Chapter 1
Introduction
Solutions

1.1. Suppose that you want to design an experiment to study the proportion of unpopped kernels of popcorn. Complete steps 1-3 of the guidelines for designing experiments in Section 1.4. Are there any major sources of variation that would be difficult to control?

Step 1 – Recognition of and statement of the problem. Possible problem statement would be – find the best combination of inputs that maximizes yield on popcorn – minimize unpopped kernels.

Step 2 – Selection of the response variable. Possible responses are number of unpopped kernels per 100 kernals in experiment, weight of unpopped kernels versus the total weight of kernels cooked.

Step 3 – Choice of factors, levels and range. Possible factors and levels are brand of popcorn (levels: cheap, expensive), age of popcorn (levels: fresh, old), type of cooking method (levels: stovetop, microwave), temperature (levels: 150C, 250C), cooking time (levels: 3 minutes, 5 minutes), amount of cooking oil (levels, 1 oz, 3 oz), etc.

1.2. Suppose that you want to investigate the factors that potentially affect cooked rice.

(a) What would you use as a response variable in this experiment? How would you measure the response?

(b) List all of the potential sources of variability that could impact the response.

(c) Complete the first three steps of the guidelines for designing experiments in Section 1.4.

Step 1 – Recognition of and statement of the problem.

Step 2 – Selection of the response variable.

Step 3 – Choice of factors, levels and range.

1.3. Suppose that you want to compare the growth of garden flowers with different conditions of sunlight, water, fertilizer and soil conditions. Complete steps 1-3 of the guidelines for designing experiments in Section 1.4.

Step 1 – Recognition of and statement of the problem.

Step 2 – Selection of the response variable.

Step 3 – Choice of factors, levels and range.

1.4. Select an experiment of interest to you. Complete steps 1-3 of the guidelines for designing experiments in Section 1.4.

1.5. Search the World Wide Web for information about Sir Ronald A. Fisher and his work on experimental design in agricultural science at the Rothamsted Experimental Station.

Sample searches could include the following:

Sir Ronald Aylmer **Fisher** (British geneticist and statistician - Britannica
www.britannica.com/EBchecked/topic/.../**Sir-Ronald**-Aylmer-**Fisher**
Sir Ronald Aylmer **Fisher**, byname R.A. **Fisher** (born February 17, 1890, **...** In 1919 **Fisher** became the statistician for the **Rothamsted Experimental Station** near **...**

Sir Ronald Fisher and the Design of **Experiments**
www.jstor.org/stable/10.2307/2528399
by F Yates - 1964 - Cited by 48 - Related articles
Rothamsted Experimental Station, Harpenden, England. **Sir Ronald Fisher** is rightly regarded as the founder of the modern methods of design and analysis of **...**

Fisher biography
www-history.mcs.st-andrews.ac.uk/Biographies/**Fisher**.html
Sir Ronald Aylmer **Fisher ... Ronald** Aylmer **Fisher** was the second of twins, but the older twin was still-born. In 1904 **...** at the Galton laboratories and he was also offered the post of statistician at the **Rothamsted** Agricultural **Experiment Station**.

The 10000 Year Explosion - **Sir Ronald Fisher**
the10000yearexplosion.com/**sir-ronald-fishers**/
Sir Ronald Aylmer **Fisher**, FRS [1890-1962] was the founder of modern statistics **...** After the war, **Fisher** accepted a job at the **Rothamsted Experimental Station**, **...**

Ronald Aylmer **Fisher**
www.csse.monash.edu.au/~lloyd/tildeImages/People/**Fisher**.RA/
courtesy of **Rothamsted Experimental Station**, Harpenden, Herts, England. **...** St Peters. There is a memorial pew (inscribed: In memory of **Sir Ronald Fisher** FRS **...**

Sir Ronald Aylmer **Fisher** Facts, information, pictures | Encyclopedia ...
www.encyclopedia.com › ... › Mathematics: Biographies
Sir Ronald Aylmer **Fisher** 1890-1962, English statistician and geneticist, b. **...** of research statistician at the **Rothamsted Experimental Station** in Hertfordshire.

1.6. Find a Web Site for a business that you are interested in. Develop a list of factors that you would use in an experimental design to improve the effectiveness of this Web Site.

1.7. Almost everyone is concerned about the rising price of gasoline. Construct a cause and effect diagram identifying the factors that potentially influence the gasoline mileage that you get in your car. How would you go about conducting an experiment to determine any of these factors actually affect your gasoline mileage?

1.8. What is replication? Why do we need replication in an experiment? Present an example that illustrates the differences between replication and repeated measures.

Repetition of the experimental runs. Replication enables the experimenter to estimate the experimental error, and provides more precise estimate of the mean for the response variable.

1.9. Why is randomization important in an experiment?

To assure the observations, or errors, are independently distributed randome variables as required by statistical methods. Also, to "average out" the effects of extraneous factors that might occur while running the experiment.

1.10. What are the potential risks of a single, large, comprehensive experiment in contrast to a sequential approach?

The important factors and levels are not always known at the beginning of the experimental process. Even new response variables might be discovered during the experimental process. By running a large comprehensive experiment, valuable information learned early in the experimental process can not likely be incorporated in the remaining experimental runs.

Experimental runs can be expensive and time consuming. If an error were to occur while running the experiment, the cost of redoing the experiment is much more manageable with one of the small sequential experiments than the large comprehensive experiment.

1.11. Have you received an offer to obtain a credit card in the mail? What "factors" were associated with the offer, such as introductory interest rate? Do you think the credit card company is conducting experiments to investigate which facors product the highest positive response rate to their offer? What potential factors in the experiment can you identify?

Interest rate, credit limit, old credit card pay-off amount, interest free period, gift points, others.

1.12. What factors do you think an e-commerce company could use in an experiment involving their web page to encourage more people to "click-through" into their site?

Font size, font type, images/icons, color, spacing, animation, sound/music, speed, others.

Chapter 2
Simple Comparative Experiments
Solutions

2.1. Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

Variable	N	Mean	SE Mean	Std. Dev.	Variance	Minimum	Maximum
Y	9	19.96	?	3.12	?	15.94	27.16

SE Mean = 1.04   Variance = 9.73

2.2. Computer output for a random sample of data is shown below. Some of the quantities are missing. Compute the values of the missing quantities.

