

# CHAPTER ONE

## Solutions for Section 1.1

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### PROBLEMS

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1. The dependent variable is  $N$ , the number of napkins used, the output of the function. The independent variable is  $C$ , the number of customers, the input of the function.
2. The dependent variable is  $P$ , the output of the function. The independent variable is  $s$ , the input of the function.
3.  $m = f(v)$ .
4.  $w = f(c)$ .
5. (a) The independent variable is  $c$ , and the dependent variable is  $T$ .  
(b) (i) This tells us that the cost of taking 3 credits is \$3000.  
(ii) This tells us that the cost of taking 12 credits is the same as the cost of taking 16 credits.
6. The point  $P$  marks the input value for  $B$ , \$20, and the point  $Q$  marks the output value for  $T$ , \$4. However, neither of these points tells us that the function connects the input with the output. The single point  $R$  tells us that  $f(20) = 4$ .
7. (a) (i) The point on the graph with  $x = 0$  is  $(0, 5)$ , so  $f(0) = 5$ .  
(ii) It is not as easy to see the point on the graph where  $x = 10$ . It is close to  $(10, 2)$ , so we estimate that  $f(10)$  is about 2.  
(iii) Using the graph of  $f$ , we estimate that  $f(16)$  is approximately 1.  
(b) (i) Evaluating  $f(x)$  at  $x = 0$ , we get

$$f(0) = 5 - \sqrt{0} = 5.$$

This is consistent with our answer in part (a).

- (ii) Evaluating  $f(x)$  at  $x = 10$ , we get

$$f(10) = 5 - \sqrt{10} = 1.838.$$

This is very close to our estimate for  $f(10)$  in part (a).

- (iii) Evaluating  $f(x)$  at  $x = 16$ , we get

$$f(16) = 5 - \sqrt{16} = 5 - 4 = 1.$$

This is consistent with our answer in part (a).

8. To evaluate when  $x = -7$ , we substitute  $-7$  for  $x$  in the function, giving  $f(-7) = -\frac{7}{2} - 1 = -\frac{9}{2}$ .
9. To evaluate when  $x = -7$ , we substitute  $-7$  for  $x$  in the function, giving  $f(-7) = (-7)^2 - 3 = 49 - 3 = 46$ .
10. We have  
(a)

$$\begin{aligned} g(2) &= (12 - 2)^2 - (2 - 1)^3 \\ &= 10^2 - 1^3 \\ &= 100 - 1 \\ &= 99. \end{aligned}$$

- (b)

$$\begin{aligned} g(5) &= (12 - 5)^2 - (5 - 1)^3 \\ &= 7^2 - 4^3 \\ &= 49 - 64 \\ &= -15. \end{aligned}$$

2 Chapter One /SOLUTIONS

(c)

$$\begin{aligned} g(0) &= (12 - 0)^2 - (0 - 1)^3 \\ &= 12^2 - (-1)^3 \\ &= 144 - (-1) \\ &= 145. \end{aligned}$$

(d)

$$\begin{aligned} g(-1) &= (12 - (-1))^2 - (-1 - 1)^3 \\ &= 13^2 - (-2)^3 \\ &= 169 - (-8) \\ &= 177. \end{aligned}$$

11. To evaluate, we substitute the input value of the function for  $x$  in  $f(x) = 2x^2 + 7x + 5$ .

(a) We substitute 3 for  $x$ :

$$f(3) = 2(3)^2 + 7(3) + 5 = 2 \cdot 9 + 21 + 5 = 18 + 26 = 44.$$

(b) We substitute  $a$  for  $x$ :

$$f(a) = 2(a)^2 + 7(a) + 5 = 2a^2 + 7a + 5.$$

(c) We substitute  $2a$  for  $x$ :

$$f(2a) = 2(2a)^2 + 7(2a) + 5 = 2 \cdot (4a^2) + 14a + 5 = 8a^2 + 14a + 5.$$

(d) We substitute  $-2$  for  $x$ :

$$f(-2) = 2(-2)^2 + 7(-2) + 5 = 2 \cdot 4 - 14 + 5 = 8 - 9 = -1.$$

12. We have

$$f(0) = \frac{2 \cdot 0 + 1}{3 - 5 \cdot 0} = \frac{1}{3}.$$

13. We have

$$g(0) = \frac{1}{\sqrt{0^2 + 1}} = \frac{1}{\sqrt{1}} = 1.$$

14. We have

$$g(-1) = \frac{1}{\sqrt{(-1)^2 + 1}} = \frac{1}{\sqrt{1 + 1}} = \frac{1}{\sqrt{2}}.$$

This can be rewritten as

$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}.$$

15. We have

$$f(10) = \frac{2 \cdot 10 + 1}{3 - 5 \cdot 10} = \frac{21}{-47} = -\frac{21}{47}.$$

16. We have

$$f(1/2) = \frac{2 \cdot \frac{1}{2} + 1}{3 - 5 \cdot \frac{1}{2}} = \frac{1 + 1}{3 - \frac{5}{2}} = \frac{2}{\frac{6}{2} - \frac{5}{2}} = \frac{2}{\frac{1}{2}} = 4.$$

17. We have

$$g(\sqrt{8}) = \frac{1}{\sqrt{(\sqrt{8})^2 + 1}} = \frac{1}{\sqrt{8+1}} = \frac{1}{\sqrt{9}} = \frac{1}{3}.$$

18. We have  $h(r) = 10 - 3r$ .

19. We have

$$\begin{aligned} h(2u) &= 10 - 3 \cdot 2u \\ &= 10 - 6u. \end{aligned}$$

20. We have

$$\begin{aligned} h(k-3) &= 10 - 3(k-3) \\ &= 10 - 3k + 9 \\ &= 19 - 3k. \end{aligned}$$

21. We have

$$\begin{aligned} h(4-n) &= 10 - 3(4-n) \\ &= 10 - 12 + 3n \\ &= -2 + 3n. \end{aligned}$$

22. We have

$$\begin{aligned} h(5t^2) &= 10 - 3 \cdot 5t^2 \\ &= 10 - 15t^2. \end{aligned}$$

23. We have

$$\begin{aligned} h(4-t^3) &= 10 - 3(4-t^3) \\ &= 10 - 12 + 3t^3 \\ &= -2 + 3t^3. \end{aligned}$$

24. We have

$$\begin{aligned} f(1-x) &= 1 - (1-x)^2 - (1-x) && \text{Applying formula for } f \\ &= 1 - (1-2x+x^2) - (1-x) && \text{Expanding} \\ &= 1 - 1 + 2x - x^2 - 1 + x && \text{Clearing parentheses} \\ &= -x^2 + 3x - 1. \end{aligned}$$

25. (a) Since  $f(T)$  gives the volume at temperature  $T$ , the 40 is the temperature, so its units are degrees Celsius ( $^{\circ}\text{C}$ ). Similarly, the 3 is the volume, so its units are liters.

(b) Since  $f(40) = 3$ , we know that when  $T = 40$  that  $f(T) = 3$ . Since  $T = 40$  at  $40^{\circ}\text{C}$ , the volume is 3 liters at  $40^{\circ}\text{C}$ .

