3.1: PROBLEM DEFINITION

Apply the grid method to cases a, b, c and d.

a.)

Situation:

Pressure values need to be converted.

Find:

Calculate the gage pressure (kPa) corresponding to 8 in. H_2O (vacuum).

Solution:

$$p = \left(\frac{6\,\text{in H}_2\text{O}}{}\right)\left(\frac{\text{Pa}}{0.00402\,\text{in-H}_2\text{O}}\right)\left(\frac{\text{kPa}}{1000\,\text{Pa}}\right)$$

$$p = 1.49\,\text{kPa-vacuum} = \boxed{-1.49\,\text{kPa-gage}}$$

b.)

Situation:

Pressure values need to be converted.

Find:

Calculate the gage pressure (psig) corresponding to 180 kPa-abs.

Properties:

$p_{\text{atm}} = 14.70$psi.

Solution:

$$p_{\text{abs}} = \left(\frac{180\text{ kPa}}{1}\right)\left(\frac{14.70\text{ psi}}{101.3\text{ kPa}}\right) = 26.12\text{ psia}$$

$$p_{\text{gage}} = p_{\text{abs}} - p_{\text{atm}} = (26.12\text{ psia}) - (14.70\text{ psia}) = 11.42\text{ psi}$$

$$\boxed{p_{\text{gage}} = 11.42\text{ psig}}$$

c.)

Situation:

Pressure values need to be converted.

Find:

Calculate the absolute pressure (psia) corresponding to a pressure of 0.4 bar (gage).

Properties:

$p_{atm} = 14.70$psi.

Solution:

$$p_{\text{gage}} = \left(\frac{0.4\text{ bar}}{1}\right)\left(\frac{14.70\text{ psi}}{1.013\text{ bar}}\right) = 5.80\text{ psig}$$

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}} = (5.80\text{ psig}) + (14.70\text{ psia}) = 20.5\text{ psia}$$

$$\boxed{p_{\text{abs}} = 20.5\text{ psia}}$$

d.)

Situation:

Pressure values need to be converted.

Find:

Calculate the pressure (kPa abs) corresponding to a blood pressure of 96 mm-Hg.

Properties:

Solution:

$$p_{\text{gage}} = \left(\frac{96 \text{ mm-Hg}}{1}\right)\left(\frac{101.3 \text{ kPa}}{760 \text{ mm-Hg}}\right) = 12.80 \text{ kPa-gage}$$

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}} = (101.3 \text{ kPa}) + (12.80 \text{ kPa-gage}) = 114.1 \text{ kPa abs}$$

$$\boxed{p_{\text{abs}} = 114.1 \text{ kPa abs}}$$

3.2: PROBLEM DEFINITION

Apply the grid method to:

a.)

Situation:
A sphere contains an ideal gas.

Find:
Calculate the density of helium at a gage pressure of 20 in. H_2O.

Properties:
From Table A.2: $R_{\text{helium}} = 2077\,\text{J/ kg}\cdot\text{K}$.

Solution:

$$p_{\text{abs}} = p_{\text{atm}} + p_{\text{gage}} = 101.3\,\text{kPa} + \left(\frac{20\text{ in. H2O}}{1}\right)\left(\frac{248.8\,\text{Pa}}{1.0\text{ in. H2O}}\right) = 106.3\,\text{kPa}$$

Ideal gas law:

$$\rho = \frac{p}{RT} = \left(\frac{106.3\,\text{kPa}}{1}\right)\left(\frac{\text{kg K}}{2077\,\text{J}}\right)\left(\frac{1}{293.2\,\text{K}}\right)\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)\left(\frac{\text{J}}{\text{N m}}\right)\left(\frac{\text{N}}{\text{Pa m}^2}\right)$$

$$\boxed{\rho = 0.175\ \text{kg/m}^3}$$

b.)

Situation:
A sphere contains an ideal gas.

Find:
Calculate the density of argon at a vacuum pressure of 3 psi.

Properties:
From Table A.2: $R_{\text{methane}} = 518\,\text{J/ kg}\cdot\text{K}$.

Solution:

$$p_{\text{abs}} = p_{\text{atm}} - p_{\text{vacuum}} = 101.3\,\text{kPa} - \left(\frac{3\text{ psi}}{1}\right)\left(\frac{101.3\,\text{kPa}}{14.696\text{ psi}}\right) = 80.62\,\text{kPa}$$

Ideal gas law:

$$\rho = \frac{p}{RT} = \left(\frac{80.62\,\text{kPa}}{1}\right)\left(\frac{\text{kg K}}{518\,\text{J}}\right)\left(\frac{1}{293.2\,\text{K}}\right)\left(\frac{1000\,\text{Pa}}{1\,\text{kPa}}\right)\left(\frac{\text{J}}{\text{N m}}\right)\left(\frac{\text{N}}{\text{Pa m}^2}\right)$$

$$\boxed{\rho = 0.531\ \text{kg/m}^3}$$

3.3: PROBLEM DEFINITION

Situation:
For the questions below, assume standard atmospheric pressure.
a. For a vacuum pressure of 30 kPa, what is the absolute pressure? Gage pressure?
b. For a pressure of 13.8 psig, what is the pressure in psia?
c. For a pressure of 200 kPa gage, what is the absolute pressure in kPa?
d. Give the pressure 100 psfg in psfa.

SOLUTION

a.)

Consulting Fig. 3.4 in EFM10e,

$$\begin{aligned} P_{abs} &= 101.3 - 30 = 71.3\,\text{kPa} \\ P_{gage} &= -30\,\text{kPa or } 30\,\text{kPa vacuum} \end{aligned}$$

b.)

Consulting Fig. 3.4 in EFM10e,

$$P_{abs} = 13.8\text{ psig} + 14.7\text{ psi } = 28.5\text{ psia}$$

c.)

Consulting Fig. 3.4 in EFM10e,

$$P_{abs} = 200\,\text{kPa gage } + 101.3\,\text{kPa} = 301.3\,\text{kPa abs}$$

d.)

Consulting Fig. 3.4 in EFM10e,

$$P_{abs} = \frac{100\,\text{lbf gage}}{\text{in}^2} + \left(\frac{14.7\,\text{lbf}}{\text{in}^2}\right)\left(\frac{144\,\text{in}^2}{\text{ft}^2}\right) = 2216.8\text{ psfa}$$

3.4: PROBLEM DEFINITION

Situation:
The local atmospheric pressure is 99.0 kPa. A gage on an oxygen tank reads a pressure of 300 kPa gage.

Find:
What is the pressure in the tank in kPa abs?

PLAN

Consult Fig. 3.4 in EFM10e

SOLUTION

$$
\begin{aligned}
P_{abs} &= P_{gage} + P_{atm} \\
P_{abs} &= 300\,\text{kPa} + 99\,\text{kPa} \\
P_{abs} &= 399\,\text{kPa abs}
\end{aligned}
$$

3.5: PROBLEM DEFINITION

Using Section 3.1 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words and equations in ways that enhance the effectiveness of your communication.