Variable	N	Mean	SE Mean	Std. Dev.	Sum
Y	16	?	0.159	?	399.851

Mean = 24.991   Std. Dev. = 0.636

2.3. Suppose that we are testing H_0: $\mu = \mu_0$ versus H_1: $\mu \neq \mu_0$. Calculate the P-value for the following observed values of the test statistic:

(a) $Z_0 = 2.25$ P-value = 0.02445

(b) $Z_0 = 1.55$ P-value = 0.12114

(c) $Z_0 = 2.10$ P-value = 0.03573

(d) $Z_0 = 1.95$ P-value = 0.05118

(e) $Z_0 = -0.10$ P-value = 0.92034

2.4. Suppose that we are testing H_0: $\mu = \mu_0$ versus H_1: $\mu > \mu_0$. Calculate the P-value for the following observed values of the test statistic:

(a) $Z_0 = 2.45$ P-value = 0.00714

(b) $Z_0 = -1.53$ P-value = 0.93699

(c) $Z_0 = 2.15$ P-value = 0.01578

(d) $Z_0 = 1.95$ P-value = 0.02559

(e) $Z_0 = -0.25$ P-value = 0.59871

2.5. Consider the computer output shown below.

One-Sample Z
Test of mu = 30 vs not = 30
The assumed standard deviation = 1.2

N	Mean	SE Mean	95% CI	Z	P
16	31.2000	0.3000	(30.6120, 31.7880)	?	?

(a) Fill in the missing values in the output. What conclusion would you draw?

Z = 4 P = 0.00006; therefore, the mean is not equal to 30.

(b) Is this a one-sided or two-sided test?

Two-sided.

(c) Use the output and the normal table to find a 99 percent CI on the mean.

CI = 30.42725, 31.97275

(d) What is the P-value if the alternative hypothesis is H_1: $\mu > 30$

P-value = 0.00003

2.6. Suppose that we are testing H_0: $\mu_1 = \mu_2$ versus H_1: $\mu_1 = \mu_2$ with a sample size of $n_1 = n_2 = 12$. Both sample variances are unknown but assumed equal. Find bounds on the P-value for the following observed values of the test statistic:

(a) $t_0 = 2.30$ Table P-value = 0.02, 0.05 Computer P-value = 0.0313

(b) $t_0 = 3.41$ Table P-value = 0.002, 0.005 Computer P-value = 0.0025

(c) $t_0 = 1.95$ Table P-value = 0.1, 0.05 Computer P-value = 0.0640

(d) $t_0 = -2.45$ Table P-value = 0.05, 0.02 Computer P-value = 0.0227

Note that the degrees of freedom is (12 +12) – 2 = 22. This is a two-sided test

2.7. Suppose that we are testing H_0: $\mu_1 = \mu_2$ versus H_1: $\mu_1 > \mu_2$ with a sample size of $n_1 = n_2 = 10$. Both sample variances are unknown but assumed equal. Find bounds on the P-value for the following observed values of the test statistic:

(a) $t_0 = 2.31$ Table P-value = 0.01, 0.025 Computer P-value = 0.01648

(b) $t_0 = 3.60$ Table P-value = 0.001, 0.0005 Computer P-value = 0.00102

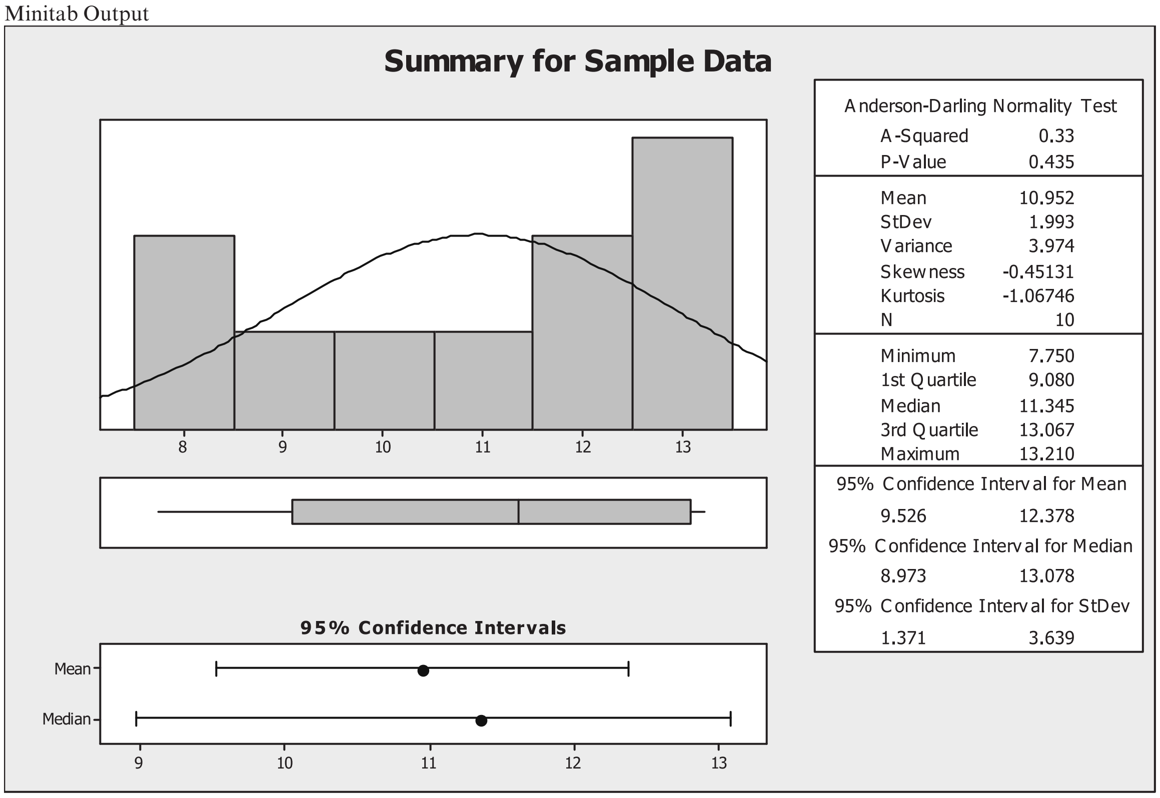
(c) $t_0 = 1.95$ Table P-value = 0.05, 0.025 Computer P-value = 0.03346

(d) $t_0 = 2.19$ Table P-value = 0.01, 0.025 Computer P-value = 0.02097

Note that the degrees of freedom is (10 +10) – 2 = 18. This is a one-sided test.

2.8. Consider the following sample data: 9.37, 13.04, 11.69, 8.21, 11.18, 10.41, 13.15, 11.51, 13.21, and 7.75. Is it reasonable to assume that this data is from a normal distribution? Is there evidence to support a claim that the mean of the population is 10?