26. Since the tax is  $0.06P$ , the total cost would be the price of the item plus the tax, or

$$C = P + 0.06P = 1.06P.$$

4 Chapter One /SOLUTIONS

27. (a) Since  $g(y)$  is 5 when  $y = 0$ , we have  $g(0) = 5$ .  
 (b) Since  $y = -5$  when  $g(y) = 0$ , we have  $g(-5) = 0$ .  
 (c)  $g(-5) = 0$   
 (d) Since  $y = -10$  when  $g(y) = -5$ , we have  $g(-10) = -5$ .
28. (a) Since  $h(t)$  is  $-2$  when  $t = 0$ , we have  $h(0) = -2$ .  
 (b) There are two  $t$  values leading to  $h(t) = 0$ , namely  $t = -2$  and  $t = 3$ . So  $h(-2) = 0$  and  $h(3) = 0$ .  
 (c)  $h(-2) = 0$   
 (d) There are two  $t$  values leading to  $h(t) = -2$ , namely  $t = 0$  and  $t = 2$ . So  $h(0) = -2$  and  $h(2) = -2$ .
29. (a) Since  $h(t)$  is  $-2$  when  $t = 0$ , we have  $2h(0) = -4$ . Thus, we need to find the value(s) of  $t$  for which  $h(t) = -4$ . There is only one  $t$  value leads to this, namely  $t = -1$ .  
 (b) Since  $h(t)$  is  $-1$  when  $t = -3$  and  $h(t)$  is  $-2$  when  $t = 2$ , we have  $2h(-3) + h(2) = -4$ . Thus, we need to find the value(s) of  $t$  for which  $h(t) = -4$ . There is only one  $t$  value that leads to this, namely  $t = -1$ .  
 (c) Since  $h(t)$  is 0 when  $t = -2$ , we have  $h(-2) = 0$ . Thus, we need to find the value(s) of  $t$  for which  $h(t) = 0$ . There are two  $t$  values leading to  $h(t) = 0$ , namely  $t = -2$  and  $t = 3$ .  
 (d) Since  $h(t)$  is  $-1$  when  $t = 1$  and  $h(t)$  is  $-2$  when  $t = 2$ , we have  $h(1) + h(2) = -3$ . Thus, we need to find the value(s) of  $t$  for which  $h(t) = -3$ . There are no  $t$  values that lead to this.
30. (a) Since the point on the graph with  $x = 0$  is  $(0, 2)$ , we have  $f(0) = 2$ .  
 (b) Since  $f(x) = 0$  when  $x = -2$  and  $x = 1$ , we have  $f(-2) = 0$  and  $f(1) = 0$ .
31. (a) Since the point on the graph with  $r = 0$  is  $(0, 1)$ , we have  $h(0) = 1$ .  
 (b) Since  $h(r)$  never equals zero, there are no values of  $r$  for which  $h(r) = 0$ .
32. (a) Since the point on the graph with  $u = 0$  is  $(0, 4)$ , we have  $g(0) = 4$ .  
 (b) Since  $g(u) = 0$  when  $u = -2$  and  $u = 2$ , we have  $g(-2) = 0$  and  $g(2) = 0$ .
33. (a) When the company charges 15 dollars for one of its products, its total sales are 112,500 dollars.  
 (b) When the company charges  $a$  dollars for one of its products, its total sales are 0 dollars.  
 (c) When the company charges 1 dollar for one of its products, its total sales are  $b$  dollars.  
 (d) The number  $c$  is the total sales, in dollars, that the company receives when it charges a price of  $p$  dollars for one of its products.
34. (a) The car takes 111 feet to stop when traveling at 30 mph.  
 (b) The car takes 10 feet to stop when traveling at  $a$  mph.  
 (c) The car takes  $b$  feet to stop when traveling at 10 mph.  
 (d) The number  $s$  is the distance it takes the car to stop when traveling at  $v$  mph.
35. (a) In Figure 1.5, we see when  $s = 60$ , the point on the graph of the function has coordinates  $(60, 37)$ . This means when the car's speed is 60 miles per hour, its highway gas mileage is about 37 miles per gallon.  
 (b) In Figure 1.5, we see the point on the graph that has the greatest value for  $H$  is approximately  $(50, 40)$ . This means the car's maximum highway gas mileage is 40 miles per gallons. This occurs when the car's speed is 50 miles per hour.
36. (a) Because  $5\% = 0.05$ , the interest each year is  $0.05 \cdot (\text{Face value})$ , so the total interest after  $t$  years is given by

$$I = (\text{Number of years}) \cdot 0.05 \cdot (\text{Face value}) = t(0.05)p = 0.05tp.$$

- (b) The payout,  $P$ , is the sum of the face value and the total interest. That is,  $P = p + pI$ . Thus, since  $I = 0.05tp$ , we have

$$P = p + 0.05tp.$$

37. The variable is  $r$  and the constant is  $\pi$ .
38. The variable is  $x$  and the constants are  $b$  and  $m$ .
39. The variable is  $t$  and the constants are  $r$  and  $A$ .
40. The variable is  $r$  and the constants are  $A$  and  $t$ .
41. We are given that  $E$  is a function of  $m$ , so  $E$  is the dependent variable and  $m$  is the independent variable. The symbol  $c$  is a constant (which stands for the speed of light).

42. (a) In this situation, we regard the tip as a function of the variable  $B$  and regard  $r$  as a constant. If we call this function  $f$ , then we can write

$$\text{Tip} = f(B) = \frac{r}{100}B.$$

- (b) Here we regard the tip as a function of the variable  $r$  and regard  $B$  as a constant. If we call this function  $g$ , then

$$\text{Tip} = g(r) = \frac{r}{100}B.$$

43. (a) Since your possibilities are all apartments of 1000 square feet,  $A$  is the constant (always 1000), and  $d$  is the variable (since you can have the same-sized apartment at different distances from the station).  
 (b) Since your possibilities are all apartments 1 mile from the station,  $d$  is the constant (always 1), and  $A$  is the variable (since you can have apartments of different sizes that are the same distance from the station).  
 (c) Here, both  $d$  and  $A$  are variables. In choosing the apartment, you must decide whether you want to be farther from the station or to have a smaller area.
44.  $r(99)$  gives the average number of times the song is downloaded in a day at a price of \$0.99 per song.
45. The current price of the song is  $p_0$ , so  $r(p_0)$  is the average number of times the song is downloaded in a day at the current price.
46.  $p_0 - 10$  means 10 cents less than the current price of the song, so  $r(p_0 - 10)$  means the average number of times the song is downloaded per day at a price 10 cents lower than the current price.
47.  $p_0 - 10$  means 10 cents less than the current price of the song, so  $r(p_0 - 10)$  means the average number of times the song is downloaded per day at a price 10 cents lower than the current price. So

$$r(p_0 - 10) - r(p_0)$$

means

$$\begin{array}{ccc} \text{Number of daily downloads} & - & \text{Current number} \\ \text{at lower price} & & \text{of daily downloads} \end{array},$$

or the change in the average number of daily downloads if the price drops 10 cents from the current price.

48.  $r(p_0)$  is the average number of daily downloads of the song at the current price. Since there are 365 days in a year,  $365r(p_0)$  is the number times the song is downloaded in one year at current price.
49.  $0.80p_0$  is 80% of the current price, or 20% less than the current price. Thus,  $r(0.80p_0)$  is the average number times the song is downloaded per day after the price drops 20%.
50.  $r(p_0)$  is the average number of times the song is downloaded per day at the current price. Since there are 24 hours in one day,  $r(p_0)/24$  is the average number of downloads every hour.
51.  $r(p_0)$  is the average number of times the song is downloaded per day at its current price of  $p_0$  cents, so

$$\begin{aligned} p_0 \cdot r(p_0) &= \text{Price of the song} \cdot \text{Number of times downloaded in a day} \\ &= \text{Average daily sales for the music store.} \end{aligned}$$

We see that multiplying the average number of times the song is downloaded in a day by the price of the song gives the average daily sales of the music store—that is, the average amount of money it brings in each day.

