1.1

1.1 If p is a pressure, V a velocity, and ρ a fluid density, what are the dimensions (in the MLT system) of **(a)** p/ρ, **(b)** $pV\rho$, and **(c)** $p/\rho V^2$?

(a) $\dfrac{p}{\rho} \doteq \dfrac{ML^{-1}T^{-2}}{ML^{-3}} \doteq \underline{\underline{L^2T^{-2}}}$

(b) $pV\rho \doteq (ML^{-1}T^{-2})(LT^{-1})(ML^{-3}) \doteq \underline{\underline{M^2L^{-3}T^{-3}}}$

(c) $\dfrac{p}{\rho V^2} \doteq \dfrac{ML^{-1}T^{-2}}{(ML^{-3})(LT^{-1})^2} \doteq M^0L^0T^0$ (\underline{\underline{dimensionless}})

1.2

1.2 Verify the dimensions, in both the *FLT* system and the *MLT* system, of the following quantities which appear in Table 1.1: **(a)** acceleration, **(b)** stress, **(c)** moment of a force, **(d)** volume, and **(e)** work.

(a) acceleration $= \dfrac{\text{velocity}}{\text{time}} \doteq \dfrac{L}{T^2} \doteq \underline{\underline{LT^{-2}}}$

(b) stress $= \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \underline{\underline{FL^{-2}}}$

Since $F \doteq MLT^{-2}$,

$\text{stress} \doteq \dfrac{MLT^{-2}}{L^2} \doteq \underline{\underline{ML^{-1}T^{-2}}}$

(c) moment of a force $=$ force $\times$ distance $\doteq \underline{\underline{FL}}$

$\doteq (MLT^{-2})L \doteq \underline{\underline{ML^2T^{-2}}}$

(d) volume $= (\text{length})^3 \doteq \underline{\underline{L^3}}$

(e) work $=$ force $\times$ distance $\doteq \underline{\underline{FL}}$

$\doteq (MLT^{-2})L \doteq \underline{\underline{ML^2T^{-2}}}$

1.3

1.3 If P is a force and x a length, what are the dimensions (in the FLT system) of **(a)** dP/dx, **(b)** d^3P/dx^3, and **(c)** $\int P\,dx$?

(a) $\dfrac{dP}{dx} \doteq \dfrac{F}{L} \doteq \underline{\underline{FL^{-2}}}$

(b) $\dfrac{d^3P}{dx^3} \doteq \dfrac{F}{L^3} \doteq \underline{\underline{FL^{-3}}}$

(c) $\int P\,dx \doteq \underline{\underline{FL}}$

1.4

1.4 Dimensionless combinations of quantities (commonly called dimensionless parameters) play an important role in fluid mechanics. Make up five possible dimensionless parameters by using combinations of some of the quantities listed in Table 1.1.

Some possible examples:

$$\underline{\underline{\frac{\text{acceleration} \times \text{time}}{\text{velocity}}}} \doteq \frac{(LT^{-2})(T)}{(LT^{-1})} \doteq L^0T^0$$

$$\underline{\underline{\text{frequency} \times \text{time}}} \doteq (T^{-1})(T) \doteq T^0$$

$$\underline{\underline{\frac{(\text{velocity})^2}{\text{length} \times \text{acceleration}}}} \doteq \frac{(LT^{-1})^2}{(L)(LT^{-2})} \doteq L^0T^0$$

$$\underline{\underline{\frac{\text{force} \times \text{time}}{\text{momentum}}}} \doteq \frac{(F)(T)}{(MLT^{-1})} \doteq \frac{(F)(T)}{(FT^2L^{-1})(LT^{-1})} \doteq F^0L^0T^0$$

$$\underline{\underline{\frac{\text{density} \times \text{velocity} \times \text{length}}{\text{dynamic viscosity}}}} \doteq \frac{(ML^{-3})(LT^{-1})(L)}{ML^{-1}T^{-1}} \doteq M^0L^0T^0$$