Minitab Output



According to the output, the Anderson-Darling Normality Test has a P-Value of 0.435. The data can be considered normal. The 95% confidence interval on the mean is (9.526,12.378). This confidence interval contains 10, therefore there is evidence that the population mean is 10.

2.9. A computer program has produced the following output for the hypothesis testing problem:

Difference in sample means: 2.35
Degrees of freedom: 18
Standard error of the difference in the sample means: ?
Test statistic: $t_0 = 2.01$
P-Value = 0.0298

(a) What is the missing value for the standard error?

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{2.35}{StdError} = 2.01$$

$$StdError = 2.35/2.01 = 1.169$$

(b) Is this a two-sided or one-sided test? One-sided test for a t_0 = 2.01 is a P-value of 0.0298.

(c) If α=0.05, what are your conclusions? Reject the null hypothesis and conclude that there is a difference in the two samples.

(d) Find a 90% two-sided CI on the difference in the means.

$$\bar{y}_1 - \bar{y}_2 - t_{\alpha/2,n_1+n_2-2}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_1 \le \bar{y}_1 - \bar{y}_2 + t_{\alpha/2,n_1+n_2-2}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$\bar{y}_1 - \bar{y}_2 - t_{0.05,18}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}} \le \mu_1 - \mu_1 \le \bar{y}_1 - \bar{y}_2 + t_{0.05,18}S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$$