52. We have

$$\begin{aligned} f(100) &= \frac{1}{2} \cdot 100(100 + 1) \\ &= 50(101) \\ &= 5050. \end{aligned}$$

6 Chapter One /SOLUTIONS

53. Working this in steps, we first find  $f(n+1)$ :

$$\begin{aligned} f(n+1) &= \frac{1}{2}(n+1)((n+1)+1) \\ &= \frac{1}{2}(n+1)(n+2). \end{aligned}$$

Now we subtract:

$$\begin{aligned} f(n+1) - f(n) &= \underbrace{\frac{1}{2}(n+1)(n+2)}_{f(n+1)} - \underbrace{\frac{1}{2}n(n+1)}_{f(n)} \\ &= \frac{1}{2}(n+1)((n+2)-n) \quad \text{factor out } \frac{1}{2}(n+1) \\ &= \frac{1}{2}(n+1) \cdot 2 \\ &= n+1. \end{aligned}$$

54. This is the number of people (in 1000s) infected by a strain whose open-air survival time is 3 minutes longer than the most common strain.
55.  $h(t_0)$  is the number eventually infected by the most common strain, and  $h(2t_0)$  is the number eventually infected by a strain that survives twice as long in open air. The ratio of these numbers gives how many times more people are infected by the longer-living strain.
56. This is the gas mileage of the car if its tire pressure is 90% of the recommended pressure, that is, if the tires are 10% underinflated.
57.  $f(P_0)$  is the mileage at the recommended pressure, and  $f(P_0 - 5)$  is the mileage if the tires are underinflated by 5 lbs/in<sup>2</sup>. So this is the difference in gas mileage between driving on properly inflated tires and tires underinflated by 5 lbs/in<sup>2</sup>.
58. We have

$$\begin{aligned} w(s-5) &= 4 - 7(s-5) \quad \text{because } w(s) = 4 - 7s \\ &= 4 - 7s + 35 \\ &= 39 - 7s. \end{aligned}$$

Thus,

$$\begin{aligned} 3w(s-5) &= 3 \underbrace{(39 - 7s)}_{w(s-5)} \\ &= 117 - 21s. \end{aligned}$$

59. We have

$$\begin{aligned} w(s^2 - 2) &= 4 - 7(s^2 - 2) \quad \text{because } w(s) = 4 - 7s \\ &= 4 - 7s^2 + 14 \\ &= 18 - 7s^2. \end{aligned}$$

Thus,

$$\begin{aligned} w(s^2 - 2) + 2 &= 18 - 7s^2 + 2 \quad \text{because } w(s^2 - 2) = 18 - 7s^2 \\ &= 20 - 7s^2. \end{aligned}$$

60. We have

$$\begin{aligned} w(7 - 4s) &= 4 - 7(7 - 4s) && \text{because } w(s) = 4 - 7s \\ &= 4 - (49 - 28s) \\ &= 4 - 49 + 28s \\ &= -45 + 28s. \end{aligned}$$

Thus,

$$\begin{aligned} 4w(7 - 4s) - 7 &= 4(-45 + 28s) - 7 && \text{because } w(7 - 4s) = -45 + 28s \\ &= -180 + 112s - 7 \\ &= -187 + 112s. \end{aligned}$$

## Solutions for Section 1.2

### IDENTIFYING ALGEBRAIC STRUCTURE

1. Increasing the value of  $x$  means we are adding a larger value to the constant  $A$ , so the value of the expression increases as  $x$  increases from 1.
2. Increasing the value of  $x$  means we are subtracting a larger value from the constant  $A$ , so the value of the expression decreases as  $x$  increases from 1.
3. Increasing the value of  $x$  means we are adding a larger value to the constant  $-A$ , so the value of the expression increases as  $x$  increases from 1.
4. Increasing the value of  $x$  means we are multiplying the positive constant  $A$  by larger value, so the value of the expression increases as  $x$  increases from 1.
5. Increasing the value of  $x$  means we are multiplying the negative number  $-A$  by a larger value. The value of the expression is therefore becoming more negative, so the expression decreases as  $x$  increases from 1.
6. Increasing the value of  $x$  means we are dividing the positive number  $A$  by a larger value, so the expression decreases as  $x$  increases from 1.
7. Increasing the value of  $x$  means we are multiplying the positive number  $1/A$  by a larger value, so the expression increases as  $x$  increases from 1.
8. Increasing the value of  $x$  means we are multiplying the negative number  $-A$  by a larger value. The value of the expression is therefore becoming more negative, so the expression decreases as  $x$  increases from 1.
9. Increasing the value of  $x$  means we are multiplying the negative number  $-A$  by a larger value. The value of the expression  $-Ax$  is therefore becoming more negative. Squaring a number that is becoming more and more negative results in a number that is becoming more and more positive, so the expression increases as  $x$  increases from 1.
10. Since the expression is equivalent to

$$A \cdot \frac{x}{x} = A,$$

increasing the value of  $x$  does not change the value of the expression. Note that we can divide by  $x$  because  $x$  is greater than or equal to 1, and thus cannot be 0.

11. Since the expression is equivalent to

$$A + (x - x) = A,$$

increasing the value of  $x$  does not change the value of the expression.

12. Subtracting  $x$  from 1 gives  $1 - x$ .
13. Subtracting 1 from  $x$  gives  $x - 1$ .

8 Chapter One /SOLUTIONS

14. Adding 3 to  $x$  gives  $x + 3$ . Doubling this result gives  $2(x + 3)$ .
15. Doubling  $x$  gives  $2x$ . Adding 3 to this result gives  $2x + 3$ .
16. Subtracting  $x$  from 1 gives  $1 - x$ . Doubling gives  $2(1 - x)$ . Adding 3 gives  $2(1 - x) + 3$ .
17. Subtracting 1 from  $x$  gives  $x - 1$ . Doubling gives  $2(x - 1)$ . Adding 3 gives  $2(x - 1) + 3$ .
18. Adding 3 to  $x$  gives  $x + 3$ . Subtracting the result from 1 gives  $1 - (x + 3)$ . Doubling gives  $2(1 - (x + 3))$ .
19. Adding 3 to  $x$  gives  $x + 3$ . Doubling gives  $2(x + 3)$ . Subtracting 1 from the result gives  $2(x + 3) - 1$ .

PROBLEMS

20. (a) We have  $G(50) = 17 - 0.05 \cdot 50 = 14.5$ , and  $G(100) = 17 - 0.05 \cdot 100 = 12$ . So  $G(50) > G(100)$ .  
 (b) Since the term  $0.05d$  is subtracted from 17 in the expression for  $G(d)$ , the value of  $G(d)$  is smaller when  $d$  is larger. This makes sense, since there is less gas left when you have driven further.
21. (a) We have  $B(35) = 30 - 480/35 = 16$  minutes, and  $B(45) = 30 - 480/45 = 19$  minutes, so  $B(45) > B(35)$ .  
 (b) Since  $v$  is in the denominator, the fraction  $480/v$  is smaller when  $v$  is larger. So when  $v$  is larger, a smaller amount is being subtracted from 30, giving a larger answer. This makes sense, since if you drive faster you have longer for breakfast.
22. (a) Table 1.1 shows values of the function, and Figure 1.1 shows the graph.

