CHAPTER 4

Section 4-2

4-1. a) $P(1 < X) = \int_1^{\infty} e^{-2x}dx = (-e^{-2x})\Big|_1^{\infty} = e^{-2} = 0.1353$

b) $P(1 < X < 2.5) = \int_1^{2.5} e^{-2x}dx = (-e^{-2x})\Big|_1^{2.5} = e^{-2} - e^{-5} = 0.1286$

c) $P(X = 3) = \int_3^{3} e^{-2x}dx = 0$

d) $P(X < 4) = \int_0^{4} e^{-2x}dx = (-e^{-2x})\Big|_0^{4} = 1 - e^{-8} = 0.9997$

e) $P(3 \le X) = \int_3^{\infty} e^{-2x}dx = (-e^{-2x})\Big|_3^{\infty} = e^{-6} = 0.0025$

f) $P(x < X) = \int_x^{\infty} e^{-2x}dx = (-e^{-2x})\Big|_x^{\infty} = e^{-2x} = 0.10$

Then, 2x = –ln(0.10) = 2.3 ⇒ x = 1.15

g) $P(X \le x) = \int_0^{x} e^{-2x}dx = (-e^{-2x})\Big|_0^{x} = 1 - e^{-2x} = 0.10$

Then, x = $\frac{-\ln(0.9)}{2} = 0.0527$

4-2. a) $P(X < 2) = \int_0^{2} \frac{3(8x - x^2)}{256}dx = \left(\frac{3x^2}{64} - \frac{x^3}{256}\right)\Big|_0^{2} = \left(\frac{3}{16} - \frac{1}{32}\right) - 0 = 0.1563$

b) $P(X < 9) = \int_0^{8} \frac{3(8x - x^2)}{256}dx = \left(\frac{3x^2}{64} - \frac{x^3}{256}\right)\Big|_0^{8} = (3-2) - 0 = 1$

c) $P(2 < X < 4) = \int_2^{4} \frac{3(8x - x^2)}{256}dx = \left(\frac{3x^2}{64} - \frac{x^3}{256}\right)\Big|_2^{4} = \left(\frac{3}{4} - \frac{1}{4}\right) - \left(\frac{3}{16} - \frac{1}{32}\right) = 0.3438$

d) $P(X > 6) = \int_6^{8} \frac{3(8x - x^2)}{256}dx = \left(\frac{3x^2}{64} - \frac{x^3}{256}\right)\Big|_6^{8} = (3-2) - \left(\frac{27}{16} - \frac{27}{32}\right) = 0.1563$

e) $P(X < x) = \int_0^{x} \frac{3(8u - u^2)}{256}du = \left(\frac{3u^2}{64} - \frac{u^3}{256}\right)\Big|_0^{x} = \left(\frac{3x^2}{64} - \frac{x^3}{256}\right) - 0 = 0.95$

Then, $x^3 - 12x^2 + 243.2 = 0$, and $x = 6.9172$

4-3. a) $P(X < 0) = \int_{-\pi/2}^{0} \cos x dx = \sin x \Big|_{-\pi/2}^{0} = 0 - (-1) = 1$

b) $P(X < -\pi/4) = \int_{-\pi/2}^{-\pi/4} \cos x dx = \sin x \Big|_{-\pi/2}^{-\pi/4} = -0.7071 - (-1) = 0.2929$

c) $P(-\pi/4 < X < \pi/4) = \int_{-\pi/4}^{\pi/4} \cos x dx = \sin x \Big|_{-\pi/4}^{\pi/4} = 0.7071 - (-0.7071) = 1.4142$

d) $P(X > -\pi/4) = \int_{-\pi/4}^{\pi/2} \cos x dx = \sin x \Big|_{-\pi/4}^{\pi/2} = 1 - (-0.7071) = 1.7071$

e) $P(X < x) = \int_{-\pi/2}^{x} \cos x dx = \sin x \Big|_{-\pi/2}^{x} = \sin x - (-1) = 0.95$

Then, sin x = -0.05, and x = -0.05 radians.

