

CHAPTER 1

Kinematics of Particles

Solutions to Tutorial Questions

Q1.1 $a=0.3 \text{ m/s}^2$, $u=0$, $v=22 \text{ m/s}$

The distance s in the acceleration phase is:

$$v^2 = u^2 + 2as \Rightarrow (22)^2 = 0 + 2 \times 0.3 \times s \Rightarrow s = 806.67 \text{ m} = 0.807 \text{ km}$$

The time in the acceleration phase is:

$$v = u + at \Rightarrow 22 = 0 + 0.3 \times t \Rightarrow t = 73.33 \text{ s} = 1 \text{ min } 13.33 \text{ s}$$

b) $t=1 \text{ min}$, $v=0$, $u=22.347 \text{ m/s}$

The acceleration a is:

$$v = u + at \Rightarrow 0 = 22 + a \times 60 \Rightarrow a = -0.367 \text{ m/s}^2$$

To calculate the distance s in the deceleration phase:

$$v^2 = u^2 + 2as \Rightarrow 0 = (22)^2 - 2 \times 0.367 \times s \Rightarrow s = 660 = 0.66 \text{ km}$$

c) Distance during the constant velocity = $11 - 0.807 - 0.66 = 9.533 \text{ km}$

Time during the constant velocity is:

$$t = \frac{s}{v_{av}} = \frac{9.533 \times 1000}{22} = 433.318 \text{ s} = 7 \text{ min } 13.318 \text{ s}$$

Total time of the journey: $1 \text{ min } 13.33 \text{ s} + 7 \text{ min } 13.318 \text{ s} + 1 \text{ min} = 9 \text{ min } 27 \text{ s}$

Q1.2.a) $u=0$, $a=4 \text{ m/s}^2$:

$$v^2 = u^2 + 2as \Rightarrow v^2 = 0 + 2 \times 4 \times 200 \Rightarrow v = 40 \text{ m/s}$$

b) $v = u + at \Rightarrow 40 = 0 + 4 \times t \Rightarrow t = 10 \text{ s}$

Q1.3. a) For the acceleration phase:

Acceleration phase, $u=0$, $v=v_1=80 \text{ m/s}$, $a=a_1=4 \text{ m/s}^2$:

$$v^2 = u^2 + 2as \Rightarrow (60)^2 = 0 + 2 \times 3 \times s \Rightarrow s = s_1 = 600 \text{ m}$$

Deceleration phase, with $u = v_1 = 60 \text{ m/s}$, $a = a_2 = -6 \text{ m/s}^2$, $t = t_2 = 5 \text{ s}$:

$$s = ut + \frac{1}{2}at^2 \Rightarrow s = 60 \times 5 + \frac{1}{2} \times (-6) \times (5)^2 \Rightarrow s = s_2 = 225 \text{ m}$$

Total distance = $s_1 + s_2 = 600 + 225 = 825 \text{ m}$

b) Acceleration phase $u = 0$, $v = v_1 = 60 \text{ m/s}$, $a = a_1 = 3 \text{ m/s}^2$, thus

$$v = u + at \Rightarrow 60 = 0 + 3 \times t \Rightarrow t = t_1 = 20 \text{ s}$$

Deceleration phase, $t_2 = 5 \text{ s}$ thus

Total time = $t_1 + t_2 = 20 + 5 = 25 \text{ s}$

c) Deceleration phase $u = v_1 = 60 \text{ m/s}$, $v = v_2 = ?$, $a = a_2 = -6 \text{ m/s}^2$, $t = t_2 = 5 \text{ s}$:

$$v = u + at \Rightarrow v = 60 - 6 \times 5 \Rightarrow v = v_2 = 30 \text{ m/s}$$