Table 1.1  
 Values of  
 $f(r) =$   
 $400/r$

$r$	$f(r)$
25	16
40	10
80	5
100	4
200	2

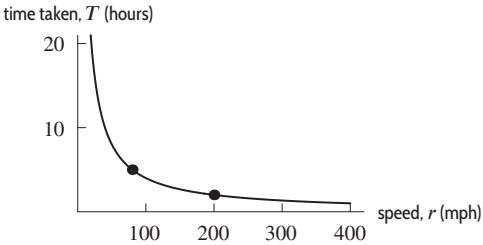


Figure 1.1: Graph of  $400/r$

- (b) The table and the graph show that larger values of the input give smaller values of the output. Since the output is read on the vertical axis, as one moves to the right on the horizontal axis, output values become smaller on the vertical axis and they get closer to 0.
- (c) In the fraction  $400/r$  as  $r$  gets larger  $400/r$  will get smaller. Since  $r$  is always positive,  $400/r$  will always be positive and will be getting closer to zero as  $r$  gets larger.
23. (a) For \$8.95, first we move the decimal point to the left to get 0.895, then double to get a tip of \$1.79. For \$23.70, we move the decimal point to the left to get 2.370, then double to get a tip of \$4.74. Pares' tip values of \$1.79 and \$23.70 are the same as the values we get when evaluating  $f(8.95)$  and  $f(23.70)$ , respectively.  
 (b) Moving the decimal point to the left is the same as multiplying by 0.1. So first Pares multiplies the bill by 0.1, then multiplies the result by 2. Her calculation of the tip is

$$2(0.1B).$$

We can simplify this expression by regrouping the multiplications:

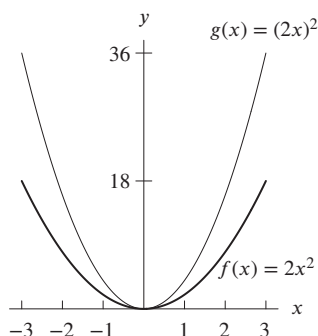
$$2(0.1B) = (2 \cdot 0.1)B = 0.2B.$$

This last expression for the tip is the same as the function  $f(B) = 0.2B$ .

24. (a) Abby's method starts with  $n$ , halves the result to get  $0.5n$ , then subtracts 10% of that to get  $0.5n - 0.1(0.5n)$ . Renato's method starts with  $n$ , subtracts 10% to get  $n - 0.1n$ , then takes half of the result to get  $0.5(n - 0.1n)$ .  
 (b) Abby's expression is equivalent to  $0.5n - 0.1(0.5n) = 0.5n - 0.05n = 0.45n$ , and Renato's is equivalent to  $0.5(n - 0.1n) = 0.5n - 0.5(0.1n) = 0.5n - 0.05n = 0.45n$ . Since the two expressions are both equivalent to  $0.45n$ , they are equivalent to each other and so the methods give the same answer.



25. (a) See Figure 1.2.



**Figure 1.2:** Comparing the graphs of  $f(x) = 2x^2$  and  $g(x) = (2x)^2$

- (b) The graphs are different, so the expressions are not equivalent. Although  $2x^2$  and  $(2x)^2$  look similar, they are different when you think about how they are constructed: the first one is 2 times the square of  $x$ , whereas the second is the square of 2 times  $x$ . So in fact the second expression is equivalent to  $4x^2$ .
26. (a) To compute  $f(x)$  we first subtract 2 from  $x$ , square the result, and then add 3 to it. So  $f(x)$  has the structure
- $$f(x) = (x - 2)^2 + 3 = \text{squared expression} + 3.$$
- (b) The squared expression  $(x - 2)^2$  is always greater or equal to zero since the square of a number is always greater or equal to zero. If  $x - 2 = 0$ , then the squared expression will equal zero. Adding a positive number to 3 results in a number that is greater than 3. Adding zero to 3 leaves it the same. So, the minimum value of  $f(x)$  is 3, which occurs when  $x = 2$ , and  $f(x)$  has no maximum value.
27. The two expressions are equivalent. We have

$$\begin{aligned} a + (2 - d) &= a + (2 + (-d)) && \text{rewriting subtraction as addition} \\ &= (a + 2) + (-d) && \text{regrouping addition} \\ &= (a + 2) - d && \text{rewriting addition as subtraction} \end{aligned}$$

28. Equivalent, since

$$3(z + w) = z + w + z + w + z + w$$

so you have 3  $z$ 's and 3  $w$ 's.

29. Not equivalent; for  $a = 2, b = 1$ , we have  $(a - b)^2 = (2 - 1)^2 = 1$ , but  $a^2 - b^2 = 2^2 - 1^2 = 3$ .
30. Not equivalent; for  $a = 3, b = 4$ , we have  $\sqrt{3^2 + 4^2} = 5$ , but  $3 + 4 = 7$ .
31. Not equivalent. If  $a = 0$ , then  $-a + 2 = 2$ , but  $-(a + 2) = -2$ .
32. Equivalent. We have  $bc - cd = cb - cd = c(b - d)$ .
33. The first two are equivalent, the third one is not equivalent to the other two.
34. Start with  $u$ , add 1, then multiply the result by 2.
35. Start with  $u$ , multiply by 2, then add 1.
36. Start with  $B$ , divide by 2, add 4 to the result, multiply the result of that by 3, then subtract the result of that from 1.
37. Start with  $s$ , add 5, multiply the result of that by 2, then subtract the result of that from 3.
38. (a) Abby's expression is  $325 - 65t$  and Leah's is  $65(5 - t)$ .
- (b) The two expressions are equivalent and define the same function.

10 Chapter One /SOLUTIONS

39. (a) Sharif's calculation starts with  $P$ , then adds  $0.05P$  to get  $P + 0.05P$ . Then he takes 5% of this to get  $0.05(P + 0.05P)$ , and adds that to  $P + 0.05P$  to get

$$P + 0.05P + 0.05(P + 0.05P).$$

Donald starts with  $P$ , multiplies it by 1.05, giving  $1.05P$ . Then he multiplies again, giving  $1.05(1.05P) = 1.05^2P$ .

- (b) Yes. Distributing and collecting like terms, we see that Sharif's function is

$$f(P) = P + 0.05P + 0.05P + 0.0025P = (1 + 0.05 + 0.05 + 0.0025)P = 1.1025P.$$

Donald's is  $g(P) = (1.05)^2P = 1.1025P$ .

40.

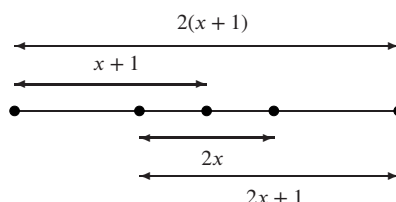


Figure 1.3

Since the interval of length  $2x+1$  is shorter than the interval of length  $2(x+1)$ , this tells us that  $2(x+1)$  is not equivalent to  $2x+1$ .