4-4. a) $P(X < 2) = \int_{1}^{2} \frac{2}{x^3} dx = (\frac{-1}{x^2}) \Big|_{1}^{2} = (\frac{-1}{4}) - (-1) = 0.75$

b) $P(X > 3) = \int_{3}^{\infty} \frac{2}{x^3} dx = (\frac{-1}{x^2}) \Big|_{3}^{\infty} = 0 - (\frac{-1}{9}) = 0.11$

c) $P(4 < X < 8) = \int_{4}^{8} \frac{2}{x^3} dx = (\frac{-1}{x^2}) \Big|_{4}^{8} = (\frac{-1}{64}) - (\frac{-1}{16}) = 0.0469$

d) $P(X < 4 or X > 8) = 1 - P(4 < X < 8)$. From part (c), $P(4 < X < 8)$ = 0.0469. Therefore, $P(X < 4 or X > 8)$ = 1 – 0.0469 = 0.9531

e) $P(X < x) = \int_{1}^{x} \frac{2}{x^3} dx = (\frac{-1}{x^2}) \Big|_{1}^{x} = (\frac{-1}{x^2}) - (-1) = 0.95$

Then, x^2 = 20, and x = 4.4721

4-5. a) $P(X < 4) = \int_{3}^{4} \frac{x}{7} dx = \frac{x^2}{14} \Big|_{3}^{4} = \frac{4^2 - 3^2}{14} = 0.5$ because $f_X(x) = 0$ for x < 3.

b) $P(X > 3.5) = \int_{3.5}^{5} \frac{x}{7} dx = \frac{x^2}{14} \Big|_{3.5}^{5} = \frac{5^2 - 3.5^2}{14} = 0.9107$ because $f_X(x) = 0$ for x > 5.

c) $P(4 < X < 5) = \int_{4}^{5} \frac{x}{7} dx = \frac{x^2}{14} \Big|_{4}^{5} = \frac{5^2 - 4^2}{14} = 0.6429$

d) $P(X < 4.5) = \int_{3}^{4.5} \frac{x}{7} dx = \frac{x^2}{14} \Big|_{3}^{4.5} = \frac{4.5^2 - 3^2}{14} = 0.8036$

e) $P(X > 4.5) + P(X < 3.5) = \int_{4.5}^{5} \frac{x}{7} dx + \int_{3}^{3.5} \frac{x}{7} dx = \frac{x^2}{14} \Big|_{4.5}^{5} + \frac{x^2}{14} \Big|_{3}^{3.5} = \frac{5^2 - 4.5^2}{14} + \frac{3.5^2 - 3^2}{14} = 0.5714$

4-6. a) $P(1 < X) = \int_5^\infty e^{-(x-5)}dx = -e^{-(x-5)}\Big|_5^\infty = 1$, because $f_X(x) = 0$ for x < 5. This can also be obtained from the fact that $f_X(x)$ is a probability density function for 5 < x.

b) $P(2 \le X \le 5) = \int_5^5 e^{-(x-5)}dx = -e^{-(x-5)}\Big|_5^5 = 0$

c) $P(5 < X) = 1 - P(X \le 5)$. From part b, $P(X \le 5) = 0$. Therefore, $P(5 < X) = 1$.

d) $P(8 < X < 12) = \int_8^{12} e^{-(x-5)}dx = -e^{-(x-5)}\Big|_8^{12} = e^{-3} - e^{-7} = 0.0489$

e) $P(X < x) = \int_5^x e^{-(x-5)}dx = -e^{-(x-5)}\Big|_5^x = 1 - e^{-(x-5)} = 0.85$

Then, x = 5 – ln(0.15) = 6.897

4-7. a) $P(0 < X) = 0.5$, by symmetry.

