)\Gamma(4.2)})(1-x)^{3.2}=\frac{(4.2)(3.2)(2.2)(1.2)\Gamma(1.2)}{(3.2)(2.2)(1.2)\Gamma(1.2)}\frac{-(1-x)^{4.2}}{4.2}\Big|_{0.5}^{1}=0+(0.5)^{4.2}=0.0544$$

c) $\mu=E(X)=\frac{\alpha}{\alpha+\beta}=\frac{1}{1+4.2}=0.1923$

$$\sigma^2=V(X)=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}=\frac{4.2}{(5.2)^2(6.2)}=0.0251$$

3-75. a) Mode $=\frac{\alpha-1}{\alpha+\beta-2}=\frac{2}{3+1.4-2}=0.8333$

$$\mu=E(X)=\frac{\alpha}{\alpha+\beta}=\frac{3}{3+1.4}=0.6818$$

$$\sigma^2=V(X)=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}=\frac{4.2}{(4.4)^2(5.4)}=0.0402$$

b) Mode $=\frac{\alpha-1}{\alpha+\beta-2}=\frac{9}{10+6.25-2}=0.6316$

$$\mu=E(X)=\frac{\alpha}{\alpha+\beta}=\frac{10}{10+6.25}=0.6154$$

$$\sigma^2=V(X)=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}=\frac{62.5}{(16.25)^2(17.25)}=0.0137$$

c) Both mean and variance from part a) are larger than part b).

3-76. a) $P(X>0.9)=\int_{0.9}^{1}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}=\int_{0.9}^{1}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}$

$$=\int_{0.9}^{1}\frac{\Gamma(11)}{\Gamma(10)\Gamma(1)})x^{9}=\frac{(10)(9)\Gamma(9)}{(9)\Gamma(9)}\frac{x^{10}}{10}\Big|_{0.9}^{1}=1-(0.9^{10})=0.6513$$

b) $P(X<0.5)=\int_0^{0.5}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}=\int_0^{0.5}\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)})x^{\alpha-1}(1-x)^{\beta-1}$

$$=\int_0^{0.5}\frac{\Gamma(11)}{\Gamma(10)\Gamma(1)})x^{9}=\frac{(10)(9)\Gamma(9)}{(9)\Gamma(9)}\frac{x^{10}}{10}\Big|_0^{0.5}=0.5^{10}=0.0010$$

c) $\mu=E(X)=\frac{\alpha}{\alpha+\beta}=\frac{10}{10+1}=0.9091$

$$\sigma^2=V(X)=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}=\frac{10}{(11)^2(12)}=0.0069$$

3-77. Let X denotes the complete time which is a proportion time of the maximum time
X = 2/2.5 = 0.8