$$2.35 - 1.734(1.169) \le \mu_1 - \mu_1 \le 2.35 + 1.734(1.169)$$

$$0.323 \le \mu_1 - \mu_1 \le 4.377$$

2.10. A computer program has produced the following output for the hypothesis testing problem:

```
Difference in sample means: 11.5
Degrees of freedom: 24
Standard error of the difference in the sample means: ?
Test statistic: t0 = -1.88
P-Value = 0.0723
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(a) What is the missing value for the standard error?

$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{-11.5}{StdError} = -1.88$$

$$StdError = -11.5/-1.88 = 6.12$$

(b) Is this a two-sided or one-sided test? Two-sided test for a t_0 = -1.88 is a P-value of 0.0723.

(c) If α=0.05, what are your conclusions? Accept the null hypothesis, there is no difference in the means.

(d) Find a 90% two-sided CI on the difference in the means.

$$\bar{y}_1 - \bar{y}_2 - t_{\alpha/2,n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_1 \le \bar{y}_1 - \bar{y}_2 + t_{\alpha/2,n_1+n_2-2} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$\bar{y}_1 - \bar{y}_2 - t_{0.05,24} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \le \mu_1 - \mu_1 \le \bar{y}_1 - \bar{y}_2 + t_{0.05,24} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

$$-11.5 - 1.711(6.12) \le \mu_1 - \mu_1 \le -11.5 + 1.711(6.12)$$

$$-21.97 \le \mu_1 - \mu_1 \le -1.03$$

2.11. A two-sample t-test has been conducted and the sample sizes are $n_1 = n_2 = 10$. The computed value of the test statistic is $t_0 = 2.15$. If the null hypothesis is two-sided, an upper bound on the P-value is

(a) 0.10

(b) <u>0.05</u>

(c) 0.025

(d) 0.01

(e) None of the above.

2.12. A two-sample t-test has been conducted and the sample sizes are $n_1 = n_2 = 12$. The computed value of the test statistic is $t_0 = 2.27$. If the null hypothesis is two-sided, an upper bound on the P-value is

(a) 0.10

(b) <u>0.05</u>

(c) 0.025

(d) 0.01

(e) None of the above.

2.13. Suppose that we are testing H_0: $\mu = \mu_0$ versus H_1: $\mu > \mu_0$ with a sample size of $n = 15$. Calculate bounds on the P-value for the following observed values of the test statistic:

(a) $t_0 = 2.35$ Table P-value = 0.01, 0.025 Computer P-value = 0.01698

(b) $t_0 = 3.55$ Table P-value = 0.001, 0.0025 Computer P-value = 0.00160

(c) $t_0 = 2.00$ Table P-value = 0.025, 0.005 Computer P-value = 0.03264

(d) $t_0 = 1.55$ Table *P*-value = 0.05, 0.10 Computer *P*-value = 0.07172

The degrees of freedom are 15 – 1 = 14. This is a one-sided test.

2.14. Suppose that we are testing H_0: $\mu = \mu_0$ versus H_1: $\mu \neq \mu_0$ with a sample size of $n = 10$. Calculate bounds on the *P*-value for the following observed values of the test statistic:

(a) $t_0 = 2.48$ Table *P*-value = 0.02, 0.05 Computer *P*-value = 0.03499

(b) $t_0 = -3.95$ Table *P*-value = 0.002, 0.005 Computer *P*-value = 0.00335

(c) $t_0 = 2.69$ Table *P*-value = 0.02, 0.05 Computer *P*-value = 0.02480

(d) $t_0 = 1.88$ Table *P*-value = 0.05, 0.10 Computer *P*-value = 0.09281

(e) $t_0 = -1.25$ Table *P*-value = 0.20, 0.50 Computer *P*-value = 0.24282

2.15. Consider the computer output shown below.

One-Sample T: Y

Test of mu = 91 vs. not = 91

Variable	N	Mean	Std. Dev.	SE Mean	95% CI	T	P
Y	25	92.5805	?	0.4675	(91.6160, ?)	3.38	0.002

(a) Fill in the missing values in the output. Can the null hypothesis be rejected at the 0.05 level? Why?

Std. Dev. = 2.3365 UCI = 93.5450
Yes, the null hypothesis can be rejected at the 0.05 level because the *P*-value is much lower at 0.002.

(b) Is this a one-sided or two-sided test?

Two-sided.

(c) If the hypothesis had been H_0: $\mu = 90$ versus H_1: $\mu \neq 90$ would you reject the null hypothesis at the 0.05 level?

Yes.

(d) Use the output and the *t* table to find a 99 percent two-sided CI on the mean.

CI = 91.2735, 93.8875

(e) What is the *P*-value if the alternative hypothesis is H_1: $\mu > 91$?

P-value = 0.001.

2.16. Consider the computer output shown below.

One-Sample T: Y

Test of mu = 25 vs > 25

Variable	N	Mean	Std. Dev.	SE Mean	95% Lower Bound	T	P
Y	12	25.6818	?	0.3360	?	?	0.034

(a) How many degrees of freedom are there on the t-test statistic?

(N-1) = (12 – 1) = 11

(b) Fill in the missing information.

Std. Dev. = 1.1639   95% Lower Bound = 2.0292

2.17. Consider the computer output shown below.

Two-Sample T-Test and CI: Y1, Y2

Two-sample T for Y1 vs Y2

	N	Mean	Std. Dev.	SE Mean
Y1	20	50.19	1.71	0.38
Y2	20	52.52	2.48	0.55

Difference = mu (X1) – mu (X2)

Estimate for difference: -2.33341

95% CI for difference: (-3.69547, -0.97135)

T-Test of difference = 0 (vs not =) : T-Value = -3.47

P-Value = 0.01 DF = 38

Both use Pooled Std. Dev. = 2.1277

(a) Can the null hypothesis be rejected at the 0.05 level? Why?

Yes, the P-Value of 0.001 is much less than 0.05.

(b) Is this a one-sided or two-sided test?

Two-sided.

(c) If the hypothesis had been H_0: μ_1 - μ_2 = 2 versus H_1: μ_1 - μ_2 ≠ 2 would you reject the null hypothesis at the 0.05 level?

Yes.

(d) If the hypothesis had been H_0: $\mu_1 - \mu_2 = 2$ versus H_1: $\mu_1 - \mu_2 < 2$ would you reject the null hypothesis at the 0.05 level? Can you answer this question without doing any additional calculations? Why?

Yes, no additional calculations are required because the test is naturally becoming more significant with the change from -2.33341 to -4.33341.

(e) Use the output and the t table to find a 95 percent upper confidence bound on the difference in means?

95% upper confidence bound = -1.21.

(f) What is the P-value if the alternative hypotheses are H_0: $\mu_1 - \mu_2 = 2$ versus H_1: $\mu_1 - \mu_2 \neq 2$?

P-value = 1.4E-07.

2.18. The breaking strength of a fiber is required to be at least 150 psi. Past experience has indicated that the standard deviation of breaking strength is $\sigma = 3$ psi. A random sample of four specimens is tested. The results are y_1=145, y_2=153, y_3=150 and y_4=147.

(a) State the hypotheses that you think should be tested in this experiment.

$$H_0\text{: } \mu = 150 \qquad H_1\text{: } \mu > 150$$

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

$n = 4$, $\sigma = 3$, $\bar{y} = 1/4\ (145 + 153 + 150 + 147) = 148.75$

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{148.75 - 150}{\frac{3}{\sqrt{4}}} = \frac{-1.25}{\frac{3}{2}} = -0.8333$$

Since $z_{0.05} = 1.645$, do not reject.

(c) Find the P-value for the test in part (b).

From the z-table: $P \cong 1 - \left[0.7967 + (2/3)(0.7995 - 0.7967)\right] = 0.2014$

(d) Construct a 95 percent confidence interval on the mean breaking strength.

The 95% confidence interval is

$$\bar{y} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{y} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$
$$148.75 - (1.96)(3/2) \le \mu \le 148.75 + (1.96)(3/2)$$

$$145.81 \le \mu \le 151.69$$

2.19. The viscosity of a liquid detergent is supposed to average 800 centistokes at 25°C. A random sample of 16 batches of detergent is collected, and the average viscosity is 812. Suppose we know that the standard deviation of viscosity is $\sigma = 25$ centistokes.

(a) State the hypotheses that should be tested.

$$H_0\text{: } \mu = 800 \qquad H_1\text{: } \mu \neq 800$$

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{812 - 800}{\frac{25}{\sqrt{16}}} = \frac{12}{\frac{25}{4}} = 1.92$$

Since $z_{\alpha/2} = z_{0.025} = 1.96$, do not reject.

(c) What is the *P*-value for the test?

(d) Find a 95 percent confidence interval on the mean.

The 95% confidence interval is

$$\bar{y} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{y} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$
$$812 - (1.96)(25/4) \leq \mu \leq 812 + (1.96)(25/4)$$
$$812 - 12.25 \leq \mu \leq 812 + 12.25$$
$$799.75 \leq \mu \leq 824.25$$

2.20. The diameters of steel shafts produced by a certain manufacturing process should have a mean diameter of 0.255 inches. The diameter is known to have a standard deviation of $\sigma = 0.0001$ inch. A random sample of 10 shafts has an average diameter of 0.2545 inches.

(a) Set up the appropriate hypotheses on the mean μ.

$$H_0\text{: } \mu = 0.255 \qquad H_1\text{: } \mu \neq 0.255$$

(b) Test these hypotheses using $\alpha = 0.05$. What are your conclusions?

$$n = 10, \quad \sigma = 0.0001, \quad \bar{y} = 0.2545$$

$$z_o = \frac{\bar{y} - \mu_o}{\frac{\sigma}{\sqrt{n}}} = \frac{0.2545 - 0.255}{\frac{0.0001}{\sqrt{10}}} = -15.81$$

Since $z_{0.025} = 1.96$, reject H_0.

(c) Find the *P*-value for this test. $P = 2.6547\text{x}10^{-56}$

(d) Construct a 95 percent confidence interval on the mean shaft diameter.

The 95% confidence interval is

$$\bar{y} - z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{y} + z_{\alpha/2}\frac{\sigma}{\sqrt{n}}$$

$$0.2545 - (1.96)\left(\frac{0.0001}{\sqrt{10}}\right) \le \mu \le 0.2545 + (1.96)\left(\frac{0.0001}{\sqrt{10}}\right)$$

$$0.254438 \le \mu \le 0.254562$$

2.21. A normally distributed random variable has an unknown mean μ and a known variance $\sigma^2 = 9$. Find the sample size required to construct a 95 percent confidence interval on the mean that has total length of 1.0.

Since $y \sim N(\mu,9)$, a 95% two-sided confidence interval on μ is

If the total interval is to have width 1.0, then the half-interval is 0.5. Since $z_{\alpha/2} = z_{0.025} = 1.96$,

$$(1.96)\left(\frac{3}{\sqrt{n}}\right) = 0.5(1.96)$$

$$\sqrt{n} = (1.96)\left(\frac{3}{0.5}\right) = 11.76$$

$$n = (11.76)^2 = 138.30 \cong 139$$

2.22. The shelf life of a carbonated beverage is of interest. Ten bottles are randomly selected and tested, and the following results are obtained:

Days	
108	138
124	163
124	159
106	134
115	139

(a) We would like to demonstrate that the mean shelf life exceeds 120 days. Set up appropriate hypotheses for investigating this claim.

H_0: $\mu = 120$ $\qquad$ H_1: $\mu > 120$

(b) Test these hypotheses using $\alpha = 0.01$. What are your conclusions?

$\bar{y} = 131$
$S^2 = 3438 / 9 = 382$
$S = \sqrt{382} = 19.54$

$$t_0 = \frac{\bar{y} - \mu_0}{S/\sqrt{n}} = \frac{131-120}{19.54/\sqrt{10}} = 1.78$$

since $t_{0.01,9} = 2.821$; do not reject H_0

Minitab Output