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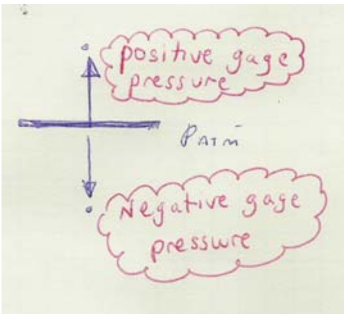
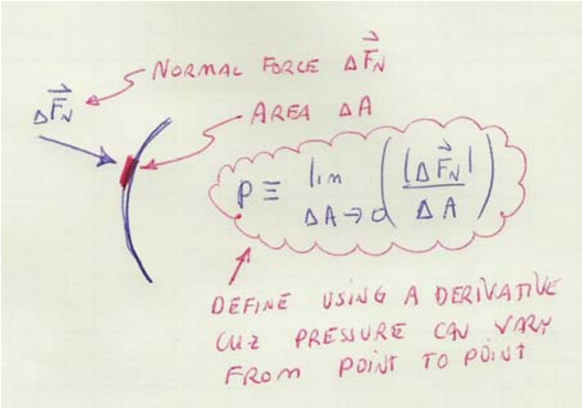
a. What are five important facts that engineers need to know about pressure?

- Pressure is often expressed using "gage pressure," where gage pressure is the difference between local atmospheric pressure and actual pressure.



- Primary dimensions of pressure are M/LT^2.
- Vacuum pressure = negative gage pressure. Negative vacuum pressure = gage pressure.
- Pressure is often expressed as length of a fluid column; e.g. the pressure of air in a duct is 10 inches of water column.
- pressure is defined using a derivative



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b. What are five common instances in which people use gage pressure?

- car tire pressure is expressed as gage pressure.
- blood pressure measured by a doctor is a gage pressure.
- the pressure inside a pressure cooker is expressed as a gage pressure.
- a Bourdon-tube pressure gage gives a pressure reading as a gage pressure.
- the pressure that a scuba diver feels is usually expressed as a gage pressure; e.g. a diver at a depth of 10 m will experience a pressure of 1 atm.

---

c. What are the most common units for pressure?

- Pa, psi, psf
- length of a column of water (in-H_20; ft-H_2O)
- length of a column of mercury (mm-Hg; in-Hg)
- bar

---

d. Why is pressure defined using a derivative?
Pressure is defined as a derivative because pressure can vary at every point along a surface.

---

e. How is pressure similar to shear stress? How does pressure differ from shear stress?

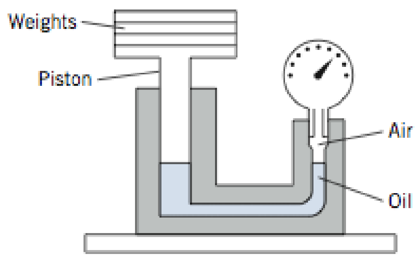
- Similarities
  - Both pressure and shear stress give a ratio of force to area.
  - Both pressure and shear stress apply at a point (they are defined using a derivative.
  - Pressure and shear stress have the same units.
  - Both pressure and shear stress are types of "stress."
- Differences: (the easy way to show differences is to make a table as shown below)

Attribute	Pressure	Shear Stress
direction of associated force	associated with force normal to area	associated with force tangent to an area
presence in a hydrostatic fluid	pressure is non-zero	shear stress is zero
typical magnitude	much larger than shear stress	much smaller than pressure
main physical cause	associated with weight of fluid & motion of fluid (non-viscous effects)	associated with motion of fluid (viscous effects)

3.6: PROBLEM DEFINITION

Situation:

A Crosby gage tester is applied to calibrate a pressure gage.
Indicated pressure on the gage is $p = 200\,\text{kPa}$ gage.
$W = 140\,\text{N}$, $D = 0.03\,\text{m}$.



Find:

Percent error in gage reading.

PLAN The oil exerts an upward force on the piston to support the weights. Thus, we can calculate the true pressure and then compare this with indicated reading to obtain the error in the gage reading. The steps are
1. Calculate the true pressure by applying force equilibrium to the piston and weights.
2. Calculate the error in the gage reading.

SOLUTION

1. Force equilibrium (apply to piston + weights)

$$\begin{aligned} F_{\text{pressure}} &= W \\ p_{\text{true}} A &= W \\ p_{\text{true}} &= \frac{W}{A} \\ &= \frac{140\,\text{N}}{(\pi/4 \times 0.03^2)\ \text{m}^2} \\ &= 198,049\,\text{Pa} \end{aligned}$$

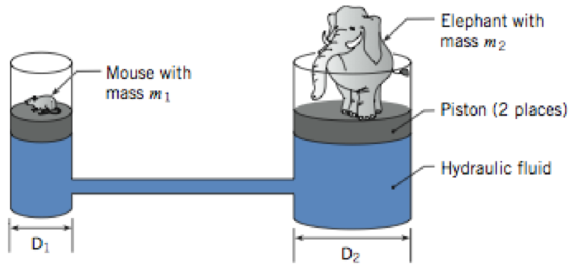
2. Percent error

$$\begin{aligned} \%\ \text{Error} &= \frac{(p_{\text{recorded}} - p_{\text{true}})\,100}{p_{\text{true}}} \\ &= \frac{(200\,\text{kPa} - 198\,\text{kPa})\,100}{198\,\text{kPa}} \\ &= 1.0101\% \end{aligned}$$

$$\boxed{\%\ \text{Error} = 1.01\%}$$

3.7: PROBLEM DEFINITION

Situation:
A hydraulic machine is used to provide a mechanical advantage.
$m_1 = 0.025\,\text{kg}$, $m_2 = 7500\,\text{kg}$.



Find:
(a) Derive an algebraic equation for the mechanical advantage.
(b) Calculate D_1 and D_2 so the mouse can support the elephant.

Assumptions:

- Neglect the mass of the pistons.
- Neglect the friction between the piston and the cylinder wall.
- The pistons are at the same elevation; thus, the pressure acting on the bottom of each piston is the same.
- A mouse can fit onto a piston of diameter $D_1 = 70$ mm.

PLAN

1. Define "mechanical advantage."
2. Derive an equation for the pressure acting on piston 1.
3. Derive an equation for the pressure acting on piston 2.
4. Derive an equation for mechanical advantage by combining steps 2 and 3.
5. Calculate D_2 by using the result of step 4.

SOLUTION

1. Mechanical advantage.

$$\left\{ \begin{array}{c} \text{Mechanical} \\ \text{advantage} \end{array} \right\} = \frac{\text{Weight "lifted" by the mouse}}{\text{Weight of the mouse}} = \frac{W_2}{W_1} \tag{1}$$

where W_2 is the weight of the elephant, and W_1 is the weight of the mouse.