41. Since  $a$  pounds of apples are purchased at \$0.99 a pound, the cost of the apples is  $0.99a$ . Similarly, the cost of the pears is  $1.25p$ . Thus the total cost is  $0.99a + 1.25p$  dollars.
42. (a) The cost of the apples is  $8a$  and the cost of the pears is  $5p$ , so the total cost in dollars is  $8a + 5p$ .  
 (b) We have  $a = 0.40$  and  $p = 0.75$ , so total cost is  $8 \cdot 0.40 + 5 \cdot 0.75 = 6.95$  dollars.
43. We have  $0.07p$ .
44. We have  $20,000r$ .
45. The price is  $p - 1000$ , so the tax is  $0.06(p - 1000)$ .
46. 10% less than  $p$  is 90% of  $p$ , which is  $0.9p$ . So the tax is  $r(0.9p) = 0.9rp$ .
47. The sum of the semester grades is  $g_1 + g_2 + g_3 + g_4$ . Then he adds  $2f$  to get  $g_1 + g_2 + g_3 + g_4 + 2f$ . Then he divides by 6 to get

$$\text{Course grade} = \frac{g_1 + g_2 + g_3 + g_4 + 2f}{6}.$$

48. (a) Using the formula, we have

$$\text{Course grade} = 0.1 \cdot 67 + 0.6 \left( \frac{80 + 96 + 82}{3} \right) + 0.3 \cdot 74 = 80.5.$$

- (b) If we add 10 points to the third test grade then it becomes  $82 + 10 = 92$ , so

$$\text{Course grade} = 0.1 \cdot 67 + 0.6 \left( \frac{80 + 96 + 92}{3} \right) + 0.3 \cdot 74 = 82.5.$$

If we add 10 points to the final then it becomes  $74 + 10 = 84$ , so

$$\text{Course grade} = 0.1 \cdot 67 + 0.6 \left( \frac{80 + 96 + 82}{3} \right) + 0.3 \cdot 84 = 83.5.$$

So it is better to get the extra points on the final.

- (c) The part of the formula for the tests can be expanded using the distributed law as

$$0.6 \left( \frac{t_1 + t_2 + t_3}{3} \right) = 0.6 \cdot \frac{1}{3}(t_1 + t_2 + t_3) = 0.2t_1 + 0.2t_2 + 0.2t_3.$$

So

$$\text{Course grade} = 0.1h + 0.2t_1 + 0.2t_2 + 0.2t_3 + 0.3f.$$

If we ignore all the terms except the one involving the third test grade,  $0.2t_3$ , we can see that the third test grade is multiplied by 0.2. This means that any extra points on the third test get multiplied by 0.2. On the other hand, the term involving the final is  $0.3f$ , so any extra points on the final get multiplied by 0.3. Since 0.3 is greater than 0.2, it is better to add the points to the final.

49. The driving time is  $870/v$  hours. Adding this to the rest time we get a total time of  $5 + 870/v$  hours. The car traveling 65 mph takes  $5 + 870/65 = 18.4$  hour, and the car traveling 75 mph takes  $5 + 870/75 = 16.6$  hour, so it gets there about 1.8 hour earlier.

50. Since the radius is one half the height, we have  $r = (1/2)h = h/2$ . Substituting into the formula for volume, we have

$$\text{Volume of this cone} = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 h = \frac{1}{3}\pi \frac{h^2}{4} h = \frac{1}{12}\pi h^3.$$

51. (a) Since substituting  $p = 0$  gives 160, this represents the number of people who attend when the concert is free.  
 (b) We expect the number who attend to decrease as the price increases.  
 (c) Since  $160 - p$  is smaller than  $175 - p$  for all values of  $p$ , more people attend the movie.  
 (d) Since  $160 - 2p$  is smaller than  $160 - p$  for all positive values of  $p$ , more people attend the concert.

52. Two of the triangles can fit inside the rectangle, and so the triangle's area is half the rectangle's area.

53. The expression  $P + Q$  is larger.

- The expression  $P + Q$  gives the total size of the two populations put together.
- The expression  $2P$  gives the size of a population twice as large as  $P$ .
- Putting the smaller population together with the larger yields more animals than merely doubling the smaller.

Another way to see this is to notice that  $2P = P + P$ , which is smaller than  $P + Q$  because adding  $P$  to  $P$  is less than adding  $Q$  to  $P$ .

54. The expression  $\frac{P + Q}{2}$  is larger.

- The total size of the two populations put together is  $P + Q$ , so the expression  $\frac{P}{P + Q}$  gives the fraction of this total belonging to  $P$ . Since  $P < Q$ , this will be a number less than 1. For instance, if  $P = 100$  and  $Q = 150$ , this fraction equals  $100/(100 + 150) = 0.4 = 40\%$ .
- The average or mean size of the two populations is their sum divided by two, or  $\frac{P + Q}{2}$ . This will be a number between  $P$  and  $Q$ , so it is larger than 1 (since  $P$  and  $Q$  describe animal populations). For instance, if  $P = 100$  and  $Q = 150$ , the average is  $(100 + 150)/2 = 125$ .

55. The expression  $Q - P/2$  is larger.

- The expression  $(Q - P)/2$  gives half the difference between  $P$  and  $Q$ . For instance, if  $Q = 150$  and  $P = 100$ , half the difference is  $(150 - 100)/2 = 25$ .
- The expression  $Q - P/2$  gives the difference between  $Q$  and a population half the size of  $P$ . For instance, if  $Q = 150$  and  $P = 100$ , this difference equals  $150 - 100/2 = 100$ .

Another way to see this is to write

$$\begin{aligned}(Q - P)/2 &= 0.5Q - 0.5P \\ Q - P/2 &= Q - 0.5P\end{aligned}$$

In the expression  $Q - P/2$ , we subtract  $0.5P$  from  $Q$ . But in  $(Q - P)/2$ , we subtract the same value,  $0.5P$ , from a smaller amount,  $0.5Q$ .

56. The expression  $Q + 50t$  is larger.

- In both expressions, the same value,  $50t$ , is added to the population.
- Since  $P < Q$ , adding  $50t$  to  $P$  results in a smaller value than adding the same amount to  $Q$ .

12 Chapter One /SOLUTIONS

57.  $A + B$  is the total number of students in both classes.  
 58.  $A - B$  is how much larger class  $A$  is than class  $B$ .  
 59.  $(A + B)/2$  is the average number of students in the two classes.  
 60.  $A/(A + B)$  is the proportion of all students that are in class  $A$ .  
 61. Since everyone belongs to one of the three categories, we see that

$$\text{Total number of people} = \underbrace{\text{Number susceptible}}_S + \underbrace{\text{Number infected}}_I + \underbrace{\text{Number recovered}}_R = S + I + R.$$

62. Since everyone belongs to one of the three categories, we see that

$$\text{Total number of people} = \underbrace{\text{Number susceptible}}_S + \underbrace{\text{Number infected}}_I + \underbrace{\text{Number recovered}}_R = S + I + R.$$

Thus, the fraction of the total population that has recovered after one month is:

$$\text{Fraction recovered} = \frac{\overbrace{\text{Number recovered}}^R}{\underbrace{\text{Total number of people}}_{S+I+R}} = \frac{R}{S + I + R}.$$

63. We see that

$$\begin{aligned} \underbrace{\text{Number infected}}_I &= 2 \times \underbrace{\text{Number recovered}}_R \\ \text{so } I &= 2R \\ \text{and } \underbrace{\text{Number susceptible}}_S &= 10 \times \underbrace{\text{Number infected}}_I \\ \text{so } S &= 10I = 10 \underbrace{(2R)}_I = 20R. \end{aligned}$$

Since everyone belongs to one of the three categories, we see that

$$\begin{aligned} \text{Total number of people} &= \underbrace{\text{Number susceptible}}_S + \underbrace{\text{Number infected}}_I + \underbrace{\text{Number recovered}}_R \\ &= S + I + R \\ &= \underbrace{20R}_S + \underbrace{2R}_I + R = 23R. \end{aligned}$$

This means:

$$\begin{aligned} \text{The percentage} &= \frac{\overbrace{\text{Number susceptible}}^{S=20R}}{\underbrace{\text{Total number}}_{23R}} = \frac{20R}{23R} = \frac{20}{23} = 0.8696 = 86.96\%. \\ \text{remaining susceptible} & \end{aligned}$$

64. Initially the entire population is susceptible. After one month, one in four, or 25%, of the population has been infected, and of these, one in five, or 20% of 25% =  $0.2(0.25) = 5\%$ , has recovered.  
 65. Increases.  
 66. Decreases.  
 67. Decreases.