1.5

1.5 A formula for estimating the volume rate of flow, Q, over the spillway of a dam is

$$Q = C\sqrt{2g}\,B(H + V^2/2g)^{3/2}$$

where C is a constant, g the acceleration of gravity, B the spillway width, H the depth of water passing over the spillway, and V the velocity of water just upstream of the dam. Would this equation be valid in any system of units? Explain.

$$Q = C\sqrt{2g}\,B\left(H + \frac{V^2}{2g}\right)^{3/2}$$

$$[L^3T^{-1}] \doteq [C][\sqrt{2}][LT^{-2}]^{1/2}[L]\left([L] + \left[\frac{L^2}{2T^2}\cdot\frac{T^2}{L}\right]\right)^{3/2}$$

$$[L^3T^{-1}] \doteq [C][\sqrt{2}][L^{3/2}T^{-1}]\left([L] + \left[\frac{L}{2}\right]\right)^{3/2}$$

$$[L^3T^{-1}] \doteq [C\sqrt{2}][L^3T^{-1}]$$

Since each term in the equation must have the same dimensions the constant $C\sqrt{2}$ must be dimensionless. Thus, the equation is a general homogeneous equation that would be valid in any consistent set of units. Yes.

1.6

1.6 The pressure difference, Δp, across a partial blockage in an artery (called a *stenosis*) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u\left(\frac{A_0}{A_1} - 1\right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity $(FL^{-2}T)$, ρ the blood density (ML^{-3}), D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the stenosis. Determine the dimensions of the constants K_v and K_u. Would this equation be valid in any system of units?

$$\Delta p = K_v \frac{\mu V}{D} + K_u\left[\frac{A_0}{A_1} - 1\right]^2 \rho V^2$$

$$[FL^{-2}] \doteq [K_v]\left[\left(\frac{FT}{L^2}\right)\left(\frac{L}{T}\right)\left(\frac{1}{L}\right)\right] + [K_u]\left[\frac{(L^2)}{(L^2)} - 1\right]^2\left[\frac{FT^2}{L^4}\right]\left[\frac{L}{T}\right]^2$$

$$[FL^{-2}] \doteq [K_v][FL^{-2}] + [K_u][FL^{-2}]$$

Since each term must have the same dimensions, K_v and K_u are dimensionless. Thus, the equation is a general homogeneous equation that would be valid in any consistent system of units. Yes.

1.7

1.7 According to information found in an old hydraulics book, the energy loss per unit weight of fluid flowing through a nozzle connected to a hose can be estimated by the formula

$$h = (0.04 \text{ to } 0.09)(D/d)^4 V^2/2g$$

where h is the energy loss per unit weight, D the hose diameter, d the nozzle tip diameter, V the fluid velocity in the hose, and g the acceleration of gravity. Do you think this equation is valid in any system of units? Explain.

$$h = (0.04 \text{ to } 0.09)\left(\frac{D}{d}\right)^4 \frac{V^2}{2g}$$

$$\left[\frac{FL}{F}\right] \doteq \left[0.04 \text{ to } 0.09\right]\left[\frac{L^4}{L^4}\right]\left[\frac{1}{2}\right]\left[\frac{L^2}{T^2}\right]\left[\frac{T^2}{L}\right]$$

$$[L] \doteq [0.04 \text{ to } 0.09][L]$$

Since each term in the equation must have the same dimensions, the constant term (0.04 to 0.09) must be dimensionless. Thus, the equation is a general homogeneous equation that is valid in any system of units. Yes.

1.9

1.9 Make use of Table 1.2 to express the following quantities in SI units: **(a)** 10.2 in./min, **(b)** 4.81 slugs, **(c)** 3.02 lb, **(d)** 73.1 ft/s², **(e)** 0.0234 lb·s/ft².