```
T-Test of the Mean

Test of mu = 120.00 vs mu > 120.00

Variable      N      Mean    StDev   SE Mean        T          P
Shelf Life   10    131.00    19.54      6.18     1.78      0.054

T Confidence Intervals

Variable      N      Mean    StDev  SE Mean        99.0 % CI
Shelf Life   10    131.00    19.54     6.18  (  110.91,   151.09)
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(c) Find the P-value for the test in part (b). P=0.054

(d) Construct a 99 percent confidence interval on the mean shelf life.

The 99% confidence interval is $\bar{y} - t_{\alpha/2, n-1}\frac{S}{\sqrt{n}} \le \mu \le \bar{y} + t_{\alpha/2, n-1}\frac{S}{\sqrt{n}}$ with $\alpha = 0.01$.

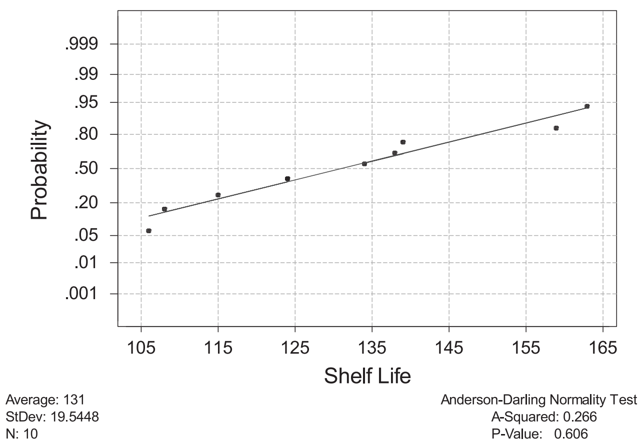
$$131 - (3.250)\left(\frac{19.54}{\sqrt{10}}\right) \le \mu \le 131 + (3.250)\left(\frac{19.54}{\sqrt{10}}\right)$$

$$110.91 \le \mu \le 151.08$$

2.23. Consider the shelf life data in Problem 2.22. Can shelf life be described or modeled adequately by a normal distribution? What effect would violation of this assumption have on the test procedure you used in solving Problem 2.22?

A normal probability plot, obtained from Minitab, is shown. There is no reason to doubt the adequacy of the normality assumption. If shelf life is not normally distributed, then the impact of this on the t-test in problem 2.22 is not too serious unless the departure from normality is severe.

Normal Probability Plot



2.24. The time to repair an electronic instrument is a normally distributed random variable measured in hours. The repair time for 16 such instruments chosen at random are as follows:

Hours			
159	280	101	212
224	379	179	264
222	362	168	250
149	260	485	170

(a) You wish to know if the mean repair time exceeds 225 hours. Set up appropriate hypotheses for investigating this issue.

$$H_0\text{: } \mu = 225 \qquad H_1\text{: } \mu > 225$$

(b) Test the hypotheses you formulated in part (a). What are your conclusions? Use $\alpha = 0.05$.

$$\bar{y} = 241.50$$
$$S^2 = 146202 / (16 - 1) = 9746.80$$

$$S = \sqrt{9746.8} = 98.73$$

$$t_o = \frac{\bar{y} - \mu_o}{\frac{S}{\sqrt{n}}} = \frac{241.50 - 225}{\frac{98.73}{\sqrt{16}}} = 0.67$$

since $t_{0.05,15} = 1.753$; do not reject H_0

Minitab Output

T-Test of the Mean

```
Test of mu = 225.0 vs mu > 225.0

Variable     N      Mean    StDev   SE Mean        T          P
Hours       16     241.5     98.7      24.7     0.67       0.26
```

T Confidence Intervals

```
Variable     N      Mean    StDev   SE Mean        95.0 % CI
Hours       16     241.5     98.7      24.7  (   188.9,    294.1)
```

(c) Find the *P*-value for this test. *P*=0.26

(d) Construct a 95 percent confidence interval on mean repair time.

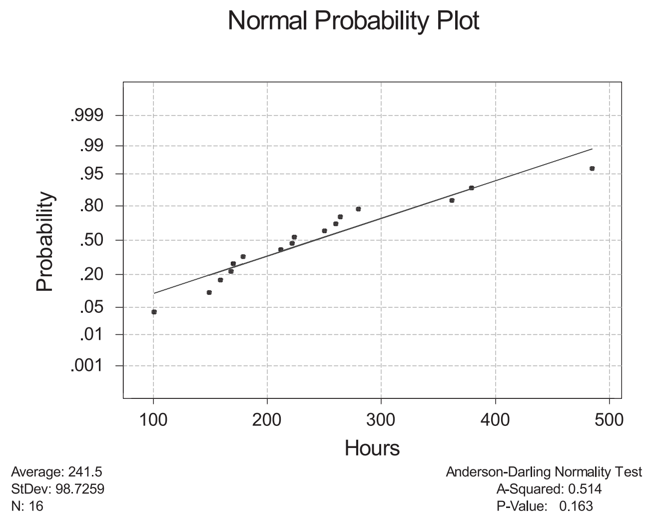
The 95% confidence interval is $\bar{y} - t_{\alpha/2,n-1}\frac{S}{\sqrt{n}} \le \mu \le \bar{y} + t_{\alpha/2,n-1}\frac{S}{\sqrt{n}}$

$$241.50 - (2.131)\left(\frac{98.73}{\sqrt{16}}\right) \le \mu \le 241.50 + (2.131)\left(\frac{98.73}{\sqrt{16}}\right)$$

$$188.9 \le \mu \le 294.1$$

2.25. Reconsider the repair time data in Problem 2.24. Can repair time, in your opinion, be adequately modeled by a normal distribution?

The normal probability plot below does not reveal any serious problem with the normality assumption.