2. Equilibrium (piston 1):

$$\begin{aligned} W_1 &= p\left(\frac{\pi D_1^2}{4}\right) \\ p &= W_1\left(\frac{4}{\pi D_1^2}\right) \qquad (2) \end{aligned}$$

3. Equilibrium (piston 2):

$$\begin{aligned} W_2 &= p\left(\frac{\pi D_2^2}{4}\right) \\ p &= W_2\left(\frac{4}{\pi D_2^2}\right) \qquad (3) \end{aligned}$$

4. Combine Eqs. (2) and (3):

$$p = W_1\left(\frac{4}{\pi D_1^2}\right) = W_2\left(\frac{4}{\pi D_2^2}\right) \qquad (5)$$

Solve Eq. (5) for mechanical advantage:

$$\boxed{\frac{W_2}{W_1} = \left(\frac{D_2}{D_1}\right)^2}$$

5. Calculate D_2.

$$\begin{aligned} \frac{W_2}{W_1} &= \left(\frac{D_2}{D_1}\right)^2 \\ \frac{(7500\,\mathrm{kg})\,(9.80\,\mathrm{m/\,s^2})}{(0.025\,\mathrm{kg})\,(9.80\,\mathrm{m/\,s^2})} &= 300000 = \left(\frac{D_2}{0.07\,\mathrm{m}}\right)^2 \\ D_2 &= 38.3\,\mathrm{m} \end{aligned}$$

The ratio of (D_2/D_1) needs to be $\sqrt{300,000}$. If $D_1 = 70\,\mathrm{mm}$, then $D_2 = 38.3\,\mathrm{m}$.

REVIEW

1. Notice. The mechanical advantage varies as the diameter ratio squared.
2. The mouse needs a mechanical advantage of 300,000:1. This results in a piston that is impractical (diameter = 38.3 m = 126 ft !).

3.8: PROBLEM DEFINITION

Situation:

To work the problem, data was recorded from a parked vehicle. Relevant information:

- Left front tire of a parked VW Passat 2003 GLX Wagon (with 4-motion).
- Bridgestone snow tires on the vehicle.
- Inflation pressure = 36 psig. This value was found by using a conventional "stick-type" tire pressure gage.
- Contact Patch: $5.88\,\text{in} \times 7.5\,\text{in}$. The 7.5 inch dimension is across the tread. These data were found by measuring with a ruler.
- Weight on the front axle = 2514 lbf. This data was recorded from a sticker on the driver side door jamb. The owners manual states that this is maximum weight (car + occupants + cargo).

Assumptions:

- The weight on the car axle without a load is 2000 lbf. Thus, the load acting on the left front tire is 1000 lbf.
- The thickness of the tire tread is 1 inch. The thickness of the tire sidewall is 1/2 inch.
- The contact path is flat and rectangular.
- Neglect any tensile force carried by the material of the tire.

Find:

Measure the size of the contact patch.
Calculate the size of the contact patch.
Compare the measurement with the calculation and discuss.

PLAN

To estimate the area of contact, apply equilibrium to the contact patch.

SOLUTION

Equilibrium in the vertical direction applied to a section of the car tire

$$p_i A_i = F_{\text{pavement}}$$

where p_i is the inflation pressure, A_i is the area of the contact patch on the inside of the tire and F_{pavement} is the normal force due to the pavement. Thus,

$$\begin{aligned} A_i &= \frac{F_{\text{pavement}}}{p_i} \\ &= \frac{1000\,\text{lbf}}{36\,\text{lbf/in}^2} \\ &= 27.8\,\text{in}^2 \end{aligned}$$

Comparison. The actual contact patch has an area $A_o = 5.88\,\text{in} \times 7.5\,\text{in} = 44.1\,\text{in}^2$. Using the assumed thickness of rubber, this would correspond to an inside contact area of $A_o = 4.88\,\text{in} \times 5.5\,\text{in} = 26.8\,\text{in}^2$.Thus, the predicted contact area $\left(27.8\,\text{in}^2\right)$ and the measured contact area $\left(26.8\,\text{in}^2\right)$ agree to within about 1 part in 25 or about 4%.

REVIEW

The comparison between predicted and measured contact area is highly dependent on the assumptions made.

3.9: PROBLEM DEFINITION

Situation:
To derive the hydrostatic equation, which of the following must be assumed? (Select all that are correct.)
a. the specific weight is constant
b. the fluid has no charged particles
c. the fluid is at equilibrium

SOLUTION

The answers are (a) and (c); see §3.2

3.10: PROBLEM DEFINITION

Situation:
Two tanks.
Tank A is filled to depth h with water.
Tank B is filled to depth h with oil.

Find:
Which tank has the largest pressure?
Why?
Where in the tank does the largest pressure occur?

SOLUTION

In both tanks, pressure increases with depth, according to $p = -\gamma z$.
At the bottom of each tank, pressure is given by $p = \gamma h$.
At the bottom of Tank A, $p = \gamma_{water} h$.
At the bottom of Tank B, $p = \gamma_{oil} h$.
Because $\gamma_{oil} < \gamma_{water}$, the pressure in Tank A has the largest pressure.
The reason is because water has a larger specific weight than oil.
The largest pressure occurs at the bottom of the tank.

3.11: PROBLEM DEFINITION

Situation:

Consider Figure 3.8 on p. 67 of §3.2 in EFM10e.

a. Which fluid has the larger density?

b. If you graphed pressure as a function of z in these two layered liquids, in which fluid does the pressure change more with each incremental change in z?

SOLUTION

a. Water has the larger density, and thus the larger specific weight.

b. To pressure as a function of fluid, you would use $p = -\gamma z$. The pressure changes more with each incremental change in z in the water than in the oil because $\gamma_{oil} < \gamma_{water}$.

Problem 3.12

Apply the grid method to calculations involving the hydrostatic equation:

$$\Delta p = \gamma \Delta z = \rho g \Delta z$$

Note: Unit cancellations are not shown in this solution.

a.)

Situation:
Pressure varies with elevation.
$\Delta z = 10\,\text{ft}$.

Find:
Pressure change (kPa).

Properties:
$\rho = 90\,\text{lb/ ft}^3$.

Solution:
Convert density to units of kg/m^3:

$$\rho = \left(\frac{90 \text{ lbm}}{\text{ft}^3}\right)\left(\frac{35.315\,\text{ft}^3}{\text{m}^3}\right)\left(\frac{1.0\,\text{kg}}{2.2046 \text{ lbm}}\right) = 1442\ \frac{\text{kg}}{\text{m}^3}$$

Calculate the pressure change:

$$\Delta p = \rho g \Delta z = \left(\frac{1442\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{10\,\text{ft}}{1.0}\right)\left(\frac{\text{m}}{3.208\,\text{ft}}\right)\left(\frac{\text{Pa}\cdot\text{m}\cdot\text{s}^2}{\text{kg}}\right)$$

$$\boxed{\Delta p = 44.1\,\text{kPa}}$$

b.)

Situation:
Pressure varies with elevation.
$\Delta z = 22\,\text{m}$, $S = 0.8$.

Find:
Pressure change (psf).