b) $P(0.5 < X) = \int_{0.5}^1 2x^2 dx = \frac{2}{3}x^3\Big|_{0.5}^1 = 0.6667 - 0.0833 = 0.5834$

c) $P(-0.5 \le X \le 0.5) = \int_{-0.5}^{0.5} 2x^2 dx = \frac{2}{3}x^3\Big|_{-0.5}^{0.5} = 0.167$

d) P(X < –2) = 0

e) P(X < 0 or X > –0.5) = 1

f) $P(x < X) = \int_x^1 2x^2 dx = \frac{2}{3}x^3\Big|_x^1 = 0.667 - 0.667x^3 = 0.05$

Then, x = 0.925

4-8. a) $P(X > 5000) = \int_{5000}^\infty \frac{e^{\frac{-x}{1000}}}{1000}dx = -e^{\frac{-x}{1000}}\Big|_{5000}^\infty = e^{-5} = 0.0067$

b) $P(1000 < X < 2000) = \int_{1000}^{2000} \frac{e^{\frac{-x}{1000}}}{1000}dx = -e^{\frac{-x}{1000}}\Big|_{1000}^{2000} = e^{-1} - e^{-2} = 0.233$

c) $P(X < 1000) = \int_0^{1000} \frac{e^{\frac{-x}{1000}}}{1000}dx = -e^{\frac{-x}{1000}}\Big|_0^{1000} = 1 - e^{-1} = 0.6321$

d) $P(X < x) = \int_0^x \frac{e^{\frac{-x}{1000}}}{1000}dx = -e^{\frac{-x}{1000}}\Big|_0^x = 1 - e^{-x/1000} = 0.10$.

Then, $e^{-x/1000} = 0.9$, and x = –1000 ln 0.9 = 105.36.

4-9. a) $P(X > 25) = \int_{25}^{25.25} 2.5dx = 2.5x\Big|_{25}^{25.25} = 0.625$

b) $P(X > x) = 0.85 = \int_{x}^{25.25} 2.5dx = 2.5x\Big|_{x}^{25.25} = 63.125 - 2.5x$

Then, 2.5x = 62.275 and x = 31.14.

4-10. a) $P(X < 74.7) = \int_{74.6}^{74.7} 1.3dx = 1.3x\Big|_{74.6}^{74.7} = 0.13$

b) P(X < 74.8 or X > 75.2) = P(X < 74.8) + P(X > 75.2) because the two events are mutually exclusive.

$$P(X < 74.8) = \int_{74.6}^{74.8} 1.3dx = 1.3x\Big|_{74.6}^{74.8} = 1.3\times 0.2 = 0.26$$

and

$$P(X < 75.2) = \int_{75.2}^{75.4} 1.3dx = 1.3x\Big|_{75.2}^{75.4} = 1.3\times 0.2 = 0.26$$

The result is 0.26 + 0.26 = 0.52.

c) $P(74.7 < X < 75.3) = \int_{74.7}^{75.3} 1.3dx = 1.3x\Big|_{74.7}^{75.3} = 1.3(0.6) = 0.780$

4-11. a) P(X < 2.25 or X > 2.75) = P(X < 2.25) + P(X > 2.75) because the two events are mutually exclusive. Then, P(X < 2.25) = 0 and

P(X > 2.75) = $\int_{2.75}^{2.9} 2dx = 2(0.15) = 0.30$.

b) If the probability density function is centered at 2.60 meters, then $f_X(x) = 2$ for 2.3 < x < 2.9 and all rods will meet specifications.