2.26. Two machines are used for filling plastic bottles with a net volume of 16.0 ounces. The filling processes can be assumed to be normal, with standard deviation of σ_1 = 0.015 and σ_2 = 0.018. The quality engineering department suspects that both machines fill to the same net volume, whether or not this volume is 16.0 ounces. An experiment is performed by taking a random sample from the output of each machine.

Machine 1		Machine 2	
16.03	16.01	16.02	16.03
16.04	15.96	15.97	16.04
16.05	15.98	15.96	16.02
16.05	16.02	16.01	16.01
16.02	15.99	15.99	16.00

(a) State the hypotheses that should be tested in this experiment.

$$H_0\colon\ \mu_1 = \mu_2 \qquad H_1\colon\ \mu_1 \neq \mu_2$$

(b) Test these hypotheses using α=0.05. What are your conclusions?

$$\bar{y}_1 = 16.015 \qquad \bar{y}_2 = 16.005$$
$$\sigma_1 = 0.015 \qquad \sigma_2 = 0.018$$
$$n_1 = 10 \qquad n_2 = 10$$

$$z_o = \frac{\bar{y}_1 - \bar{y}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{16.015 - 16.018}{\sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}}} = 1.35$$

$z_{0.025}$ = 1.96; do not reject

(c) What is the P-value for the test? P = 0.1770

(d) Find a 95 percent confidence interval on the difference in the mean fill volume for the two machines.

The 95% confidence interval is

$$\bar{y}_1 - \bar{y}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{y}_1 - \bar{y}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(16.015 - 16.005) - (1.96)\sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}} \le \mu_1 - \mu_2 \le (16.015 - 16.005) + (1.96)\sqrt{\frac{0.015^2}{10} + \frac{0.018^2}{10}}$$

$$-0.0045 \le \mu_1 - \mu_2 \le 0.0245$$

2.27. Two types of plastic are suitable for use by an electronic calculator manufacturer. The breaking strength of this plastic is important. It is known that $\sigma_1 = \sigma_2 = 1.0$ psi. From random samples of $n_1 = 10$ and $n_2 = 12$ we obtain $\bar{y}_1 = 162.5$ and $\bar{y}_2 = 155.0$. The company will not adopt plastic 1 unless its breaking strength exceeds that of plastic 2 by at least 10 psi. Based on the sample information, should they use plastic 1? In answering this questions, set up and test appropriate hypotheses using $\alpha = 0.01$. Construct a 99 percent confidence interval on the true mean difference in breaking strength.

H_0: $\mu_1 - \mu_2 = 10$ $\quad$ H_1: $\mu_1 - \mu_2 > 10$

$$\begin{array}{ll} \bar{y}_1 = 162.5 & \bar{y}_2 = 155.0 \\ \sigma_1 = 1 & \sigma_2 = 1 \\ n_1 = 10 & n_2 = 10 \end{array}$$

$$z_o = \frac{\bar{y}_1 - \bar{y}_2 - 10}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{162.5 - 155.0 - 10}{\sqrt{\frac{1^2}{10} + \frac{1^2}{12}}} = -5.84$$

$z_{0.01}$ = 2.325; do not reject

The 99 percent confidence interval is

$$\bar{y}_1 - \bar{y}_2 - z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{y}_1 - \bar{y}_2 + z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$(162.5 - 155.0) - (2.575)\sqrt{\frac{1^2}{10} + \frac{1^2}{12}} \le \mu_1 - \mu_2 \le (162.5 - 155.0) + (2.575)\sqrt{\frac{1^2}{10} + \frac{1^2}{12}}$$

$$6.40 \le \mu_1 - \mu_2 \le 8.60$$

2.28. The following are the burning times (in minutes) of chemical flares of two different formulations. The design engineers are interested in both the means and variance of the burning times.

Type 1		Type 2	
65	82	64	56
81	67	71	69
57	59	83	74
66	75	59	82
82	70	65	79

(a) Test the hypotheses that the two variances are equal. Use $\alpha = 0.05$.

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 \neq \sigma_2^2$$

Do not reject.

(b) Using the results of (a), test the hypotheses that the mean burning times are equal. Use $\alpha =$ 0.05. What is the P-value for this test?

Do not reject.

From the computer output, t=0.05; do not reject. Also from the computer output P=0.96

Minitab Output

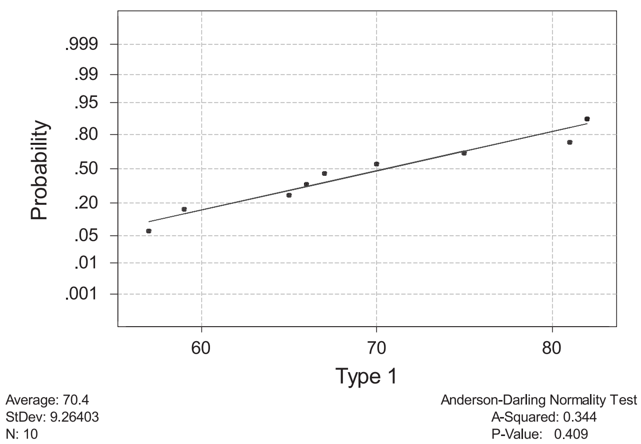
Two Sample T-Test and Confidence Interval

```
Two sample T for Type 1 vs Type 2

         N      Mean     StDev   SE Mean
Type 1  10     70.40      9.26       2.9
Type 2  10     70.20      9.37       3.0

95% CI for mu Type 1 - mu Type 2: ( -8.6,  9.0)
T-Test mu Type 1 = mu Type 2 (vs not =): T = 0.05   P = 0.96   DF = 18
Both use Pooled StDev = 9.32
```

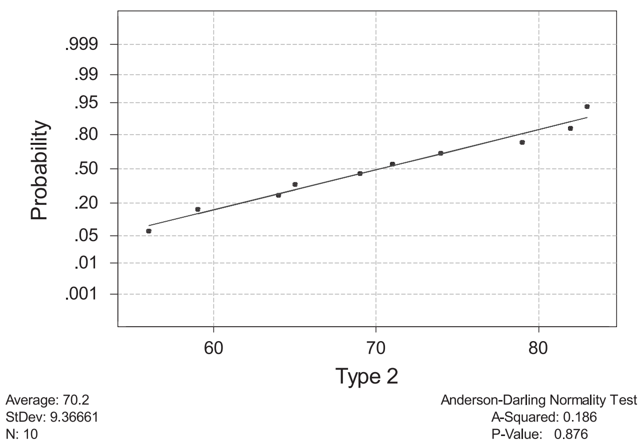
(c) Discuss the role of the normality assumption in this problem. Check the assumption of normality for both types of flares.