Properties:
$\gamma = 62.4\,\text{lbf/ ft}^3$.

Solution:

$$\Delta p = \gamma \Delta z = S\ \gamma_{H2O}\ \Delta z = \left(\frac{(0.8 \cdot 62.4) \text{ lbf}}{\text{ft}^3}\right)\left(\frac{22\,\text{m}}{1.0}\right)\left(\frac{3.2808\,\text{ft}}{\text{m}}\right)$$

$$\boxed{\Delta p = 3600 \text{ psf}}$$

c.)

Situation:
Pressure varies with elevation.
$\Delta z = 1000\,\text{ft}$.

Find:
Pressure change (in H_2O).

Properties:
air, $\rho = 1.2\,\text{kg/m}^3$.

Solution:

$$\Delta p = \rho g \Delta z = \left(\frac{1.2\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{1000\,\text{ft}}{1.0}\right)\left(\frac{\text{m}}{3.281\,\text{ft}}\right)\left(\frac{\text{Pa}\cdot\text{m}\cdot\text{s}^2}{\text{kg}}\right)\left(\frac{\text{in.-H2O}}{249.1\,\text{Pa}}\right)$$

$$\boxed{\Delta p = 14.4 \text{ in } H_2O}$$

d.)

Situation:
Pressure varies with elevation.
$\Delta p = 1/6\,\text{atm}$, $S = 13$.

Find:
Elevation change (mm).

Properties:
$\gamma = 9810\,\text{N/m}^3$, $p_{atm} = 101.3\,\text{kPa}$.

Solution:
d. Calculate Δz (mm) corresponding to $S = 13$ and $\Delta p = 1/6\,\text{atm}$.

$$\Delta z = \frac{\Delta p}{\gamma} = \frac{\Delta p}{S\gamma_{H2O}} = \left(\frac{1/6\,\text{atm}}{1.0}\right)\left(\frac{\text{m}^3}{(13\cdot 9810)\ \text{N}}\right)\left(\frac{101.3\times 10^3\,\text{Pa}}{\text{atm}}\right)\left(\frac{1000\,\text{mm}}{1.0\,\text{m}}\right)$$

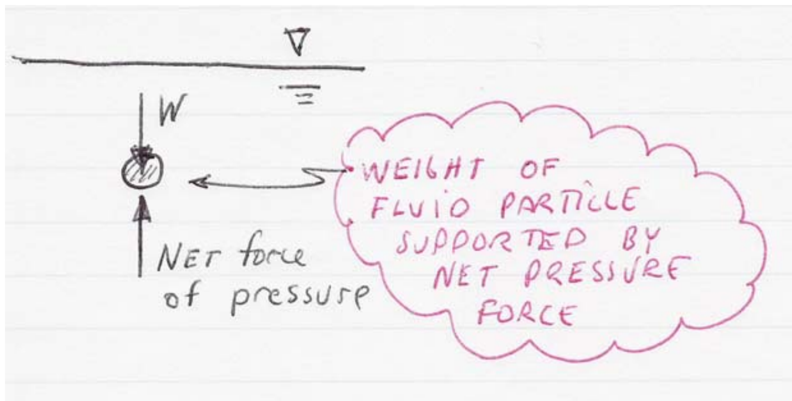
$$\boxed{\Delta z = 132\,\text{mm}}$$

Problem 3.13

Using Section 3.2 and other resources, answer the questions below. Strive for depth, clarity, and accuracy while also combining sketches, words and equations in ways that enhance the effectiveness of your communication.

a. What does hydrostatic mean? How do engineers identify if a fluid is hydrostatic?

- Each fluid particle within the body is in force equilibrium(z-direction) with the net force due to pressure balancing the weight of the particle. Here, the z-direction is aligned with the gravity vector.



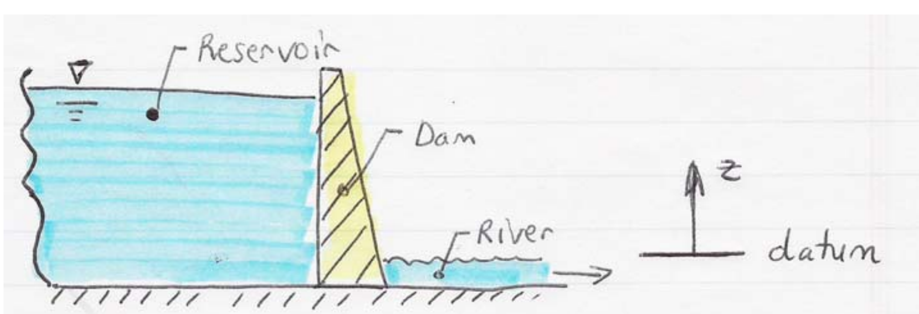
- Engineers establish hydrostatic conditions by analyzing the forces acting in the z-direction.

b. What are common forms of the hydrostatic equation? Are the forms equivalent or are they different?

- There are three common forms; these are given in Table F.2 (front of book).
- These equations are equivalent because you can start with any of the equations and derive the other two.

c. What is a datum? How do engineers establish a datum?

- A datum is a fixed reference point from which elevations are measured.



- Engineers select a datum that makes calculations easy. For example, select a datum on the free surface of a river below a dam so that all elevations are positive.

---

d. What are the main ideas of Eq. (3.10) EFM10e? That is, what is the meaning of this equation?

$$p_z = p + \gamma z = \text{constant}$$

This equation means that the sum of $(p + \gamma z)$ has the same numerical value at every location within a body of fluid.

---

e. What assumptions need to be satisfied to apply the hydrostatic equation?

$$p_z = p + \gamma z = \text{constant}$$

This equation is valid when

- the density of the fluid is constant at all locations.
- equilibrium is satisfied in the z-direction (net force of pressure balances weight of the fluid particle.

Problem 3.14

Apply the grid method to each situation below. Unit cancellations are not shown in these solutions.

a.)

Situation:

Pressure varies with elevation.

$\Delta z = 10\,\text{ft}$.

Find:

Pressure change (Pa).

Properties:

air, $\rho = 1.2\,\text{kg/m}^3$.

Solution:

$$\Delta p = \rho g \Delta z$$

$$\begin{aligned}\Delta p &= \rho g \Delta z \\ &= \left(\frac{1.2\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{10\,\text{ft}}{1.0}\right)\left(\frac{\text{m}}{3.281\,\text{ft}}\right)\left(\frac{\text{Pa}\cdot\text{m}\cdot\text{s}^2}{\text{kg}}\right)\end{aligned}$$

$$\boxed{\Delta p = 35.9\,\text{Pa}}$$

b.)

Situation:

Pressure increases with depth in the ocean.

Pressure reading is 2.5 atm gage.

Find:

Water depth (m).

Properties:

Seawater, Table A.4, $S = 1.03$, $\gamma = 10070\,\text{N/m}^3$.

