1 Solutions to Chapter 1 problems

Problem 1.1: This problem requires the reading of Table 1.1 on which the units and dimensions of important concepts and properties are given. Let us take one example. Let us verify the dimension of pressure in terms of M, L and T; it is given as [p] = [M L^{-1} T^{-2}] in the table. Recall that the unit of pressure in SI is N/m^2. Recall also that a unit of force of 1 Newton, N, is equal to 1 kg m/s^2. Thus, the unit of pressure can also be expressed as kg m^{-1} s^{-2} and, hence, the dimension of pressure can also be expressed as follows: [M L^{-1} T^{-2}]. This is what is given in the table.

Problem 1.2: The answer, as given in the text, is one divided by the correct answer. Based on the formula given (which is correct), the unit of G must be $[G]$ = [M^{-1} T^{-2} L^3]. The gravitational constant can be found in any physics text; it is $G = 6.670 \times 10^{-11}$ N m^2 kg^{-2} or $G = 6.670 \times 10^{-11}$ kg^{-1} m^2 s^{-2}. Hence, the unit of G must be $[G]$ = [M^{-1} T^{-2} L^3].

Problem 1.3: This problem is the dimensional analysis of a pendulum. The period of oscillation is τ. The length mass of the object attached to the end of the pendulum is m. The length of the pendulum is l. The acceleration of gravity is g. Let us examine the expression

$$\tau = Cm^a l^b g^c$$

The dimensions of each parameter is as follow:

$$[\tau] = C[m]^a [l]^b [g]^c$$

$$[T] = C\,[M]^a\,[L]^b \left[\frac{L}{T^2}\right]^c$$

The exponents equate as follows: $a = 0$, $b + c = 0$ and $1 = -2c$. Thus, $c = -1/2$, $b = 1/2$ and $a = 0$. This means that

$$\tau = Cm^0 l^{1/2} g^{-1/2} = C\sqrt{l/g},$$

where C is a constant. This illustrates that τ is independent of the mass, m.

Problem 1.4: We want to examine the power required to rotate a disk in a viscous fluid. We assume (based on the problem statement) that the power is related to the other properties as follows:

$$P = f(D, \omega, \rho, \nu)$$

The unit of power is N m s^{-1} or kg m^2 s^{-3}. Thus the dimension of power and its relationship with the dimensions of the other properties are as follows:

$$[P] = C\,[D]^a\,[\omega]^b\,[\rho]^c\,[\nu]^d$$

$$\left[\frac{ML^2}{T_3}\right] = C\,[L]^a \left[\frac{1}{T}\right]^b \left[\frac{M}{L^3}\right]^c \left[\frac{L^2}{T}\right]^d$$

Equating the exponents we get: $1 = c$, $2 = a - 3c + d$ and $-3 = -b - 2d$. Let d be the undetermined parameter. Hence, $c = 1$, $a = 5 - d$ and $b = 3 - 2d$. Thus,

$$P = CD^{5-d}\omega^{3-2d}\rho\nu^d$$

This can be rearranged as follows:

$$P = \rho\omega^3 D^5 \left(\frac{\nu}{D\omega^2}\right)^d$$

or

$$P = \rho\omega^3 D^5 f\left(\frac{\nu}{D\omega^2}\right)$$

or

$$C_P = \frac{P}{\rho\omega^3 D^5} = f\left(\frac{\nu}{D\omega^2}\right)$$

This is one of a number of forms that can be written for this relationship. Part (b) provides other nondimensional quantities that can be written from dimensional analysis. Of course, in this case, the power coefficient reduces to a function of only one independent parameter.

Problem 1.5: The terminal velocity of a sphere is to be examined by dimensional analysis. Assume V depends on the diameter and density of the sphere, D and σ, respectively. Also assume it depends on g, the acceleration of gravity, and ν, the kinematic viscosity of the fluid. Thus, we assume

$$V = CD^a\sigma^b\rho^c g^d\nu^e$$

The dimensions of each factor in this equation are

$$\left[\frac{L}{T}\right] = CL^a\left[\frac{M_s}{L^3}\right]^b\left[\frac{M}{L^3}\right]^c\left[\frac{L}{T^2}\right]^d\left[\frac{L^2}{T}\right]^e$$

Equating the coefficients, we get: $1 = a - 3b - 3c + d + 2e$, $b = -c$ and $-1 = -2d - e$. If we let a be the undetermined parameter, we get

$$V = C\left(\nu g\right)^{1/3} f\left(\frac{\sigma}{\rho}\right) h\left(Dg^{1/3}\nu^{-2/3}\right)$$

where the variable in the function h is the dimensionless variable to the power of a as determined by this application of dimensional analysis. Another construction of this dimensional relationship is given in the problem statement.