The assumption of normality is required in the theoretical development of the t-test. However, moderate departure from normality has little impact on the performance of the t-test. The normality assumption is more important for the test on the equality of the two variances. An indication of nonnormality would be of concern here. The normal probability plots shown below indicate that burning time for both formulations follow the normal distribution.

Normal Probability Plot



Normal Probability Plot



2.29. An article in *Solid State Technology*, "Orthogonal Design of Process Optimization and Its Application to Plasma Etching" by G.Z. Yin and D.W. Jillie (May, 1987) describes an experiment to determine the effect of C_2F_6 flow rate on the uniformity of the etch on a silicon wafer used in integrated circuit manufacturing. Data for two flow rates are as follows:

C_2F_6	Uniformity Observation					
(SCCM)	1	2	3	4	5	6
125	2.7	4.6	2.6	3.0	3.2	3.8
200	4.6	3.4	2.9	3.5	4.1	5.1

(a) Does the C_2F_6 flow rate affect average etch uniformity? Use $\alpha = 0.05$.

No, C_2F_6 flow rate does not affect average etch uniformity.

Minitab Output

```
Two Sample T-Test and Confidence Interval

Two sample T for Uniformity

Flow Rat     N      Mean     StDev   SE Mean
125          6     3.317     0.760      0.31
200          6     3.933     0.821      0.34

95% CI for mu (125) - mu (200): ( -1.63,  0.40)
T-Test mu (125) = mu (200) (vs not =): T = -1.35  P = 0.21  DF = 10
Both use Pooled StDev = 0.791
```

(b) What is the *P*-value for the test in part (a)? From the *Minitab* output, P=0.21

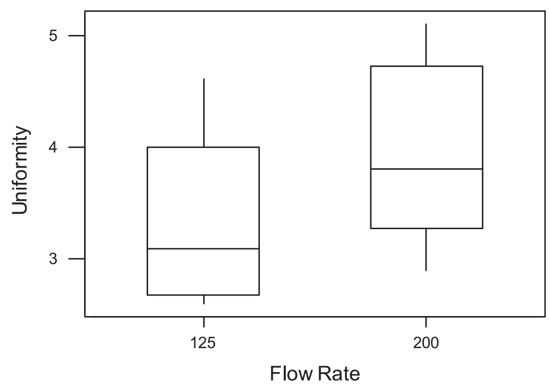
(c) Does the C_2F_6 flow rate affect the wafer-to-wafer variability in etch uniformity? Use $\alpha = 0.05$.

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 \neq \sigma_2^2$$
$$F_{0.025,5,5} = 7.15$$
$$F_{0.975,5,5} = 0.14$$
$$F_0 = \frac{0.5776}{0.6724} = 0.86$$

Do not reject; C_2F_6 flow rate does not affect wafer-to-wafer variability.

(d) Draw box plots to assist in the interpretation of the data from this experiment.

The box plots shown below indicate that there is little difference in uniformity at the two gas flow rates. Any observed difference is not statistically significant. See the *t*-test in part (a).



2.30. A new filtering device is installed in a chemical unit. Before its installation, a random sample yielded the following information about the percentage of impurity: $\bar{y}_1 = 12.5$, $S_1^2 = 101.17$, and $n_1 = 8$. After installation, a random sample yielded $\bar{y}_2 = 10.2$, $S_2^2 = 94.73$, $n_2 = 9$.

(a) Can you conclude that the two variances are equal? Use $\alpha = 0.05$.

$$H_0 : \sigma_1^2 = \sigma_2^2$$
$$H_1 : \sigma_1^2 \neq \sigma_2^2$$
$$F_{0.025,7,8} = 4.53$$
$$F_0 = \frac{S_1^2}{S_2^2} = \frac{101.17}{94.73} = 1.07$$

Do not reject. Assume that the variances are equal.

(b) Has the filtering device reduced the percentage of impurity significantly? Use $\alpha = 0.05$.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 > \mu_2$$
$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{(8-1)(101.17)+(9-1)(94.73)}{8+9-2} = 97.74$$
$$S_p = 9.89$$
$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{12.5-10.2}{9.89\sqrt{\frac{1}{8}+\frac{1}{9}}} = 0.479$$
$$t_{0.05,15} = 1.753$$

Do not reject. There is no evidence to indicate that the new filtering device has affected the mean.

2.31. Photoresist is a light-sensitive material applied to semiconductor wafers so that the circuit pattern can be imaged on to the wafer. After application, the coated wafers are baked to remove the solvent in the photoresist mixture and to harden the resist. Here are measurements of photoresist thickness (in kÅ) for eight wafers baked at two different temperatures. Assume that all of the runs were made in random order.

95 ºC	100 ºC
11.176	5.623
7.089	6.748
8.097	7.461
11.739	7.015
11.291	8.133
10.759	7.418
6.467	3.772
8.315	8.963

(a) Is there evidence to support the claim that the higher baking temperature results in wafers with a lower mean photoresist thickness? Use $\alpha = 0.05$.

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 > \mu_2$$
$$S_p^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2} = \frac{(8-1)(4.41)+(8-1)(2.54)}{8+8-2} = 3.48$$
$$S_p = 1.86$$
$$t_0 = \frac{\bar{y}_1 - \bar{y}_2}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{9.37-6.89}{1.86\sqrt{\frac{1}{8}+\frac{1}{8}}} = 2.65$$
$$t_{0.05,14} = 1.761$$

Since $t_{0.05,14} = 1.761$, reject H_0. There appears to be a lower mean thickness at the higher temperature. This is also seen in the computer output.