Solution:

$$\Delta z = \frac{\Delta p}{\gamma} = \left(\frac{2.5\,\text{atm}}{1.0}\right)\left(\frac{\text{m}^3}{10070\,\text{N}}\right)\left(\frac{101.3\times10^3\,\text{Pa}}{\text{atm}}\right)\left(\frac{\text{N}}{\text{Pa}\,\text{m}^2}\right)$$

$$\boxed{\Delta z = 25.15\,\text{m}}$$

c.)

Situation:

Pressure decreases with elevation in the atmosphere.

$\Delta z = 1200\,\text{ft}$.

Find:

Pressure (mbar).

Assumptions:
Density of air is constant.

Properties:
Air, $\rho = 1.1\,\text{kg/m}^3$.

Solution:

$$\Delta p = \rho g \Delta z = \left(\frac{1.1\,\text{kg}}{\text{m}^3}\right)\left(\frac{9.81\,\text{m}}{\text{s}^2}\right)\left(\frac{-1200\,\text{ft}}{1.0}\right)\left(\frac{\text{m}}{3.281\,\text{ft}}\right)\left(\frac{\text{Pa}\cdot\text{m}\cdot\text{s}^2}{\text{kg}}\right) = -3947\,\text{Pa}$$

Pressure at summit:

$$p_{\text{summit}} = p_{\text{base}} + \Delta p = 940\,\text{mbar} - \left(\frac{3947\,\text{Pa}}{1.0}\right)\left(\frac{10^{-2}\,\text{mbar}}{\text{Pa}}\right)$$

$$\boxed{p_{\text{summit}} = 901\,\text{mbar (absolute)}}$$

d.)

Situation:
Pressure increases with depth in a lake.
$\Delta z = 350\,\text{m}$.

Find:
Pressure (MPa).

Properties:
Water, $\gamma = 9810\,\text{N/m}^3$.

Solution:

$$\begin{aligned}\Delta p &= \gamma \Delta z \\ &= \left(\frac{9810\,\text{N}}{\text{m}^3}\right)\left(\frac{350\,\text{m}}{1.0}\right)\left(\frac{\text{Pa}\cdot\text{m}^2}{\text{N}}\right)\left(\frac{\text{MPa}}{10^6\,\text{Pa}}\right)\end{aligned}$$

$$\boxed{p_{\max} = 3.4\,\text{MPa (gage) [about 34 atmospheres]}}$$

e.)

Situation:
Pressure increase with water depth in a standpipe.
$\Delta z = 70\,\text{m}$.

Find:
Pressure (kPa).

Properties:
Water, $\gamma = 9810\,\text{N/m}^3$.

Solution:

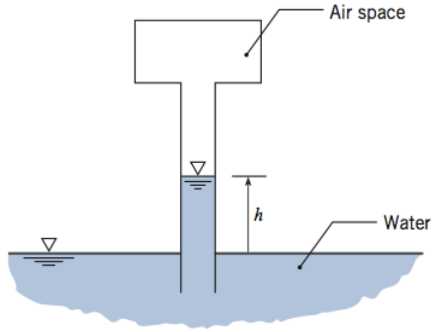
$$\begin{aligned}\Delta p &= \gamma \Delta z \\ &= \left(\frac{9810\,\mathrm{N}}{\mathrm{m}^3}\right)\left(\frac{70\,\mathrm{m}}{1.0}\right)\left(\frac{\mathrm{Pa}\cdot\mathrm{m}^2}{\mathrm{N}}\right)\left(\frac{\mathrm{kPa}}{10^3\,\mathrm{Pa}}\right)\end{aligned}$$

$$\boxed{p_{\max} = 687\,\mathrm{kPa}\ \text{(gage) [nearly 7 atmospheres]}}$$

3.15: PROBLEM DEFINITION

Situation:

Air above a long tube is pressurized.
Initial state: $p_{\text{air1}} = 50\,\text{kPa}$-vacuum
Final state: $p_{\text{air2}} = 25\,\text{kPa}$-vacuum.



Find:

Will h increase or decrease?
The change in water column height (Δh) in meters.

Assumptions:

Atmospheric pressure is 100 kPa.
Surface tension can be neglected.

Properties:

Water (20 °C), Table A.5, $\gamma = 9790\,\text{N/m}^3$.

PLAN

Since pressure increases, the water column height will decrease. Use absolute pressure in the hydrostatic equation.

1. Find h (initial state) by applying the hydrostatic equation.
2. Find h (final state) by applying the hydrostatic equation.
3. Find the change in height by $\Delta h = h(\text{final state}) - h\,(\text{initial state})$.

SOLUTION

1. Initial State. Locate point 1 on the reservoir surface; point 2 on the water surface inside the tube:

$$\begin{aligned}
\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\
\frac{100\,\text{kPa}}{9790\,\text{N/m}^3} + 0 &= \frac{50\,\text{kPa}}{9790\,\text{N/m}^3} + h \\
h\,(\text{initial state}) &= 5.107\,\text{m}
\end{aligned}$$

2. Final State:

$$\begin{aligned}
\frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\
\frac{100\,\text{kPa}}{9790\,\text{N/m}^3} + 0 &= \frac{75\,\text{kPa}}{9790\,\text{N/m}^3} + h \\
h\,(\text{final state}) &= 2.554\,\text{m}
\end{aligned}$$

3. Change in height:

$$\begin{aligned}
\Delta h &= h(\text{final state}) - h\,(\text{initial state}) \\
&= 2.554\,\text{m} - 5.107\,\text{m} = -2.55\,\text{m}
\end{aligned}$$

The height has decreased by 2.55 m.

REVIEW

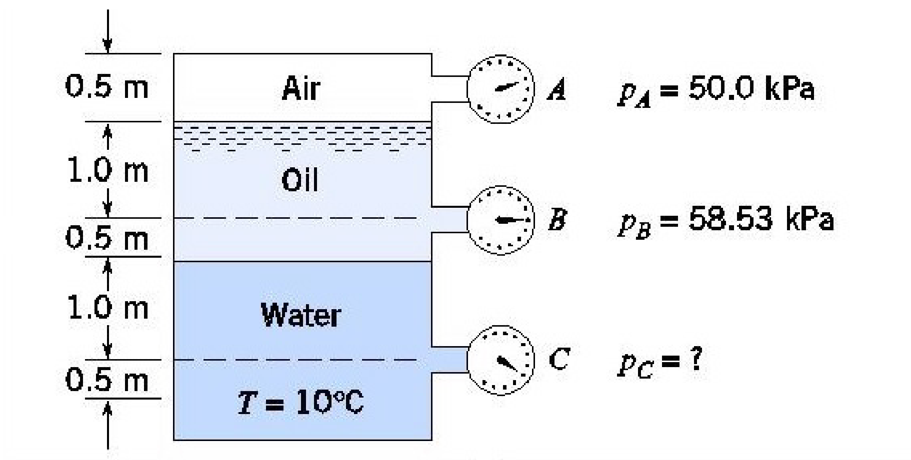
Tip! In the hydrostatic equation, use gage pressure or absolute pressure. Using vacuum pressure will give a wrong answer.