Q1.4. a) $u = 60 \text{ km/hr}$, $a = 6000 \text{ km/hr}^2$, $v = 110 \text{ km/hr}$

$$v = u + at \Rightarrow 110 = 60 + 6000 \times t \Rightarrow t = 8.33 \times 10^{-3} \text{ hr} = 30 \text{ s}$$

$$\text{b) } v^2 = u^2 + 2as \Rightarrow 110^2 = 60^2 + 2 \times 6000 \times s \Rightarrow s = 0.708 \text{ km} = 708 \text{ m}$$

Q1.5 a) During period of constant velocity $v = 40 \text{ m/s}$, $t = 3 \text{ s}$:

$$s = vt = 40 \times 3 = 120 \text{ m}$$

b) Turning point at $u = 40 \text{ m/s}$, $v = 0$, $a = -10 \text{ m/s}^2$:

$$v^2 = u^2 + 2as \Rightarrow 0 = (40)^2 + 2 \times (-10) \times s_{\max} \Rightarrow s_{\max} = 80 \text{ m}$$

Time to turning point C:

$$v = u + at \Rightarrow 0 = 40 - 10 \times t \Rightarrow t = 4 \text{ s}$$

c) The distance from A to C = $120 + 80 = 200 \text{ m}$

Time to travel back to A, $u = 0$, $a = -10 \text{ m/s}^2$ and $s = -200 \text{ m}$,

$$s = ut + \frac{1}{2}at^2 \Rightarrow -200 = 0 + \frac{1}{2} \times (-10) \times (t)^2 = 6.325 \text{ s} = 6.3 \text{ s}$$

Its velocity when it returns point A:

$$v = u + at \Rightarrow v = 0 - 10 \times 6.325 \Rightarrow v = -63.25 \text{ m/s}$$

d) Total distance $s = 2 \times (120 + 80) = 400 \text{ m}$

Total time:

$$t = 3 + 4 + 6.3 = 13.3 \text{ s}$$

Q1.6. a) $u = 20 \text{ km/hr}$, $t = 12 \text{ s}$, $v = 100 \text{ km/hr}$

$$v = u + at \Rightarrow 100 = 20 + a \times \frac{12}{60 \times 60} \Rightarrow a = 24000 \text{ km/hr}^2$$

$$\text{b) } v^2 = u^2 + 2as \Rightarrow 100^2 = 20^2 + 2 \times 24000 \times s \Rightarrow s = 0.2 \text{ km} = 200 \text{ m}$$

$$\text{Q1.7 } s = t^3 - \frac{15}{2}t^2 + 18t + 5$$

$$v = \frac{ds}{dt} = 3t^2 - 15t + 18$$

$$\text{And acceleration } a = \frac{dv}{dt} = 6t - 15$$

$$\text{a) At } v=0, v = 3t^2 - 15t + 18 = 0 \Rightarrow t^2 - 5t + 6 = 0 \Rightarrow (t-2)(t-3) = 0, \text{ thus } t=2 \text{ s or } 3 \text{ s}$$

$$\text{b) } t = 2 \Rightarrow s = 2^3 - \frac{15}{2} \times 2^2 + 18 \times 2 + 5 = 19 \text{ m}$$

$$t = 3 \Rightarrow s = 3^3 - \frac{15}{2} \times 3^2 + 18 \times 3 + 5 = 18.5 \text{ m}$$

$$\text{c) } t = 2 \Rightarrow a = 6 \times 2 - 15 = -3 \text{ m/s}^2$$

$$t = 3 \Rightarrow a = 6 \times 3 - 15 = 3 \text{ m/s}^2$$

$$\text{At } t = 4 \Rightarrow s = 4^3 - \frac{15}{2} \times 4^2 + 18 \times 4 + 5 = 21 \text{ m}$$

$$\text{Q1.8 } a = 5.8 + 0.021s$$

$$a = v \frac{dv}{ds} = (5.8 + 0.021s) \Rightarrow v dv = (5.8 + 0.021s) ds$$

$$\text{Integrating: } \int v dv = \int (5.8 + 0.021s) ds$$

$$\frac{v^2}{2} = 5.8s + 0.021 \frac{s^2}{2} \Rightarrow v^2 = 11.6s + 0.021s^2$$