4-12. a) $P(X < 90) = 0$ because the pdf is not defined in the range $(-\infty, 90)$.

b)

$$P(100 < X \le 300) = \int_{100}^{300} (-5.56\times 10^{-4} + 5.56\times 10^{-6}x)dx = (-5.56\times 10^{-4}x + 2.78\times 10^{-6}x^2)\Big|_{100}^{300}$$

$$= (-5.56\times 10^{-4}\times 300 + 2.78\times 10^{-6}\times 300^2) - (-5.56\times 10^{-4}\times 100 + 2.78\times 10^{-6}\times 100^2)$$

$$= 0.111$$

c)

$$P(X > 800) = \int_{800}^{1000} (4.44\times 10^{-3} - 4.44\times 10^{-6}x)dx = (4.44\times 10^{-3}x - 2.22\times 10^{-6}x^2)\Big|_{800}^{1000}$$

$$= (4.44\times 10^{-3}\times 10^3 - 2.22\times 10^{-6}\times 10^6) - (4.44\times 10^{-3}\times 800 - 2.22\times 10^{-6}\times 800^2)$$

$$= 0.0888 \cong 0.09$$

d) Find a such that $P(X > a) = 0.1$

$$P(X > a) = \int_a^{1000} (4.44\times10^{-3} - 4.44\times10^{-6}x)dx = (4.44\times10^{-3}x - 2.22\times10^{-6}x^2)\Big|_a^{1000} = 0.1$$

$$(4.44\times10^{-3}\times10^3 - 2.22\times10^{-6}\times10^6) - (4.44\times10^{-3}\times a - 2.22\times10^{-6}\times a^2) = 0.1$$

$$(2.22) - (4.44\times10^{-3}\times a - 2.22\times10^{-6}\times a^2) = 0.1$$

Then, $a \cong 787.76$

Section 4-3

4-13. a) P(X<2.8) = P(X ≤ 2.8) because X is a continuous random variable.
Then, P(X<2.8) = F(2.8) = 0.3(2.8) = 0.84.

b) $P(X > 1.5) = 1 - P(X \le 1.5) = 1 - 0.3(1.5) = 0.55$

c) $P(X < -2) = F_X(-2) = 0$

d) $P(X > 6) = 1 - F_X(6) = 0$

4-14. a) $P(X < 1.7) = P(X \le 1.7) = F_X(1.7)$ because X is a continuous random variable. Then,
$F_X(1.7) = 0.2(1.7) + 0.5 = 0.84$

b) $P(X > -1.5) = 1 - P(X \le -1.5) = 1 - 0.2 = 0.8$

c) P(X < -2) = 0.1

d) $P(-1 < X < 1) = P(-1 < X \le 1) = F_X(1) - F_X(-1) = 0.7 - 0.3 = 0.4$

4-15. Now, $f(x) = e^{-2x}$ for 0 < x and $F_X(x) = \int_0^x e^{-2x}dx = -e^{-2x}\Big|_0^x = 1 - e^{-2x}$

for 0 < x. Then, $F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-2x}, & x > 0 \end{cases}$

4-16. Now, $f(x) = \dfrac{3(8x - x^2)}{256}$ for 0 < x < 8 and

$$F_X(x) = \int_0^x \frac{3(8u - u^2)}{256}du = \left(\frac{3u^2}{64} - \frac{u^3}{256}\right)\Bigg|_0^x = \frac{3x^2}{64} - \frac{x^3}{256} \text{ for } 0 < x.$$

Then, $F_X(x) = \begin{cases} 0, \; x \le 0 \\ \dfrac{3x^2}{64} - \dfrac{x^3}{256}, \; 0 \le x < 8 \\ 1, \; x \ge 8 \end{cases}$

4-17. Now, $f(x) = \cos x$ for -π/2 < x < π/2 and

$$F_X(x) = \int_{-\pi/2}^{x} \cos u\,du = \sin x\Big|_{-\pi/2}^{x} = \sin x + 1$$

Then, $F_X(x) = \begin{cases} 0, & x \le -\pi/2 \\ \sin x + 1, & -\pi/2 \le x < \pi/2 \\ 1, & x \ge \pi/2 \end{cases}$

4-18. Now, $f(x) = \dfrac{2}{x^3}$ for x > 1 and

$$F_X(x) = \int_1^x \frac{2}{u^3} du = \left(\frac{-1}{u^2}\right)\Bigg|_1^x = \left(\frac{-1}{x^2}\right) + 1$$