3.16: PROBLEM DEFINITION

Situation:
A closed tank contains air, oil, and water.

Find:
Specific gravity of oil.
Pressure at C (kPa-gage).

Sketch:



Properties:
Water (10 °C), Table A.5, $\gamma = 9810\,\text{N}/\,\text{m}^3$.

PLAN

1. Find the oil specific gravity by applying the hydrostatic equation from A to B.
2. Apply the hydrostatic equation to the water.
3. Apply the hydrostatic equation to the oil.
4. Find the pressure at C by combining results for steps 2 and 3.

SOLUTION

1. Hydrostatic equation (from oil surface to elevation B):

$$\begin{aligned} p_A + \gamma z_A &= p_B + \gamma z_B \\ 50,000\text{ N/m}^2 + \gamma_{\text{oil}}\,(1\text{ m}) &= 58,530\text{ N/m}^2 + \gamma_{\text{oil}}\,(0\text{ m}) \\ \gamma_{\text{oil}} &= 8530\text{ N/m}^3 \end{aligned}$$

Specific gravity:

$$S = \frac{\gamma_{\text{oil}}}{\gamma_{\text{water}}} = \frac{8530\text{ N/m}^3}{9810\text{ N/m}^3}$$

$$\boxed{S_{\text{oil}} = 0.87}$$

2. Hydrostatic equation (in water):

$$p_c = (p_{\text{btm of oil}}) + \gamma_{\text{water}} (1\,\text{m})$$

3. Hydrostatic equation (in oil):

$$p_{\text{btm of oil}} = (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m})$$

4. Combine equations:

$$\begin{aligned} p_c &= (58,530\,\text{Pa} + \gamma_{\text{oil}} \times 0.5\,\text{m}) + \gamma_{\text{water}} (1\,\text{m}) \\ &= \left(58,530\,\text{Pa} + 8530\,\text{N/m}^3 \times 0.5\,\text{m}\right) + 9810\,\text{N/m}^3 (1\,\text{m}) \\ &= 72,605\ \text{N/m}^2 \end{aligned}$$

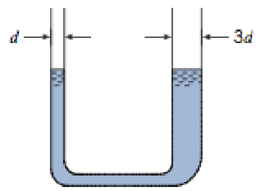
$$\boxed{p_c = 72.6\ \text{kPa-gage}}$$

3.17: PROBLEM DEFINITION

Situation:

A manometer is described in the problem statement.

$d_{\text{left}} = 1\,\text{mm}$, $d_{\text{right}} = 3\,\text{mm}$.



Find:

Water surface level in the left tube as compared to the right tube.

SOLUTION

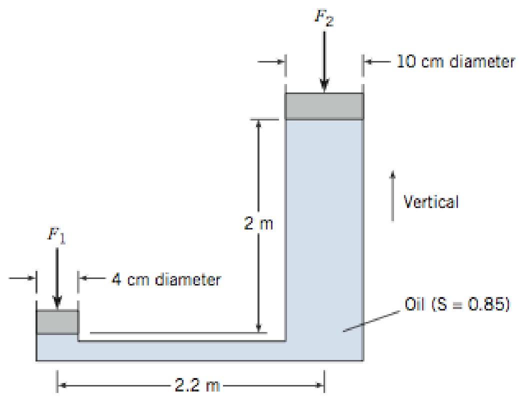
(a) The water surface level in the left tube will be higher because of greater surface tension effects for that tube.

3.18: PROBLEM DEFINITION

Situation:

A force is applied to a piston.

$F_1 = 200\,\text{N}$, $d_1 = 4\,\text{cm}$, $d_2 = 10\,\text{cm}$.



Find:

Force resisted by piston.

Assumptions:

Neglect piston weight.

PLAN

Apply the hydrostatic equation and equilibrium.

SOLUTION

1. Equilibrium (piston 1)

$$\begin{aligned} F_1 &= p_1 A_1 \\ p_1 &= \frac{F_1}{A_1} \\ &= \frac{4 \times 200\,\text{N}}{\pi \cdot (0.04\,\text{m})^2\ \text{m}^2} \\ &= 1.592 \times 10^5\,\text{Pa} \end{aligned}$$

2. Hydrostatic equation

$$\begin{aligned} p_2 + \gamma z_2 &= p_1 + \gamma z_1 \\ p_2 &= p_1 + (S\gamma_{\text{water}})(z_1 - z_2) \\ &= 1.592 \times 10^5\,\text{Pa} + \left(0.85 \times 9810\,\text{N/m}^3\right)(-2\,\text{m}) \\ &= 1.425 \times 10^5\,\text{Pa} \end{aligned}$$

$$
\begin{aligned}
&= \left(1.425 \times 10^5 \, \mathrm{N/m^2}\right) \left(\frac{\pi \, (0.1 \, \mathrm{m})^2}{4}\right) \\
&= 1119 \, \mathrm{N}
\end{aligned}
$$

$$
\boxed{F_2 = 1120 \ \mathrm{N}}
$$

3.19: PROBLEM DEFINITION

Situation:

Regarding the hydraulic jack in Problem 3.18 (EFM 10e), which ideas were used to analyze the jack? (select all that apply)

a. pressure = (force)(area)

b. pressure increases linearly with depth in a hydrostatic fluid

c. the pressure at the very bottom of the 4-cm chamber is larger than the pressure at the very bottom of the 10-cm chamber

d. when a body is stationary, the sum of forces on the object is zero

e. when a body is stationary, the sum of moments on the object is zero

f. pressure = (weight/volume)(change in elevation)

SOLUTION

Correct answers are a, b, d, e and f.
Statement c is incorrect because the two chambers are connected, therefore the pressure at the flat bottom is everywhere the same. Pressure is a scalar, and is transferred continuously in all directions. It increases with depth; however at the same depth (of a fluid with constant density that is not being accelerated to the left or right) it is the same.

3.20: PROBLEM DEFINITION

Situation:

A diver goes underwater.

$\Delta z = 50\,\text{m}$.

Find:

Gage pressure (kPa).

Ratio of pressure to normal atmospheric pressure.

Properties:

Water (20 °C), Table A.5, $\gamma = 9790\,\text{N/m}^3$.

PLAN

1. Apply the hydrostatic equation.
2. Calculate the pressure ratio (use absolute pressure values).

SOLUTION

1. Hydrostatic equation

$$\begin{aligned} p &= \gamma \Delta z = 9790\,\text{N/m}^3 \times 50\,\text{m} \\ &= 489,500\ \text{N/m}^2 \end{aligned}$$

$$\boxed{p = 490\ \text{kPa gage}}$$

2. Calculate pressure ratio

$$\frac{p_{50}}{p_{\text{atm}}} = \frac{489.5\,\text{kPa} + 101.3\,\text{kPa}}{101.3\,\text{kPa}}$$

$$\boxed{\frac{p_{50}}{p_{\text{atm}}} = 5.83}$$

3.21: PROBLEM DEFINITION

Situation:

Water and kerosene are in a tank.
$z_{\text{water}} = 0.8\,\text{m}$, $z_{\text{kerosene}} = 0.3\,\text{m}$.