$$\text{For } s=3000 \text{ m}$$

$$v^2 = 11.6 \times 3000 + 0.021 \times 3000^2 \Rightarrow v = 473.1 \text{ m/s}$$

$$\text{Q1.9 a) } v = 4s^2 + 3$$

$$a = v \frac{dv}{ds} = (4s^2 + 3)(8s) = 32s^3 + 24s$$

$$\text{b) For } s=3 \text{ m}$$

$$v = 4s^2 + 3 = 4 \times 3^2 + 3 = 39 \text{ m/s}$$

$$a = v \frac{dv}{ds} = (4s^2 + 3)(8s) = 39 \times 8 \times 3 = 936 \text{ m/s}^2$$

$$\text{Q1.10 } a = -12t$$

Integrating over time

$$v = -6t^2 + 20$$

Integrating again

$$s = -2t^3 + 20t$$

$$\text{at } v=0, 0 = -6t^2 + 20 \Rightarrow t = 1.8257\text{s}$$

$$s = -2 \times 1.8257^3 + 20 \times 1.8257 = 24.35\text{m}$$

Q1.11 $v = t^2 + \frac{t}{2}$

Integrating over time

$$s = \frac{t^3}{3} + \frac{t^2}{4} + C$$

Initial conditions $t=0, s_0=0$:

$$s = \frac{t^3}{3} + \frac{t^2}{4}$$

When $t=3\text{s}$:

$$s = \frac{3^3}{3} + \frac{3^2}{4} = 11.25\text{ m}$$

Acceleration:

$$a = \frac{dv}{dt} = (2t^2 + 0.5) = 2 \times 3^2 + 0.5 = 6.5 \text{ m/s}^2$$

Q1.12 Acceleration:

$$a = \frac{dv}{dt} = -0.5v^3 \Rightarrow \frac{dv}{-0.5v^3} = dt$$

Integrating:

$$\frac{1}{v^2} = t + C$$

Applying initial condition at $t=0, v_0=100 \text{ m/s}$:

$$\frac{1}{100^2} = 0 + C \Rightarrow C = 10^{-4}$$

At $t=3\text{s}$:

$$\frac{1}{v^2} = t + 10^{-4} \Rightarrow \frac{1}{v^2} = 3 + 10^{-4} \Rightarrow v = 0.577 \text{ m/s}$$

Q1.13 $x = 2t^3 - 10t^2 + 2t + 2, y = 3t^3 - 4t^2 + 5t + 3$



a) Differentiating with respect to time to derive velocity and acceleration:

$$\dot{x} = 6t^2 - 20t + 2 \qquad \dot{y} = 9t^2 - 8t + 5$$

$$\ddot{x} = 12t - 20 \qquad \ddot{y} = 18t - 8$$

When $t=4$ s

$$\dot{x} = 6 \times 4^2 - 20 \times 4 + 2 = 18 \text{ m/s} \qquad \dot{y} = 9 \times 4^2 - 8 \times 4 + 5 = 117 \text{ m/s}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(18)^2 + (117)^2} = 118.38 \text{ m/s}$$

$$\text{b) } \ddot{x} = 12 \times 4 - 20 = 28 \text{ m/s}^2 \qquad \ddot{y} = 18 \times 4 - 8 = 64 \text{ m/s}^2$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(28)^2 + (64)^2} = 69.86 \text{ m/s}^2$$