68. Increases.
69. Since  $a + x - (2 + a) = x - 2$ , the value remains unchanged.
70. The expression  $10 + t^2$  is larger. Since  $t^2$  is positive (or zero), the first expression is no smaller than 10, whereas the second is no larger than 9.
71. The expression  $k^2 + 3$  is larger. Since  $k^2$  is positive (or zero),  $k^2 + 3$  is no less than 3. This means  $\frac{6}{k^2 + 3}$  equals 6 divided by 3 (or more), so this expression can't be larger than 2.
72. The expression  $(s^2 + 2)(s^2 + 3)$  is larger. We know  $s^2 + 3$  is always larger than  $s^2 + 1$ , and the other factor in each product is  $s^2 + 2$ , so the first product is larger than the second product.
73. The expression  $\frac{12}{z^2 + 3}$  is larger. We know  $z^2 + 4$  is always larger than  $z^2 + 3$ . Thus, dividing 12 by  $z^2 + 4$  always results in a smaller number than dividing 12 by  $z^2 + 3$ .
74. Yes, with  $a = 3$  and  $x = y$  (or vice versa).
75. Yes, with  $a = b^2$  and  $x = \theta^2$  (or vice versa).
76. Yes, with  $a = 2$  and  $x = 4y$  (or vice versa).
77. Yes, with  $a = 5$  and  $x = y^2 + 3$  (or vice versa).

## Solutions for Section 1.3

### IDENTIFYING ALGEBRAIC STRUCTURE

1.  $x$  is the number when multiplied by 3 gives 15. Since  $3 \cdot 5 = 15$ , we see  $x = 5$  is a solution.
2. 10 subtracted by  $y$  must give 2, so  $y = 8$  is a solution.
3. If  $w$  divided by 4 equals 3 divided by 4, then  $w = 3$  must be a solution.
4. Since the square root of 49 is 7,  $p$  is the number that is added to 1 to get 49. So  $p = 48$  is a solution.
5. Since 0 is the only number added to 3 that gives 3, we know  $a^2$  must be 0. Since  $0^2 = 0$ , we see  $a = 0$  is a solution.
6. If the fraction on the left-hand side of the equation equals  $-1$ , the denominator,  $q + 1$  must equal  $-7$ . Since adding  $-8$  and 1 gives  $-7$ ,  $q = -8$  is a solution.
7. Since the square root of a number cannot be negative, this equation has no solutions.
8. Since the square of a number can never be negative, we cannot multiply  $-2$  and  $x^2$  to get a positive result.
9. A fraction equals zero when its numerator is zero and its denominator is not zero. Since the numerator of the fraction is never zero, it cannot equal zero.
10. Adding 1 to a number gives a different result than adding 7 to the number, so this equation has no solutions.
11. Since  $y^2$  is greater than or equal to 0, the  $10 + y^2$  must be greater than or equal to 10. Therefore this equation has no solutions.
12. Since 5 raised to any power results in a positive number,  $5^x$  can never result in a negative value, and this equation has no solutions.
13. Divide by  $-2$ :

$$\begin{aligned} 13 &= -2z \\ \frac{13}{-2} &= \frac{-2z}{-2} \\ -6.5 &= z. \end{aligned}$$

14. Dividing both sides by 0.5 gives  $x = 6$ , so the solution is 6.

14 Chapter One /SOLUTIONS

15. Subtracting  $6x$  from both sides and collecting like terms on the left gives  $x = -6$ , so the solution is  $-6$ .  
 16. Multiply by  $\frac{7}{3}$ :

$$\begin{aligned}\frac{3}{7}M &= \frac{4}{3} \\ \frac{7}{3}\left(\frac{3}{7}M\right) &= \frac{7}{3}\left(\frac{4}{3}\right) \\ M &= \frac{28}{9}.\end{aligned}$$

## PROBLEMS

17. No, because  $3 + 3 = 6$  and  $3^2 + 9 = 18$ , so the two sides are not equal when  $t = 3$ .  
 18. Yes, because  $-3 + 3 = 0$ , and  $(-3)^2 - 9 = 9 - 9 = 0$ , so both sides are zero when  $x = -3$ .  
 19. Not a solution. When  $a = 0$ , the left-hand side is  $\frac{3}{-3} = -1$ , which is not equal to the right-hand side.  
 20. A solution. When  $a = 0$ , the left-hand side is  $\frac{3+0}{3-0} = \frac{3}{3} = 1$ , which is equal to the right-hand side.  
 21. The graph contains the points  $(20, 60)$ ,  $(100, 60)$ , so the solutions are  $x = 20, 100$ .  
 22. The graph contains the point  $(60, 20)$ , so  $v(60) = 20$ .  
 23. (a) Looking at the graph, we see that  $t$  is close to 7 years when tuition is \$3700.  
 (b) We substitute  $t = 7$  into the equation:

$$3000 + 100t = 3700.$$

The left-hand side becomes

$$3000 + 100t = 3000 + 100 \cdot 7 = 3000 + 700 = 3700.$$

Since the two sides are equal,  $t = 7$  is a solution.

24. (a) Looking at the graph, we see that  $s$  is close to 50 when profits per package are \$8000.  
 (b) We substitute  $s = 50$  into the equation:

$$10,000 - \frac{100,000}{s} = 8000.$$

The left-hand side becomes

$$10,000 - \frac{100,000}{s} = 10,000 - \frac{100,000}{50} = 10,000 - 2000 = 8000.$$

Since the two sides are equal,  $s = 50$  is a solution.

25. We have

$$\begin{aligned}3z &= 22 \\ \frac{3z}{3} &= \frac{22}{3} \\ z &= \frac{22}{3} = 7.333.\end{aligned}$$

26. We undo the operations on  $x$  by first subtracting 12 from both sides and then dividing both sides by 5.

$$5x + 12 = 90$$

$$5x = 78 \quad \text{subtract 12 from both sides}$$

$$x = 15.6 \quad \text{divide both sides by 5.}$$

27. We undo the operations on  $x$  by first subtracting 10 from both sides and then dividing both sides by  $-2$ .

$$\begin{aligned} 10 - 2x &= 60 \\ -2x &= 50 && \text{subtract 10 from both sides} \\ x &= -25 && \text{divide both sides by } -2. \end{aligned}$$

28. We undo the operations on  $x$  by first dividing both sides by 3 and then adding 5 to both sides.

$$\begin{aligned} 3(x - 5) &= 12 \\ x - 5 &= 4 && \text{divide both sides by 3} \\ x &= 9 && \text{add 5 to both sides.} \end{aligned}$$

29. We undo the operations on  $x$  by first multiplying both sides by 5 and then subtracting 2 from both sides.

$$\begin{aligned} \frac{x + 2}{5} &= 10 \\ x + 2 &= 50 && \text{multiply both sides by 5} \\ x &= 48 && \text{subtract 2 from both sides.} \end{aligned}$$

30. We have

$$\begin{aligned} 2x + 5 &= 4x - 9 \\ 2x &= 4x - 14 \\ -2x &= -14 \\ x &= 7. \end{aligned}$$

31. Instead of distributing the 2 first, Scott could have divided both sides of the equation by 2. This would have simplified the equation, as illustrated below.

$$\begin{aligned} 2(x + 3) &= 8 \\ x + 3 &= 4 \\ x &= 1. \end{aligned}$$

Other answers are possible.