Find:

Gage pressure at bottom of tank (kPa-gage).

Properties:

Water (20 °C), Table A.5, $\gamma_w = 9790\ \text{N/m}^3$.
Kerosene (20 °C), Table A.4, $\gamma_k = 8010\ \text{N/m}^3$.

SOLUTION

Manometer equation (add up pressure from the top of the tank to the bottom of the tank).

$$p_{\text{atm}} + \gamma_{\text{k}}\ (0.3\,\text{m}) + \gamma_{\text{w}}\ (0.8\,\text{m}) = p_{\text{btm}}$$

Solve for pressure

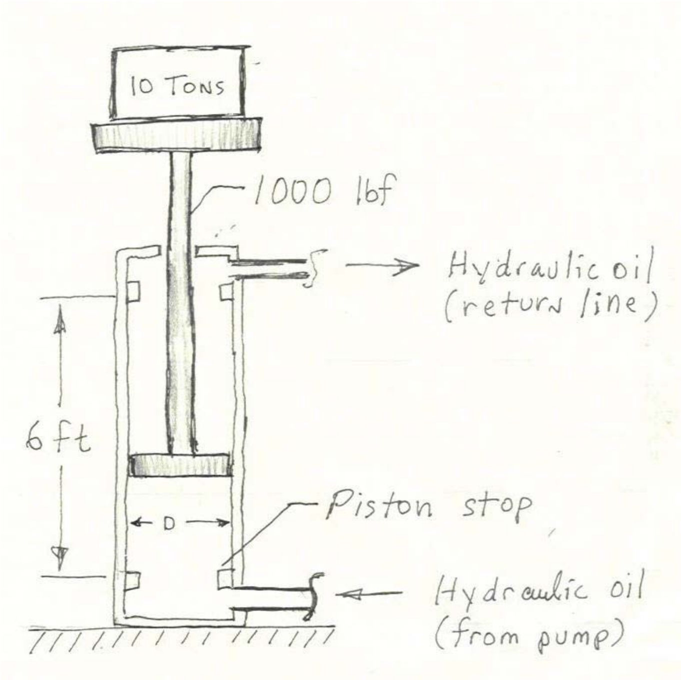
$$\begin{aligned} p_{\text{btm}} &= 0 + \gamma_{\text{k}}\ (0.3\,\text{m}) + \gamma_{\text{w}}\ (0.8\,\text{m}) \\ &= \left(8010\,\text{N/ m}^3\right)(0.3\,\text{m}) + \left(9790\,\text{N/ m}^3\right)(0.8\,\text{m}) \\ &= 10.3\,\text{kPa} \end{aligned}$$

$$\boxed{p_{\text{btm}} = 10.3\,\text{kPa gage}}$$

3.22: PROBLEM DEFINITION

Situation:

A hydraulic lift is being designed.
$W_{\max} = 10$ ton $= 20000\,\text{lbf}$, $W_{parts} = 1000\,\text{lbf}$.
$\Delta L = 6\,\text{ft}$, $\Delta t = 20\,\text{s}$.
Diameter range: $2 - 8\,\text{in}$.
Pressure range: $200 - 3000$ psig.
Available pumping capacity: 5, 10, 15 gpm.



Find:

Select a hydraulic pump capacity (gpm).
Select a cylinder diameter (D).

PLAN

Apply equilibrium to find the smallest bore diameter (D) that works. Then find the largest bore diameter that works by considering the lift speed requirement. Select bore and pump combinations that meet the desired specifications.

SOLUTION

Equilibrium (piston)

$$F = pA$$

where $F = 21,000\,\text{lbf}$ is the load that needs to be lifted and p is the pressure on the bottom of the piston. Maximum pressure is 3000 psig so minimum bore area is

$$\begin{aligned} A_{\min} &= \frac{F}{p_{\max}} \\ &= \frac{21,000\,\text{lbf}}{3000\,\text{in}^2} \\ &= 7.0\ \text{in}^2 \end{aligned}$$

Corresponding minimum bore diameter is

$$\begin{aligned} D &= \sqrt{\frac{4}{\pi}A} \\ D_{\min} &= 2.98\ \text{in} \end{aligned}$$

The pump needs to provide enough flow to raise the lift in 20 seconds.

$$A\Delta L = \dot{V}\Delta t$$

where A is the bore area, ΔL is stroke (lift height), $\dot{V}$ is the volume/time of fluid provided by the pump, and Δt is the time. Thus, the maximum bore area is

$$A_{\max} = \frac{\dot{V}\Delta t}{\Delta L}$$

Conversion from gallons to cubic feet $\left(\text{ft}^3\right)$: 7.48 gal=1 ft^3. Thus, the maximum bore diameter for three pumps (to meet the lift speed specification) is given in the table below.

pump (gpm)	pump (cfm)	A (ft^2)	D_{max} (in)
5	0.668	0.037	2.61
10	1.337	0.074	3.68
15	2.01	0.116	4.61

Since the minimum bore diameter is 2.98 in., the 5 gpm pump will not work. The 10 gpm pump can be used with a 3 in. bore. The 15 gpm pump can be used with a 3 or 4 in. bore.

1.) The 10 gpm pump will work with a bore diameter between 3.0 and 3.6 inches.

2.) The 15 gpm pump will work with a bore diameter between 3.0 and 4.6 inches.

REVIEW

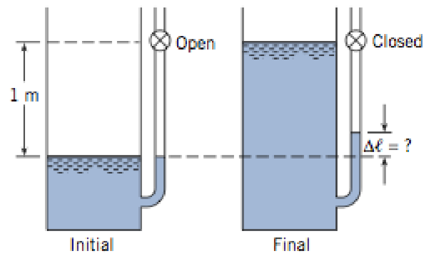
1. These are preliminary design values. Other issues such as pressure drop in the hydraulic lines and valves would have to be considered.
2. We recommend selecting the 15 gpm pump and a 4.5 inch bore to provide latitude to handle pressure losses, and to reduce the maximum system pressure.

3.23: PROBLEM DEFINITION

Situation:

Initial State: Water levels as shown. Valve in open.

Final State: Water is added to the tank with the valve closed.



Find:

Increase of water level $\Delta\ell$ in manometer (in meters).

Properties:

Water (20 °C), Table A.5, γ_w=9790 N/m^3.

$p_{atm} = 100\,\text{kPa}$.

Assumptions: Ideal gas.

PLAN

Apply the hydrostatic equation and the ideal gas law.