Q1.14 $x = 8t$, $y = 0.03x^2 = 0.03 \times (8t)^2 = 1.92t^2$

a) Differentiate to derive velocity and acceleration:

$$\dot{x} = 8 \qquad \dot{y} = 3.84t$$

$$\ddot{x} = 0 \qquad \ddot{y} = 3.84$$

When $t=3$ s

$$\dot{x} = 8 \text{ m/s} \qquad \dot{y} = 3.84 \times 3 = 11.52 \text{ m/s}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(8)^2 + (11.52)^2} = 14.03 \text{ m/s}$$

$$\text{b) } \theta = \tan^{-1} \frac{\dot{y}}{\dot{x}} = \tan^{-1} \frac{11.52}{8} \Rightarrow \theta = 55.22^\circ$$

$$\text{c) } \ddot{x} = 0 \text{ m/s}^2 \qquad \ddot{y} = 3.84 \text{ m/s}^2$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(0)^2 + (3.84)^2} = 3.84 \text{ m/s}^2$$

Q1.15 a) $x = 3t^2$, $y = 0.06t^3$

When $t=10$ s:

$$x = 3t^2 = 3 \times 10^2 = 300 \text{ m}$$

$$y = 0.06t^3 = 0.06 \times 10^3 = 60 \text{ m}$$

$$L = \sqrt{x^2 + y^2} = \sqrt{(300)^2 + (60)^2} = 305.94 \text{ m}$$

b) Differentiate to derive velocity and acceleration:

$$\dot{x} = 6t$$

$$\dot{y} = 0.18t^2$$

$$\ddot{x} = 6$$

$$\ddot{y} = 0.36t$$

When $t=10$ s

$$\dot{x} = 6 \times 10 = 60 \text{ m/s}$$

$$\dot{y} = 0.18 \times 10^2 = 18 \text{ m/s}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(60)^2 + (18)^2} = 62.64 \text{ m/s}$$

$$\text{c) } \ddot{x} = 6 \text{ m/s}^2$$

$$\ddot{y} = 0.36 \times 10 = 3.6 \text{ m/s}^2$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(6)^2 + (3.6)^2} = 7 \text{ m/s}^2$$

Q1.16 a) $(y - 50)^2 = 169x$,

When $y=100$ m:

$$x = \frac{(100 - 50)^2}{169} = 14.7829 \text{ m}$$

$$L = \sqrt{x^2 + y^2} = \sqrt{(14.7829)^2 + (100)^2} = 101.088 \text{ m}$$

b) Differentiate to derive velocity and acceleration:

$$2\dot{y}y - 100\dot{y} = 169\dot{x}$$

$$2\ddot{y}y + 2\dot{y}^2 - 100\ddot{y} = 169\ddot{x}$$

When $y=100$ m

$$\dot{x} = \frac{2 \times 200 \times 100 - 100 \times 200}{169} = 118.343 \text{ m/s}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(118.343)^2 + (200)^2} = 232.39 \text{ m/s}$$

$$\text{c) } \ddot{x} = \frac{0 + 2 \times 200^2 - 0}{169} = 473.37 \text{ m/s}^2$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(0)^2 + (473.37)^2} = 473.37 \text{ m/s}^2$$

Q1.17 $\dot{y}_O = v_O \sin 30 = 9 \times \sin 30 = 4.5 \text{ m/s}$

$$a) \ y = y_o + \dot{y}_o t - \frac{1}{2} g t^2 \Rightarrow -1 = 0 + 4.5 \times t - \frac{1}{2} \times 9.81 \times t^2 \Rightarrow t = 1.1023 \text{ s}$$

$$b) \ x = x_o + \dot{x}_o t \Rightarrow L = 0 + 9 \times \cos 30 \times 1.1023 = 8.59 \text{ m}$$

$$c) \ \dot{y}^2 = \dot{y}_o^2 - 2g(y - y_o) \Rightarrow 0 = (9 \times \sin 30)^2 - 2 \times 9.81 \times (h - 1)$$

From which $h = 2.03 \text{ m}$

$$Q1.18 \ a) \ x = x_o + \dot{x}_o t \Rightarrow x = x_o + v_o \cos \theta \times t \Rightarrow 9 = 0 + 15 \times \cos 20 \times t$$