Then, $F_X(x) = \begin{cases} 0, & x \le 1 \\ 1 - \dfrac{1}{x^2}, & x > 1 \end{cases}$

4-19. Now, $f(x) = x/7$ for 3 < x < 5 and $F_X(x) = \int_3^x \frac{u}{7} du = \frac{u^2}{14}\Big|_3^x = \frac{x^2 - 9}{14}$

for 0 < x. Then, $F_X(x) = \begin{cases} 0, & x < 3 \\ \dfrac{x^2 - 9}{14}, & 3 \le x < 5 \\ 1, & x \ge 5 \end{cases}$

4.20. Now, $f(x) = \dfrac{e^{-x/1000}}{1000}$ for 0 < x and

$$F_X(x) = \frac{1}{1000} \int_0^x e^{-x/1000} dy = -e^{-y/1000} \Big|_0^x = 1 - e^{-x/1000}$$ for 0 < x.

Then, $F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-x/1000}, & x > 0 \end{cases}$

P(X>3000) = 1- P(X ≤ 3000) = 1- F(3000) = $e^{-3000/1000}$ = 0.5

4-21. Now, f(x) = 2 for 2.3 < x < 2.9 and $F(x) = \int_{2.3}^x 2dy = 2x - 4.6$

for 2.3 < x < 2.9. Then,

$$F(x) = \begin{cases} 0, & x < 2.3 \\ 2x - 4.6, & 2.3 \le x < 2.9 \\ 1, & 2.9 \le x \end{cases}$$

$P(X > 2.7) = 1 - P(X \le 2.7) = 1 - F(2.7) = 1 - 0.8 = 0.2$ because X is a continuous random variable.

4-22. Now, $f(x) = \frac{e^{-x/10}}{10}$ for 0 < x and

$$F_X(x) = 1/10\int_0^x e^{-x/10}dx = -e^{-x/10}\Big|_0^x = 1 - e^{-x/10} \text{ for } 0 < x.$$

Then, $F_X(x) = \begin{cases} 0, & x \le 0 \\ 1 - e^{-x/10}, & x > 0 \end{cases}$

a) $P(X<30) = F(30) = 1 - e^{-3} = 1 - 0.0498 = 0.9502$

b) $1/10\int_{15}^{30} e^{-x/10}dx = e^{-1.5} - e^{-3} = 0.173343$

c) $P(X_1>40) + P(X_1<40 \text{ and } X_2>40) = e^{-4} + (1 - e^{-4})\, e^{-4} = 0.0363$

d) $P(20<X<40) = F(40) - F(20) = e^{-2} - e^{-4} = 0.117$

4-23. $F(x) = \int_0^x 1.5x\,dx = \frac{1.5x^2}{2}\Big|_0^x = 0.75x^2$ for 0 < x < 2. Then,

$$F(x) = \begin{cases} 0, & x < 0 \\ 0.75x^2, & 0 \le x < 2 \\ 1, & 2 \le x \end{cases}$$

4-24. $f_X(x) = 2e^{-3x}, \; x > 0$

4-25. $f(x) = \begin{cases} 0.25, & 0 < x < 4 \\ 0.04, & 4 \le x < 9 \end{cases}$

4-26. $f_X(x) = \begin{cases} 0.5, & -2 < x < 1 \\ 0.75, & 1 \le x < 1.5 \end{cases}$

<u>Section 4-4</u>

4-27. $E(X) = \int_0^4 0.3x\,dx = 0.3\frac{x^2}{2}\Big|_0^4 = 2.4$

$$V(X) = \int_0^4 0.3(x-2.4)^2 dx = 0.3\frac{(x-2.4)^3}{3}\Big|_0^4 = 0.4096 + 1.3824 = 1.792$$

4-28. $E(X)=\int_0^4 0.13x^2dx = 0.13\left.\frac{x^3}{3}\right|_0^4 = 2.7733$

$V(X)=\int_0^4 0.13x(x-\tfrac{8}{3})^2dx = 0.13\int_0^4 (x^3-\tfrac{16}{3}x^2+\tfrac{64}{9}x)dx$