SOLUTION

Ideal gas law (mole form; apply to air in the manometer tube)

$$pV\!\!\!\!-= n\Re T$$

Because the number of moles (n) and temperature (T) are constants, the ideal gas reduces to Boyle's equation.

$$p_1 V\!\!\!\!-_1 = p_2 V\!\!\!\!-_2 \quad (1)$$

State 1 (before air is compressed)

$$\begin{aligned} p_1 &= 100,000 \text{ N/m}^2 \text{ abs} \\ V\!\!\!\!-_1 &= 1 \text{ m} \times A_{\text{tube}} \end{aligned} \quad \text{(a)}$$

State 2 (after air is compressed)

$$\begin{aligned} p_2 &= 100,000 \text{ N/m}^2 + \gamma_w(1 \text{ m} - \Delta\ell) \\ V\!\!\!\!-_2 &= (1 \text{ m} - \Delta\ell)A_{\text{tube}} \end{aligned} \quad \text{(b)}$$

Substitute (a) and (b) into Eq. (1)

$$\begin{aligned}
p_1 V_1 &= p_2 V_2 \\
\left(100,000\,\mathrm{N/\,m^2}\right)(1\ \mathrm{m} \times A_{\mathrm{tube}}) &= \left(100,000\ \mathrm{N/m^2} + \gamma_{\mathrm{w}}(1\ \mathrm{m} - \Delta\ell)\right)(1\ \mathrm{m} - \Delta\ell)A_{\mathrm{tube}} \\
100,000\,\mathrm{N/\,m^2} &= \left(100,000\,\mathrm{N/\,m^2} + 9790\,\mathrm{N/\,m^3}\,(1 - \Delta\ell)\right)(1 - \Delta\ell)
\end{aligned}$$

Solving for $\Delta\ell$

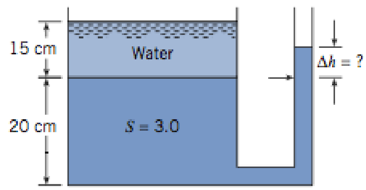
$$\boxed{\Delta\ell = 0.0824\ \mathrm{m}}$$

3.24: PROBLEM DEFINITION

Situation:

A tank is fitted with a manometer.

$S = 3$, $z_1 = 0.15\,\text{m}$.



Find:

Deflection of the manometer (cm).

Properties:

γ_{water}=9810 N/m^3.

PLAN

Apply the hydrostatic principle to the water and then to the manometer fluid.

SOLUTION

1. Hydrostatic equation (location 1 is on the free surface of the water; location 2 is the interface)

$$\begin{aligned}
\frac{p_1}{\gamma_{\text{water}}} + z_1 &= \frac{p_2}{\gamma_{\text{water}}} + z_2 \\
\frac{0\,\text{Pa}}{9810\,\text{N/m}^3} + 0.15\,\text{m} &= \frac{p_2}{9810\,\text{N/m}^3} + 0\,\text{m} \\
p_2 &= (0.15\,\text{m})\left(9810\,\text{N/m}^3\right) \\
&= 1471.5\,\text{Pa}
\end{aligned}$$

2. Hydrostatic equation (manometer fluid; let location 3 be on the free surface)

$$\begin{aligned}
\frac{p_2}{\gamma_{\text{man. fluid}}} + z_2 &= \frac{p_3}{\gamma_{\text{man. fluid}}} + z_3 \\
\frac{1471.5\,\text{Pa}}{3\,(9810\,\text{N/m}^3)} + 0\,\text{m} &= \frac{0\,\text{Pa}}{\gamma_{\text{man. fluid}}} + \Delta h
\end{aligned}$$

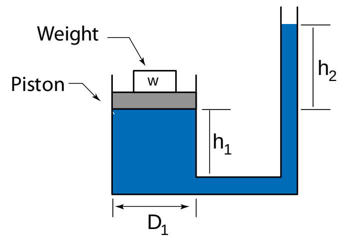
3. Solve for Δh

$$\begin{aligned}
\Delta h &= \frac{1471.5\,\text{Pa}}{3\,(9810\,\text{N/m}^3)} \\
&= 0.0500\,\text{m}
\end{aligned}$$

$$\boxed{\Delta h = 5.00\text{ cm}}$$

3.25: PROBLEM DEFINITION

Situation:
A mass sits on top of a piston situated above a reservoir of oil.



Find:
Derive an equation for h_2 in terms of the specified parameters.

Assumptions:
Neglect the mass of the piston.
Neglect friction between the piston and the cylinder wall.
The pressure at the top of the oil column is 0 kPa-gage.

PLAN

1. Relate w to pressure acting on the bottom of the piston using equilibrium.
2. Related pressure on the bottom of the piston to the oil column height using the hydrostatic equation.
3. Find h_2 by combining steps 1 and 2.

SOLUTION

1. Equilibrium (piston):

$$w = p_1 \left(\frac{\pi D_1^2}{4} \right) \quad (1)$$

2. Hydrostatic equation. (point 1 at btm of piston; point 2 at top of oil column):

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{p_1}{S\gamma_{\text{water}}} + 0 &= 0 + h_2 \\ p_1 &= S\ \gamma_{\text{water}}\ h_2 \end{aligned} \quad (2)$$

3. Combine Eqs. (1) and (2):

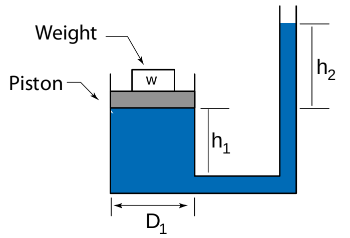
$$mg = S\ \gamma_{\text{water}}\ h_2 \left(\frac{\pi D_1^2}{4} \right)$$

Answer: $$h_2 = \frac{4w}{(S)\left(\gamma_{\text{water}}\right)\left(\pi D_1^2\right)}$$

REVIEW

1. Notice. Column height h_2 increases linearly with increasing weight w.Similarly, h_2 decreases linearly with S and decreases quadratically with D_1.
2. Notice. The apparatus involved in the problem could be used to create an instrument for weighing an object.

$D_1 = 120\,\text{mm},\ D_2 = 5\,\text{mm}.$



Find:

Calculate h_2 (m).

Assumptions:

Neglect the mass of the piston.
Neglect friction between the piston and the cylinder wall.
The pressure at the top of the oil column is 0 kPa-gage.

PLAN

1. Relate mass m to pressure acting on the bottom of the piston using equilibrium.
2. Related pressure on the bottom of the piston to the oil column height using the hydrostatic equation.
3. Find h_2 by combining steps 1 and 2.

SOLUTION

1. Equilibrium (piston):

$$mg = p_1 \left(\frac{\pi D_1^2}{4} \right) \tag{1}$$

2. Hydrostatic equation. (point 1 at btm of piston; point 2 at top of oil column):

$$\begin{aligned} \frac{p_1}{\gamma} + z_1 &= \frac{p_2}{\gamma} + z_2 \\ \frac{p_1}{S\gamma_{\text{water}}} + 0 &= 0 + h_2 \\ p_1 &= S\ \gamma_{\text{water}}\ h_2 \end{aligned} \tag{2}$$

3. Combine Eqs. (1) and (2):

$$mg = S\ \gamma_{\text{water}}\ h_2 \left(\frac{\pi D_1^2}{4} \right)$$