From which $t = 0.64 \text{ s}$

$$b) \ \dot{y} = \dot{y}_o - g t \Rightarrow \dot{y} = 15 \times \sin 20 - 9.81 \times 0.64 = -1.148 \text{ m/s}$$

$$\dot{x} = \dot{x}_o = 15 \times \cos 20 = 14.095 \text{ m/s}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(14.095)^2 + (-1.148)^2} = 14.14 \text{ m/s}$$

$$\theta = \tan^{-1} \frac{-1.148}{14.095} = -4.66^\circ$$

$$c) \ y = y_o + \dot{y}_o t - \frac{1}{2} g t^2 = 1 + 15 \times \sin 20 \times 0.64 - \frac{1}{2} \times 9.81 \times 0.64^2$$

From which:

$$y = 2.27 \text{ m}$$

$$Q1.19 \ a) \ \dot{x}_o = v_o \cos \theta = 4 \times \cos 60 = 2 \text{ m/s}$$

$$\dot{y}_o = v_o \sin \theta = 4 \times \sin 60 = 3.4641 \text{ m/s}$$

$$x = x_o + \dot{x}_o t \Rightarrow x = 0 + 2 \times t \Rightarrow x = 2t \quad (1)$$

$$y = y_o + \dot{y}_o t - \frac{1}{2} g t^2 \Rightarrow 0.012x^2 = 0 + 3.4641t - \frac{1}{2} \times 9.81 \times t^2 \quad (2)$$

Substitute Equation (1) into Equation (2) and solve for t :

$$t = 0.6994 \text{ s} = 0.7 \text{ s}$$

b) Substitute by t in (1):

$$x = 2t = 2 \times 0.6994 = 1.398 \text{ m}$$

And

$$y = 0.012 x^2 = 0.012 \times 1.398^2 = 0.02345 \text{ m}$$

Q1.20 $r = 3t^2 + 3$

$$\theta = \frac{t}{2} \sin \frac{\pi t}{4}$$

Differentiate to derive velocity and acceleration:

$$\dot{r} = 6t$$

$$\dot{\theta} = \frac{1}{2} \sin \frac{\pi t}{4} + \frac{t}{2} \times \frac{\pi}{4} \cos \frac{\pi t}{4} = \frac{1}{2} \sin \frac{\pi t}{4} + \frac{\pi t}{8} \cos \frac{\pi t}{4}$$

$$\ddot{r} = 6 \text{ m/s}^2$$

$$\ddot{\theta} = \frac{\pi}{8} \cos \frac{\pi t}{4} + \frac{\pi}{8} \cos \frac{\pi t}{4} - \frac{\pi t}{8} \times \frac{\pi}{4} \sin \frac{\pi t}{4} = \frac{\pi}{4} \cos \frac{\pi t}{4} - \frac{\pi^2 t}{32} \sin \frac{\pi t}{4}$$

When $t=2$ s

a) $r = 15 \text{ m}$

$$\dot{r} = 12 \text{ m/s}$$

$$\dot{\theta} = \frac{1}{2} \sin \frac{\pi \times 2}{4} + \frac{\pi \times 2}{8} \cos \frac{\pi \times 2}{4} = 0.5 \text{ rad/s}$$

$$\ddot{r} = 6$$

$$\ddot{\theta} = \frac{\pi}{4} \cos \frac{\pi \times 2}{4} - \frac{\pi^2 \times 2}{32} \sin \frac{\pi \times 2}{4} = -\frac{\pi^2}{16} = -0.617 \text{ rad/s}^2$$

b) Radial and tangential velocities:

$$v_r = \dot{r} = 12 \text{ m/s}$$

$$v_t = r \dot{\theta} = 15 \times 0.5 = 7.5 \text{ m/s}$$

Total magnitude of velocity:

$$v = \sqrt{v_r^2 + v_t^2} = \sqrt{(12)^2 + (7.5)^2} = 14.15 \text{ m/s}$$

c) Radial and tangential velocities and accelerations:

$$a_r = \ddot{r} - r \dot{\theta}^2 = 6 - 15 \times (0.5)^2 = 2.25 \text{ m/s}^2$$

$$a_t = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 15 \times -\frac{\pi^2}{16} + 2 \times 12 \times (0.5) = 2.747 \text{ m/s}^2$$