Problem 1.6: The model $R_{em} = V_m c_m/\nu_m$. The full-scale value is $R_{ep} = V_p c_p/\nu_p$. For dynamic similarity the Reynolds numbers must be equal, i.e.,

$$\frac{V_m c_m}{\nu_m} = \frac{V_p c_p}{\nu_p}$$

Thus,

$$V_p = \frac{\nu_p}{c_p}\frac{V_m c_m}{\nu_m}$$

The lift coefficient is

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 A}$$

This also must be equal for model and full-scale conditions. Hence,

$$\frac{L_m}{\frac{1}{2}\rho_m V_m^2 A_m} = \frac{L_p}{\frac{1}{2}\rho_p V_p^2 A_p}$$

Thus,

$$L_m = \frac{1}{2}\rho_m V_m^2 A_m \frac{L_p}{\frac{1}{2}\rho_p V_p^2 A_p}$$

This problem was graphically solved by applying the following MATLAB script:

```
% Problem 1.6
W = 60000; % N
b = 17; % m
scale = 0.1; % size of model
p = 15 * 101000; % N/m^2
T0 = 273;
T = 15 + T0; % K
R = 287; % KJ/kg/K
rho = p/(R*T);
muo = 0.0000171; % kg/m/s
mu = muo * (T/T0)^.75;
% Assume the viscosity if
num = mu/rho;
rhop = 1.2256;
nup = mu/rhop;
Vm = [20 21 22 23 24];
Vp = Vm*nup*scale/num;
Lmx = [2960 3460 4000 4580 5200];
 plot(Vm,Lmx)
% title('Model test data given in Problem 1.6')
% xlabel('Speed at maximum lift, m/s'),
% ylabel('Maximum lift, N')
Lm  = rho*Vm.^2.*scale^2.*W./(rhop*Vp.^2);
 hold on
plot(Vm,Lm,'r')
figure(2)
plot(Vm,Vp),grid
xlabel('Model speed, m/s'),ylabel('Prototype speed, m/s')
```

The first figure shows that the speed at maximum lift corresponding to the lift that is for the prototype at take-off. The second figure indicates that the speed of take-off of the full scale is $V_p = 33$ m/s.

Problem 1.7: The change in pressure over the first 60% of the chord is $-.4$. It linearly drops to zero over the last 40% of the chord. The lift is the integral of this over the chord. The lift is $C_L = (0.4 * 0.6 + 0.2 * 0.4)\cos(4 * \pi/180) = 0.3192$. The moment about the leading edge is $C_M = -0.3 * 0.4 * 0.6 - (0.6 + .4/3) * 0.2 * 0.4 = -0.1307$.

Problem 1.8: We are given the pressure distribution around a cylinder. The diameter and the upstream speed are given. The objective is to integrate the pressure to compute the drag. This was done using the following MATLAB script. The results are, with density $\rho = 1.23$ kg/m^3, $C_D = 0.9$ and $D = 75$ N m^{-1}.

```
% Problem 1.8
clear;clc
dtheta = 10;
theta = 0:dtheta:180; % degrees
thetac = theta + dtheta/2;
p = [569 502 301 -57 -392 -597 -721 -726 -707 -660 -626 ...
    -588 -569 -569 -569 -569 -569 -569 -569]; % N/m/m/
%plot(theta,p)
D = 150/1000; % m
V = 30; % m/s
R = D/2;
```

```
Dr = 0;
for n = 1:length(theta)-1;
    dAx(n) = R * (dtheta*pi/180) * cos( thetac(n)*pi/180 );
    dN(n) = dAx(n) * ( p(n) + p(n+1) ) / 2;
    Dr = Dr + dN(n);
end
Dr = 2*Dr
CD = Dr/(1.23*30^2*D/2)
plot(thetac(1:end-1),dAx)
figure(2)
plot(thetac(1:end-1),dN)
```

Problem 1.9: This problem is an engineering problem. Thus, we should not expect to get numerical results that are exactly like the answers given in the text. Three-significant figure accuracy could be expected. However, for the calculation of the moment, it depends on the calculation of $\bar{c}_A$ and, hence, on your interpretation of the meaning of the taper ratio. How do you handle the fact that there is a fuselage between the half wings as illustrated in Figure 1.5 on page 27 in the text? How do you take into account that the fuselage, which is not part of the two wings, and how do you handle the lift associated with the fuselage are engineering issues? These questions require an exercise of engineering judgment and, hence, require engineering guesses. The authors' solution to this problem is as follows:

```
%
% Homework problem 1.9: Sailplane
%
clear;clc
b = 18;
AR = 16;
S = b^2/AR;
cm = b/AR;
sig = 0.7;
rhoR = 1.22;
rho = sig*rhoR;
V = 115*1000/3600;
L = 3500;
CL = L/(rho*V^2*S/2)
D = 145;
CD = D/(rho*V^2*S/2)
CM = -0.03;
%
% AMC calculation:
%
dx = .01;
s = b/2;
xf = 0.3;
x = xf:dx:s;
TR = .3;
c_over_co = ( 1 -(1-TR)*(x-xf)/(s-xf) );
SS = 2*(sum(c_over_co(2:end-1))+ (c_over_co(1)+c_over_co(end))/2)*dx;
```

```
co = S/SS; % co is selected so that the wing area is equal to S.
c = co*c_over_co;
 c2 = c.^2;
 ca = ( sum(c2(2:end-1))+ (c2(1)+c2(end))/2 ) ...
     /( sum(c(2:end-1)) + (c(1)+c(end))/2 )
 %plot(x/s,c)
 M = CM*(rho*V^2*S/2)*ca
```

Executing this script we get $C_L = 0.3967$, $C_D = 0.0164$, $M = -337.8$ N m, and $\bar{c}_A = 1.276$ m.