32. (a) We want to know when  $15 - d/20$  is equal to 10, so we want to solve

$$15 - d/20 = 10.$$

(b) Table 1.2 shows the number of gallons, and Figure 1.4 shows the plotted points and the solution at  $d = 100$ .

**Table 1.2** Gallons of gas left 40–140 miles after leaving a gas station

$d$ (miles)	40	60	80	100	120	140
$g$ (gallons)	13	12	11	10	9	8

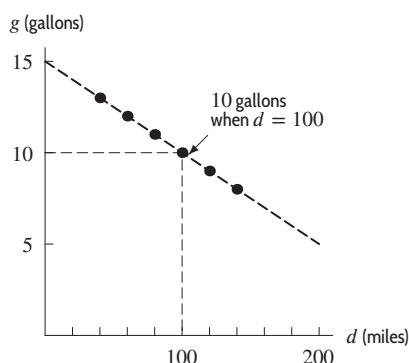


Figure 1.4

33. (a) We want to know when  $P = 50$ , so we want to solve

$$30 + 2t = 50.$$

- (b) Figure 1.5 shows the graphs of

$$P = 30 + 2t$$

and

$$P = 50.$$

The value of  $t$  at the point where these two graphs intersect is the value of  $t$  such that

$$30 + 2t = 50.$$

Since the graphs intersect when  $t = 10$ , the town's population will reach 50,000 people 10 years after its incorporation.

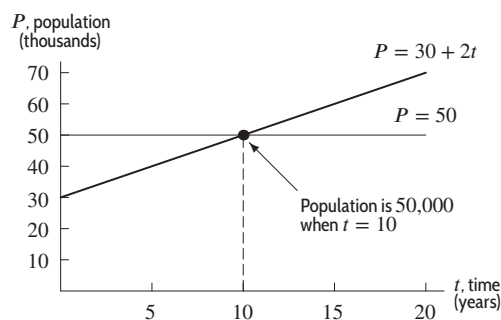


Figure 1.5: Population of a town

- (c) Looking at the algebraic structure of the equation, notice  $2t$  is the number we need to add to 30 in order to get 50. Since  $30 + 20 = 50$ , we have

$$2t = 20.$$

Next, we need to answer the question what value multiplied by 2 gives 20? Since  $2 \cdot 10 = 20$ , we see

$$t = 10$$

is a solution. This matches the solution we found in part (b).

Equivalently, we can find the solution by performing the following steps:

$$30 + 2t = 50$$

$$2t = 20 \quad \text{subtract 30 from both sides}$$

$$t = 10 \quad \text{divide both sides by 2.}$$



34. We want to know when the water reaches a height of 50 inches, so we want the solution to

$$27 + 2.1t = 50.$$

Figure 1.6 shows that the river appears to reach a height of 50 inches when  $t = 11$ .

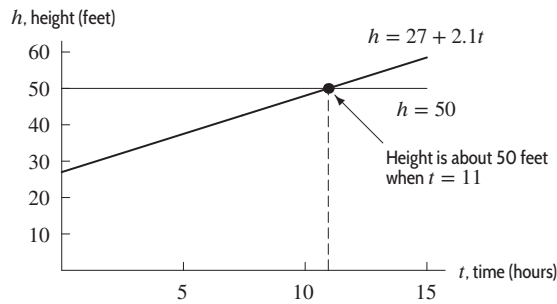


Figure 1.6: Height of a river  $t$  hours after flooding starts

However, checking numerically, we see that

$$27 + 2.1 \cdot 11 = 50.1,$$

so the river reaches the top of the levee before 11 hours are up. To get the exact solution we look at the structure of the expression on the left. It is the sum of 27 and  $2.1t$ , and want the answer to be 50. Since

$$27 + 23 = 50,$$

we want  $2.1t$  to be equal to 23. So

$$t = \frac{23}{2.1} = 10.95.$$

Since there are 60 minutes in an hour, 10.95 hours is 10 hours plus  $0.95 \cdot 60 = 57$  minutes, so the sandbags must arrive in 10 hours, 57 minutes, or 3 minutes before 11 hours (in fact more than 3 minutes, since there must be time to deploy them).

35. (a) We want to find the value of  $t$  that makes  $P(t) = Q(t)$ , so we must solve the equation

$$600 + 100(t - 2010) = 200 + 300(t - 2010).$$

(b) From Table 1.3, we see that the two populations are equal in the year  $t = 2012$ .

Table 1.3

$t$	2010	2011	2012	2013	2014
$P(t)$	600	700	800	900	1000
$Q(t)$	200	500	800	1100	1400

The table shows that  $P(2012) = Q(2012) = 800$ .

36. The equation  $f(x) = g(x)$  is satisfied if  $x = -1, 1, 2$ .
37. The equation  $f(x) = h(x)$  is satisfied if  $x = 0, 1$ .
38. The equation  $g(x) = h(x)$  is satisfied if  $x = 1$ .
39. If we let  $c$  represent the number of cones, then the amount he spends on cones is  $\$1.25c$ . Since he plans to spend all  $\$20$  on cones, we want

$$1.25c = 20.$$

The solution is  $c = 20/1.25 = 16$ , so Eric can buy 16 cones.

40. If we let  $n$  represent the number of Calories at breakfast, we want

$$1200 + n + n = 1800 \quad \text{or} \quad 1200 + 2n = 1800.$$

We want  $2n = 1800 - 1200 = 600$  so the solution is  $n = 300$ . Dennis can consume 300 Calories at breakfast and 300 Calories at lunch.

41. If  $x$  represents the maximum number of firemen that could be employed, then  $600,000 = 40,000x + 20,000x$  or  $600,000 = 60,000x$ . Thus  $x = 10$ , so ten firemen could be employed.
42. (a) For each hour, we subtract 2 gallons from Antonio's total and 6 from Lucia's total, giving Table 1.4. However, we note that in hour 5, Lucia runs out of gas and fills up to 30 gallons again.

Table 1.4

Hours driven	0	1	2	3	4	5	6	7
Antonio (gallons)	14	12	10	8	6	4	2	0
Lucia (gallons)	30	24	18	12	6	0	24	18

- (b) We see that after 4 hours, they each have 6 gallons of gas.
- (c) Antonio gets to San Diego first, since Lucia has to stop for gas.
- (d) Antonio arrives in San Diego, since he does not need to get gas. Lucia, on the other hand, needs gas after 5 hours, so she gets stuck in the desert.
- (e) (i) The two vehicles have the same amount of gas when  $14 - 2t = 30 - 6t$ , so  $t = 4$ , as we read in the table.  
(ii) Antonio runs out of gas when  $14 - 2t = 0$ , which is when  $t = 7$ .  
(iii) Lucia runs out of gas when  $30 - 6t = 0$ , which is when  $t = 5$ .
43. (a) Let  $p$  be the tag price, in dollars, of a pair of pants. With the discount certificate you pay  $p - 10$  dollars, and with the store discount you pay 75% of the price, or  $0.75p$  dollars. Table 1.5 compares the two amounts and shows that you pay the same price with either discount method when the tag price is \$40.