Total magnitude of acceleration:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(2.25)^2 + (2.747)^2} = 3.55 \text{ m/s}^2$$

Q1.21 a) $v_t = r \dot{\theta} \Rightarrow 15 = 50 \times \dot{\theta} \Rightarrow \dot{\theta} = 0.3 \text{ rad/s}$

b) $a_r = \ddot{r} - r \dot{\theta}^2 = 0 - 50 \times (0.3)^2 = -4.5 \text{ m/s}^2$

Q1.22 a) $x = r \cos \theta$

$$y = r \sin \theta$$

$$v_r = \dot{r} = 1 \text{ m/s}$$

$$v_t = r \dot{\theta} = 2 \times \left(20 \times \frac{2\pi}{360} \right) = 0.698 \text{ m/s}$$

Referring to Figure 1.22 in Chapter 1, the velocity components in the Cartesian co-ordinate system are:

$$\dot{x} = v_r \cos \theta - v_t \sin \theta = 1 \times \cos 30 - 0.698 \times \sin 30 = 0.517 \text{ m/s}$$

$$\dot{y} = v_r \sin \theta + v_t \cos \theta = 1 \times \sin 30 + 0.698 \times \cos 30 = 1.105 \text{ m/s}$$

$$\text{b) } a_r = \ddot{r} - r \dot{\theta}^2 = 3 - 2 \times \left(20 \times \frac{2\pi}{360} \right)^2 = 2.756 \text{ m/s}^2$$

$$a_t = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 2 \times \left(50 \times \frac{2\pi}{360} \right) + 2 \times 1 \times \left(20 \times \frac{2\pi}{360} \right) = 2.44 \text{ m/s}^2$$

Referring to Figure 1.23 in Chapter 1, the acceleration components in the Cartesian co-ordinate system are:

$$\ddot{x} = a_r \cos \theta - a_t \sin \theta = 2.756 \times \cos 30 - 2.44 \times \sin 30 = 1.167 \text{ m/s}^2$$

$$\ddot{y} = a_r \sin \theta + a_t \cos \theta = 2.756 \times \sin 30 + 2.44 \times \cos 30 = 3.49 \text{ m/s}^2$$

Q 1.23 a) $v_t = r \dot{\theta} = 180 \times 0.03 = 5.4 \text{ m/s}$

$$v_r = 0 \text{ m/s}$$

Total magnitude of velocity:

$$v = \sqrt{v_r^2 + v_t^2} = \sqrt{(0)^2 + (5.4)^2} = 5.4 \text{ m/s}$$

c) Radial and tangential velocities and accelerations:

$$a_r = \ddot{r} - r \dot{\theta}^2 = 0 - 180 \times (0.03)^2 = -0.162 \text{ m/s}^2$$

$$a_t = r \ddot{\theta} + 2 \dot{r} \dot{\theta} = 180 \times -0.001 + 0 = 0.18 \text{ m/s}^2$$

Total magnitude of acceleration:

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(-0.162)^2 + (0.18)^2} = 0.242 \text{ m/s}^2$$