Problem 1.10: After studying Section 1.6.3 on pressure distribution within the context of force and moment acting on an airfoil as well as C_L and C_M described in Section 1.6.4 the student is expected to recognize that at zero angle of attack the symmetric airfoil lift is zero. Obviously, the essay of any student addressing this question is the student's first guess and, hence, all the opinion reported (unless sloppy) is considered correct in terms of what is the correct answer to this question.

Problem 1.11: This problem could be approached by dimensional analysis. The properties of importance are a, speed of sound, V, speed, c, chord length, ρ, density of fluid and ν, the kinematic viscosity of the fluid. Thus, show that

$$L = \frac{1}{2}\rho V^2 c\, f\left(\frac{Vc}{\nu}\right)\, g\left(\frac{V}{a}\right), \quad D = \frac{1}{2}\rho V^2 c\, h\left(\frac{Vc}{\nu}\right)\, k\left(\frac{V}{a}\right)$$

where $Re = Vc/\nu$ and $M = V/a$ are the Reynolds number and Mach number, respectively. Under ideal circumstances you would test the model at the same Re and M. This is usually quite difficult. How the students discuss this matter at this stage of their investigation would be interesting.

Problem 1.12: A pint is an eighth of a gallon and, hence, 28.8 cubic inches (this number is not given correctly in the problem statement). This is equal to 0.0167 cubic feet. The density of fresh water is 1.93 $\mathrm{lb}_f\ \mathrm{s}^2\ \mathrm{ft}^{-4}$. The reference value of gravitational acceleration is $g = 32.174$ ft/s/s. Hence, the weight of a pint is 1.0349 pounds force. Thus, the rhyme is not bad; it favors the receiver of the liquid by 3.5%.

Problem 1.13: One kg is equal to one N $\mathrm{s}^2\ \mathrm{m}^{-1}$. It is a unit of mass. A mass of one kg weighs 9.81 N because weight is a force and it is equal to the mass of an object times the acceleration of gravity. The weight of 1 kg just given is the weight at sea level, where $g = 9.81\ \mathrm{m\ s}^{-2}$. There are a number of concepts reviewed in this problem that are useful for the student to clarify in their own mind.

Problem 1.14: At lift off assume the lift is equal the weight and the drag is opposite the thrust. Since the plot is a straight line with a constant slope equal to 3, the L/D is a constant for all planes on the chart and it is equal to 3. This assumes the plane at takeoff is not accelerating. If the fuselage is considered to contribute to half the drag, then the lift to drag ration for the wings is about 6 and it is constant for the airplanes on the chart.

Problem 1.15: This is a project for the students that is open-ended. Hence, in this case it forces the students to include economics in the problem at the beginning of their investigation of aerodynamics and the design of aircraft.

Problem 1.16: An aircraft of wing area S and drag coefficient C_D is flying at speed V in air of density ρ. It is propelled by a single airscrew of disk area A. It produces a thrust $T = D$, where D is the magnitude of the drag. Show that the speed V_s in the slipstream is given by

$$V_s = V\sqrt{1 + \frac{S}{A}C_D}$$

according to Froude's momentum theory.
Solution: Froude's theory leads to

$$T = \dot{m}\left(V_s - V\right) = \rho\frac{V_s + V}{2}A\left(V_s - V\right)$$

Noting that the magnitude of the thrust coefficient is the same as the drag coefficient, where

$$C_D = \frac{D}{\frac{1}{2}V^2 S}$$

Rearranging the formula for thrust and applying the definition of the drag coefficient, we get the formula sought (note that in the first printing of the text there is a typographical error in the formula.

Problem 1.17: The slipstream diameter of a cooling fan is given as $D = 0.5$ m. The slipstream speed far downstream of the fan is $V_s = 3$ m/s.Assume the flow through the fan and the slipstream geometry act like the flow through an ideal actuator disk. Estimate the fan diameter and the power input required.
Solution: The thrust on the fan is, according to Froude's momentum theory,

$$T = \dot{m}\left(V_s - V\right) = \dot{m}V_s = \rho\frac{V_s}{2}A_f V_s = \frac{1}{2}\rho A_f V_s^2$$

From continuity of flow through the slipstream we have

$$\rho\frac{V_s}{2}A_f - \rho A_s V_s$$

Thus,

$$\frac{D_f^2}{2} = 2D^2$$

Hence, $D_f^2 = 2/4 = 1/2$ m. Thus, $D_f = 1/\sqrt{2} = 0.707$ m. The power is equal to $P_{WR} = TV_s/2 = \rho A_f V_s^3/4 = 3.24$ W.

Problem 1.18: This problem is a direct application of the Froude momentum theory described in Section 1.8.2.