Table 1.5 Comparison of two discount methods

Tag price, $p$ (dollars)	20	30	40	50	60
Price with discount certificate, $p - 10$	10	20	30	40	50
Price with store discount, $0.75p$	15	22.50	30	37.50	45

- (b) We want to know what values of  $p$  satisfy

$$p - 10 = 0.75p.$$

Looking at the structure of the equation, this equation has terms involving the variable on both sides. So our first step is to subtract  $0.75p$  from both sides.

$$\begin{aligned} p - 10 &= 0.75p \\ 0.25p - 10 &= 0 && \text{subtract } 0.75p \text{ from both sides} \\ 0.25p &= 10 && \text{add 10 to both sides} \\ p &= 40 && \text{divide both sides by 0.25.} \end{aligned}$$

44. If we let  $c$  represent the price of one pound of coffee last year, then this year's price would be 25% more, which is

$$c + 0.25c = 1.25c.$$

Two pounds of coffee would cost  $2(1.25c)$ . When we subtract the discount coupon, the cost would be  $2(1.25c) - 3$ . Since we spent \$17.00, our equation is

$$2(1.25c) - 3 = 17,$$

or

$$2.5c - 3 = 17.$$

Since subtracting 3 from  $2.5c$  yields 17, we know that  $2.5c$  must be 20; that is,

$$2.5c = 17 + 3 = 20.$$

So since multiplying  $c$  by 2.5 gives 20,  $c$  must be 8:

$$c = 20/2.5 = 8.$$

This means that the price of coffee last year was \$8 per pound.

45. The two expressions are not equivalent. If you divide one side of the equation by 2, you must also divide the other side of the equation by 2.
46. The two expressions are not equivalent. If you subtract 5 from one side of the equation, you must also subtract 5 (not add 5) to the other side of the equation.
47. The two expressions are equivalent. We added  $3x$  to both sides.
48. The two expressions are not equivalent. We can't divide both sides by  $x$  since  $x$  might be zero. In fact,  $x = 0$  is a solution to the first equation but not the second.
49. The two expressions are equivalent. We added 6 to both sides.
50. The two expressions are not equivalent. If we subtract  $5x$  from the right side, we need to subtract  $5x$  from the left, which gives  $-3x$  rather than  $3x$ .
51. The two expressions are not equivalent. We can't multiply both sides by  $x$  since  $x$  might be zero. In fact,  $x = 0$  is a solution to the second equation but not the first (unless  $a$  is zero).
52. The two expressions are equivalent. We multiplied both sides by 3.
53. An equation must include an equal sign, so (a), (b), and (d) are equations.
54. (a) We multiply both sides of  $x/3 + 1/2 = 4x$  by 6 to obtain  $2x + 3 = 24x$ . Therefore, both equations have the same solution.  
 (b)  $x/3 + 1/2$  is not equivalent to  $2x + 3$ . If we substitute  $x = 0$  into both equations we obtain  $1/2$  and 3 respectively.
55. (a) If we divide both sides of  $8x - 4 = 12$  by 4, we obtain  $2x - 1 = 3$ . Therefore, both have the same solution.  
 (b) If we choose  $x = 0$  and substitute it into  $8x - 4$ , we obtain  $-4$ . Substituting  $x = 0$  into  $2x - 1$  produces  $-1$ . Therefore, the expressions are not equivalent.
56. (a) No; removing the parentheses on the left side of the original equation would give

$$2x - x - 3 = 4 + \frac{x-3}{10}.$$

- (b) Yes; expressing the fraction on the right side of the original equation as separate terms requires dividing each term in the numerator by 10, which is equivalent to multiplying it by 0.1:

$$\begin{aligned} 2x - (x + 3) &= 4 + \frac{x-3}{10} \\ 2x - (x + 3) &= 4 + 0.1(x - 3) \\ 2x - (x + 3) &= 4 + 0.1x - 0.3. \end{aligned}$$

- (c) Yes; expanding and collecting terms on both sides of the original equation gives

$$\begin{aligned} 2x - x - 3 &= 4 + 0.1x - 0.3 \\ x - 3 &= 3.7 + 0.1x. \end{aligned}$$

- (d) No; multiplying both sides of the original equation by 10 involves multiplying the 4 by 10 as well, so the result should be

$$20x - 10(x + 3) = 40 + x - 3.$$

20 Chapter One /SOLUTIONS

57. We have  $2p = 18$ .  
 58. We have  $3p + 5 = 32$ .  
 59. We have  $p + 0.20p = 10.80$ .  
 60. We have  $2p + 0.20(2p) = 21.60$ .  
 61. Yes. The value of  $2x - 3$  increases through all values as  $x$  increases, so there is some  $x$ -value for which  $2x - 3$  equals 7.  
 62. Yes. Since  $x^2 = 4$ , there are two solutions.  
 63. No. In the fraction on the left, the denominator is always 1 less than the numerator, no matter what the value of  $x$ , so the fraction can never equal 1.  
 64. No. Since  $5 + x^2$  is always greater than or equal to 5, there can be no solution.  
 65. Yes. Multiplying both sides by  $2x + 5$  gives  $x + 3 = 2x + 5$ , which has a solution.  
 66. No. The values of  $x + 3$  and  $5 + x$ , or  $x + 5$ , are never equal, so the fraction  $(x + 3)/(5 + x)$  is never equal to 1.  
 67. No. Since  $2x + 6 = 2(x + 3)$ , the fraction on the left side of the equation is always  $1/2$ :

$$\frac{x + 3}{2x + 6} = \frac{x + 3}{2(x + 3)} = \frac{1}{2}.$$

Thus the fraction  $(x + 3)/(2x + 6)$  is never equal to 1.

68. No. In the fraction on the left, the numerator is always one more than half the denominator, so the fraction can never equal  $1/2$ .  
 69. Increases. The larger  $a$  is, the larger  $x$  must be to give 0 when  $a$  is subtracted from it.  
 70. Decreases. The larger  $a$  is, the smaller  $x$  must be to give 1 when it is multiplied by  $a$ .  
 71. Remains unchanged. This equation is obtained from the equation  $x = 1$  by multiplying both sides by  $a$ . So the solution is always the same,  $x = 1$ .  
 72. Increases. The larger  $a$  is, the larger  $x$  must be to give 1 when it is divided by  $a$ .  
 73. We have

$$\begin{aligned} 1(1 - 1)(1 - 2)(1 - 3)(1 - 4) &= 1(0)(-1)(-2)(-3) = 0 \\ 2(2 - 1)(2 - 2)(2 - 3)(2 - 4) &= 2(1)(0)(-1)(-2) = 0 \\ 3(3 - 1)(3 - 2)(3 - 3)(3 - 4) &= 3(2)(1)(0)(-1) = 0, \end{aligned}$$

so  $t = 1, 2, 3$  are solutions. Judging from the pattern, other solutions include  $t = 0$  and  $t = 4$ :

$$\begin{aligned} 0(0 - 1)(0 - 2)(0 - 3)(0 - 4) &= 0(-1)(-2)(-3)(-4) = 0 \\ 4(4 - 1)(4 - 2)(4 - 3)(4 - 4) &= 4(3)(2)(1)(0) = 0. \end{aligned}$$

74. • We see that zero is not a solution because

$$\begin{aligned} -x\sqrt{7+x} &= 2 \quad \text{Trying } x = 0 \\ -0 \cdot \sqrt{7+0} &= 2 \quad \text{False.} \end{aligned}$$

- We see that  $x$  cannot be positive, because  $\sqrt{7+x}$  must be positive (or zero). Thus, if  $x > 0$ , the left-hand side is negative:

$$\begin{aligned} -x\sqrt{7+x} &= -1 \times (\text{A positive number}) \times (\text{Another positive number}) \\ &= \text{A negative number.} \end{aligned}$$

Since the right-hand side is positive, this is not possible.

- We conclude the solutions must be negative. They are approximately  $x = -6.916, -0.803$ , as you can check.