Minitab Output

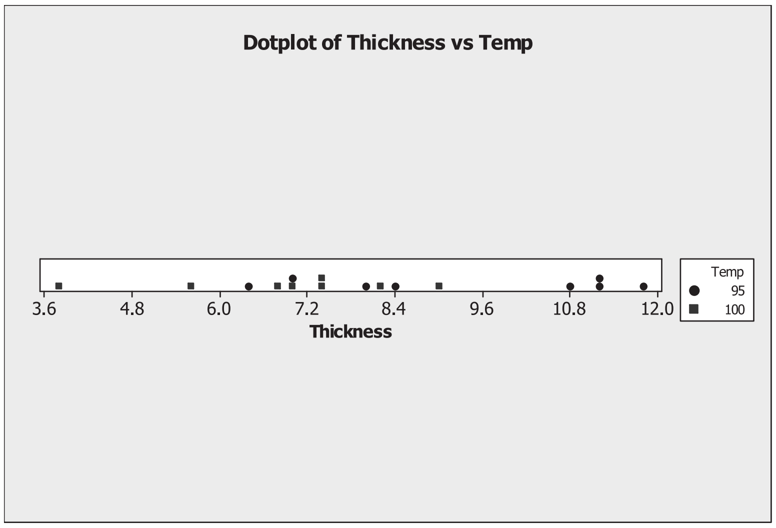
```
Two-Sample T-Test and CI: Thickness, Temp

Two-sample T for Thick@95 vs Thick@100

           N      Mean     StDev   SE Mean
Thick@95   8      9.37      2.10      0.74
Thick@10   8      6.89      1.60      0.56

Difference = mu Thick@95 - mu Thick@100
Estimate for difference:  2.475
95% lower bound for difference: 0.833
T-Test of difference = 0 (vs >): T-Value = 2.65  P-Value = 0.009  DF = 14
Both use Pooled StDev = 1.86
```

(b) What is the P-value for the test conducted in part (a)? $P = 0.009$

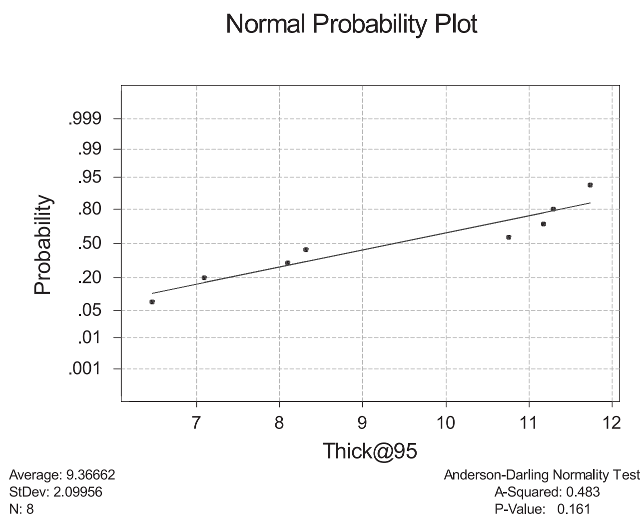
(c) Find a 95% confidence interval on the difference in means. Provide a practical interpretation of this interval.

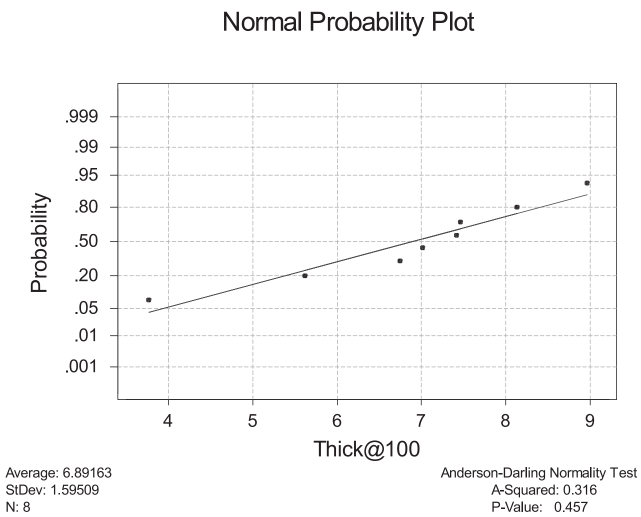
From the computer output the 95% lower confidence bound is $0.833 \le \mu_1 - \mu_2$. This lower confidence bound is greater than 0; therefore, there is a difference in the two temperatures on the thickness of the photoresist.

(d) Draw dot diagrams to assist in interpreting the results from this experiment.



(e) Check the assumption of normality of the photoresist thickness.





There are no significant deviations from the normality assumptions.

(f) Find the power of this test for detecting an actual difference in means of 2.5 kÅ.

Minitab Output

```
Power and Sample Size

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)
Calculating power for mean 1 = mean 2 + difference
Alpha = 0.05  Sigma = 1.86

              Sample
Difference      Size   Power
       2.5         8  0.7056
```

(g) What sample size would be necessary to detect an actual difference in means of 1.5 kÅ with a power of at least 0.9?.

Minitab Output

```
Power and Sample Size

2-Sample t Test

Testing mean 1 = mean 2 (versus not =)
Calculating power for mean 1 = mean 2 + difference
Alpha = 0.05  Sigma = 1.86

              Sample  Target  Actual
Difference      Size   Power   Power
       1.5        34  0.9000  0.9060
```

This result makes intuitive sense. More samples are needed to detect a smaller difference.

2.32. Front housings for cell phones are manufactured in an injection molding process. The time the part is allowed to cool in the mold before removal is thought to influence the occurrence of a particularly troublesome cosmetic defect, flow lines, in the finished housing. After manufacturing, the housings are inspected visually and assigned a score between 1 and 10 based on their appearance, with 10 corresponding to a perfect part and 1 corresponding to a completely defective part. An experiment was conducted using two cool-down times, 10 seconds and 20 seconds, and 20 housings were evaluated at each level of cool-down time. All 40 observations in this experiment were run in random order. The data are shown below.

10 Seconds		20 Seconds	
1	3	7	6
2	6	8	9
1	5	5	5
3	3	9	7
5	2	5	4
1	1	8	6
5	6	6	8
2	8	4	5
3	2	6	8
5	3	7	7

(a) Is there evidence to support the claim that the longer cool-down time results in fewer appearance defects? Use $\alpha = 0.05$.

From the analysis shown below, there is evidence that the longer cool-down time results in fewer appearance defects.

Minitab Output

Two-Sample T-Test and CI: 10 seconds, 20 seconds

```
Two-sample T for 10 seconds vs 20 seconds

           N      Mean     StDev   SE Mean
10 secon  20      3.35      2.01      0.45
20 secon  20      6.50      1.54      0.34

Difference = mu 10 seconds - mu 20 seconds
Estimate for difference:  -3.150
95% upper bound for difference: -2.196
T-Test of difference = 0 (vs <): T-Value = -5.57  P-Value = 0.000  DF = 38
Both use Pooled StDev = 1.79
```

(b) What is the *P*-value for the test conducted in part (a)? From the *Minitab* output, $P = 0.000$

(c) Find a 95% confidence interval on the difference in means. Provide a practical interpretation of this interval.

From the *Minitab* output, $\mu_1 - \mu_2 \leq -2.196$. This lower confidence bound is less than 0. The two samples are different. The 20 second cooling time gives a cosmetically better housing.