Q1.24 a) $v_r = 0 \text{ m/s}$

$$v_t = r \dot{\theta} = 3 \times 0.2 = 0.6 \text{ m/s}$$



$$\text{b) } a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 3 \times (0.2)^2 = -0.12 \text{ m/s}^2$$

$$a_t = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 3 \times 0 + 0 = 0 \text{ m/s}^2$$

$$\text{Q1.25 } v_t = r\dot{\theta} \Rightarrow 20 \times \cos 30 = \frac{866}{\cos 30} \times \dot{\theta} \Rightarrow \dot{\theta} = 0.0173 \text{ rad/s} \times \frac{180}{\pi} = 1^\circ/\text{s}$$

$$\text{Q1.26 a) } \rho = \frac{\left[1 + \left(\frac{dy}{dx} \right)^2 \right]^{3/2}}{\left| \frac{d^2y}{dx^2} \right|} = \frac{\left[1 + (0.08x)^2 \right]^{3/2}}{|0.08|}$$

$$\rho = \frac{\left[1 + (0.08 \times 15)^2 \right]^{3/2}}{|0.08|} = 47.64 \text{ m}$$

$$\text{b) } a_n = \frac{v^2}{\rho} = \frac{5.5^2}{47.64} = 0.635 \text{ m/s}^2$$

$$\text{c) } a = \sqrt{a_t^2 + a_n^2} = \sqrt{2.2^2 + 0.635^2} = 2.29 \text{ m/s}^2$$

$$\text{Q1.27 a) } a_n = \frac{v^2}{\rho} = \frac{9^2}{100} = 0.81 \text{ m/s}^2$$

$$\text{b) } v = u + at \Rightarrow 9 = 0 + 2t \Rightarrow t = 4.5 \text{ s}$$

$$\text{Q1.28 } \rho = \frac{\left[1 + (0.6 \times 6)^2 \right]^{3/2}}{|0.6|} = 86.93 \text{ km}$$

$$a_n = \frac{v^2}{\rho} = \frac{220^2}{86.93 \times 10^3} = 0.5567 \text{ m/s}^2$$

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.5^2 + 0.5567^2} = 0.748 \text{ m/s}^2$$

$$\text{Q1.29 a) } v = u + at \Rightarrow 30 = 14 + a_t \times 4 \Rightarrow a_t = 4 \text{ m/s}^2$$

$$\text{b) } a_n = \frac{v^2}{\rho} = \frac{25^2}{250} = 2.5 \quad \text{m/s}^2$$

$$\text{Q1.30 a) } a_t = 15 \times \cos 60 = 7.5 \text{ m/s}^2$$

$$\text{b) } a_n = 15 \times \sin 60 = 12.99 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} \Rightarrow 12.99 = \frac{22^2}{\rho} \Rightarrow \rho = 37.26 \text{ m}$$

$$\text{Q1.31 a) } \dot{v} = 1.1t$$

Integrating:

$$v = 0.55t^2$$

When $t=3$ s, $v=4.95$ m/s

$$\text{b) } a_n = \frac{v^2}{\rho} = \frac{4.95^2}{11} = 2.2275 \text{ m/s}^2$$

$$a_t = \dot{v} = 1.1 \times 3 = 3.3 \text{ m/s}^2$$

$$\text{c) } \dot{\theta} = \frac{v}{r} = \frac{0.55t^2}{11} = 0.05t^2$$

$$\dot{\theta} = 0.05 \times 3^2 = 0.45 \text{ rad/s}$$

d) Integrating $\dot{\theta}$:

$$\theta = \frac{0.05t^3}{3}$$

$$\theta = \frac{0.05 \times 3^3}{3} = 0.45 \text{ rad} = 0.45 \times \frac{180}{\pi} = 25.78^\circ$$

$$\text{Q1.32 } a_n = \frac{v^2}{\rho} \Rightarrow 2 = \frac{5000^2}{(h + 12713/2) \times 10^3} \Rightarrow h = 6143.5 \text{ m}$$

$$\text{Q1.33 } a_n = \frac{v^2}{\rho} = \frac{5000^2}{(7000 + 12713/2) \times 10^3} = 1.87 \text{ m/s}^2$$