$=0.13(\tfrac{x^4}{4}-\tfrac{16}{3}\tfrac{x^3}{3}+\tfrac{64}{9}\cdot\tfrac{1}{2}x^2)\Big|_0^4 = 0.92444$

4-29. $E(X)=\int_{-1}^1 2x^3dx = 2\left.\frac{x^4}{4}\right|_{-1}^1 = 0$

$V(X)=\int_{-1}^1 2x^2(x-0)^2dx = 2\int_{-1}^1 x^4dx$

$=2\frac{x^5}{5}\Big|_{-1}^1 = 0.8$

4-30. $E(X)=\int_3^5 x\frac{x}{4}dx = \left.\frac{x^3}{12}\right|_3^5 = \frac{5^3-3^3}{12} = 8.167$

$V(X)=\int_3^5 (x-8.167)^2\frac{x}{4}dx = \int_3^5\left(\frac{x^3}{4}-\frac{16.334x^2}{4}+\frac{66.6999x}{4}\right)dx$

$=\frac{1}{4}\left(\frac{x^4}{4}-\frac{18.334x^3}{3}+\frac{66.6999x^2}{2}\right)\Bigg|_3^5 = 34.0055$

4-31. $E(X)=\int_0^8 x\frac{3(8x-x^2)}{256}dx = (\frac{x^3}{32}-\frac{3x^4}{1024})\Big|_0^8 = (16-12)-0 = 4$

$V(X)=\int_0^8 (x-4)^2\frac{3(8x-x^2)}{256}dx = \int_0^8\left(\frac{-3x^4}{256}+\frac{3x^3}{16}-\frac{15x^2}{16}+\frac{3x}{2}\right)dx$

$V(X)=\left(\frac{-3x^5}{1280}+\frac{3x^4}{64}-\frac{5x^3}{16}+\frac{3x^2}{4}\right)\Bigg|_0^8 = (\frac{-384}{5}+192-160+48) = 3.2$

4-32. $E(X)=\int_{2.3}^{2.9} 2xdx = x^2\Big|_{2.3}^{2.9} = 2.9^2-2.3^2 = 3.12$

$V(X)=\int_{2.3}^{2.9} 2(x-3.12)^2dx = \int_{2.3}^{2.9}(2x^2-12.48x+19.4688)dx = (\frac{2}{3}x^3-6.24x^2+19.4688x)\Big|_{2.3}^{2.9} = 0.36$

4-33. $E(X)=\int_1^\infty x*\frac{3}{2}x^{-3}dx = -\frac{3}{2}x^{-1}\Big|_1^\infty = \frac{3}{2}$

4-34. a)

$$E(X)=\int_{1710}^{1720} x0.1dx = 0.05x^2\Big|_{1710}^{1720} = 1715$$

$$V(X)=\int_{1710}^{1720}(x-1715)^2 0.1dx = 0.1\frac{(x-1715)^3}{3}\Bigg|_{1710}^{1720} = 8.333$$

$$Therefore, \quad \sigma_x = \sqrt{V(X)} = 2.887$$

b) Clearly, centering the process at the center of the specifications results in the greatest proportion of cables within specifications.

$$P(1705 < X < 1715) = P(1710 < X < 1715) = \int_{1710}^{1715} 0.1dx = 0.1x\Big|_{1710}^{1715} = 0.5$$

4-35. a) $E(X)=\int_{100}^{120} x\frac{500}{x^2}dx = 500\ln x\Big|_{100}^{120} = 91.16$

$$V(X)=\int_{100}^{120}(x-91.16)^2\frac{500}{x^2}dx = 500\int_{100}^{120} 1-\frac{2(91.16)}{x}+\frac{(91.16)^2}{x^2}dx$$

$$= 500(x-182.32\ln x - 91.16^2x^{-1})\ \Big|_{100}^{120} = 304.688$$

b.) Average cost per part = \$0.50*91.16 = \$45.58.

4-36. (a) $E(X)=\int_{0.5}^{35} xf(x)dx = \int_{0.5}^{35}\frac{70}{69x}dx = \frac{70}{69}\ln x\,|\,\begin{matrix}35\\0.5\end{matrix} = 4.3101$

$$E(X^2)=\int_{0.5}^{35} x^2 f(x)dx = \int_{0.5}^{35}\frac{70}{69}dx = 35$$

Var(X) = $E(X^2)-(EX)^2 = 35 - \ 18.5770 = 16.423$

(b) 3*4.3101 = 12.9303

(c) $P(X>25)=\int_{25}^{35} f(x)dx = 0.0116$

4-37. a) $E(X)=\int_{5}^{\infty} x10e^{-10(x-5)}dx$.

Using integration by parts with $u = x$ and $dv = 10e^{-10(x-5)}dx$, we obtain

$$E(X) = -\,xe^{-10(x-5)}\Big|_5^{\infty} + \int_5^{\infty} e^{-10(x-5)}dx = 5 - \frac{e^{-10(x-5)}}{10}\Bigg|_5^{\infty} = 5.1$$

Now, $V(X)=\int_5^{\infty}(x-5.1)^2 10e^{-10(x-5)}dx$. Using the integration by parts with $u=(x-5.1)^2$ and

$dv = 10e^{-10(x-5)}$, we obtain $V(X) = -(x-5.1)^2e^{-10(x-5)}\Big|_5^{\infty} + 2\int_5^{\infty}(x-5.1)e^{-10(x-5)}dx$.

From the definition of E(X) the integral above is recognized to equal 0. Therefore,
$V(X) = (5-5.1)^2 = 0.01$.

b) $P(X > 5.1) = \int_{5.1}^{\infty} 10e^{-10(x-5)}dx = -e^{-10(x-5)}\Big|_{5.1}^{\infty} = e^{-10(5.1-5)} = 0.3679$

Section 4-5

4-38. a) E(X) = (5.5 + 2.5)/2 = 4

$$V(X) = \frac{(5.5-2.5)^2}{12} = 0.75, \text{ and } \sigma_x = \sqrt{0.75} = 0.866$$

b) $P(X < 2.5) = \int_{2.5}^{2.5} 0.25dx = 0.25x\Big|_{2.5}^{2.5} = 0$

c) $$F(x) = \begin{cases} 0, & x < 2.5 \\ 0.25x - 0.375, & 2.5 \le x < 5.5 \\ 1, & 5.5 \le x \end{cases}$$

4-39. a) E(X) = (-2+3)/2 = 0.5,

$$V(X) = \frac{(3-(-2))^2}{12} = 2.083, \; and \; \sigma_x = 1.443$$

b) $P(-x < X < x) = \int_{-x}^{x} \tfrac{1}{5} dt = 0.2t\Big|_{-x}^{x} = 0.2(2x) = 0.4x$,

$0.4x = 0.9$. Therefore, x should equal 2.25.

c) $$F(x) = \begin{cases} 0, & x < -2 \\ 0.2x + 0.4, & -2 \le x < 3 \\ 1, & 3 \le x \end{cases}$$

4.40. a) *f*(*x*) = 2.0 for 49.75 < x < 50.25.

E(X) = (50.25 + 49.75)/2 = 50.0,

$$V(X) = \frac{(50.25-49.75)^2}{12} = 0.0208, \text{ and } \sigma_x = 0.144.$$

b) $F(x) = \int_{49.75}^{x} 2.0dy$ for 49.75 < x < 50.25. Therefore,

$$F(x) = \begin{cases} 0, & x < 49.75 \\ 2x - 99.5, & 49.75 \le x < 50.25 \\ 1, & 50.25 \le x \end{cases}$$

c) $P(X < 50.1) = F(50.1) = 2(50.1) - 99.5 = 0.7$