(a) $10.2\ \frac{in.}{min} = \left(10.2\ \frac{in.}{min}\right)\left(2.540\times10^{-2}\ \frac{m}{in.}\right)\left(\frac{1\ min}{60\ s}\right)$

$= 4.32\times10^{-3}\ \frac{m}{s} = \underline{\underline{4.32\ \frac{mm}{s}}}$

(b) $4.81\ slugs = \left(4.81\ slugs\right)\left(1.459\times10\ \frac{kg}{slug}\right) = \underline{\underline{70.2\ kg}}$

(c) $3.02\ lb = \left(3.02\ lb\right)\left(4.448\ \frac{N}{lb}\right) = \underline{\underline{13.4\ N}}$

(d) $73.1\ \frac{ft}{s^2} = \left(73.1\ \frac{ft}{s^2}\right)\left(3.048\times10^{-1}\ \frac{\frac{m}{s^2}}{\frac{ft}{s^2}}\right) = \underline{\underline{22.3\ \frac{m}{s^2}}}$

(e) $0.0234\ \frac{lb\cdot s}{ft^2} = \left(0.0234\ \frac{lb\cdot s}{ft^2}\right)\left(4.788\times10\ \frac{\frac{N\cdot s}{m^2}}{\frac{lb\cdot s}{ft^2}}\right)$

$= \underline{\underline{1.12\ \frac{N\cdot s}{m^2}}}$

1.10

1.10 Make use of Table 1.3 to express the following quantities in BG units: **(a)** 14.2 km, **(b)** 8.14 N/m³, **(c)** 1.61 kg/m³, **(d)** 0.0320 N·m/s, **(e)** 5.67 mm/hr.

(a) $14.2\ km = (14.2\times10^3\ m)\left(3.281\ \frac{ft}{m}\right) = \underline{\underline{4.66\times10^4\ ft}}$

(b) $8.14\ \frac{N}{m^3} = \left(8.14\ \frac{N}{m^3}\right)\left(6.366\times10^{-3}\ \frac{\frac{lb}{ft^3}}{\frac{N}{m^3}}\right) = \underline{\underline{5.18\times10^{-2}\ \frac{lb}{ft^3}}}$

(c) $1.61\ \frac{kg}{m^3} = \left(1.61\ \frac{kg}{m^3}\right)\left(1.940\times10^{-3}\ \frac{\frac{slugs}{ft^3}}{\frac{kg}{m^3}}\right) = \underline{\underline{3.12\times10^{-3}\ \frac{slugs}{ft^3}}}$

(d) $0.0320\ \frac{N\cdot m}{s} = \left(0.0320\ \frac{N\cdot m}{s}\right)\left(7.376\times10^{-1}\ \frac{\frac{ft\cdot lb}{s}}{\frac{N\cdot m}{s}}\right)$

$= \underline{\underline{2.36\times10^{-2}\ \frac{ft\cdot lb}{s}}}$

(e) $5.67\ \frac{mm}{hr} = \left(5.67\times10^{-3}\ \frac{m}{hr}\right)\left(3.281\ \frac{ft}{m}\right)\left(\frac{1\ hr}{3600\ s}\right)$

$= \underline{\underline{5.17\times10^{-6}\ \frac{ft}{s}}}$

1.11

1.11 Clouds can weigh thousands of pounds due to their liquid water content. Often this content is measured in grams per cubic meter (g/m^3). Assume that a cumulus cloud occupies a volume of one cubic kilometer, and its liquid water content is 0.2 g/m^3. **(a)** What is the volume of this cloud in cubic miles? **(b)** How much does the water in the cloud weigh in pounds?

(a) $\forall\text{olume} = 1\,(km)^3 = 10^9\,m^3$

Since $1\,m = 3.281\,ft$

$$\forall\text{olume} = \frac{(10^9\,m^3)\left(3.281\,\frac{ft}{m}\right)^3}{\left(5.280\times10^3\,\frac{ft}{mi}\right)^3}$$

$$= \underline{\underline{0.240\,mi^3}}$$

(b) $\mathcal{W} = \gamma \times \forall\text{olume}$

$$\gamma = \rho g = \left(0.2\,\frac{g}{m^3}\right)\left(10^{-3}\,\frac{kg}{g}\right)\left(9.81\,\frac{m}{s^2}\right) = 1.962\times10^{-3}\,\frac{N}{m^3}$$

$$\mathcal{W} = \left(1.962\times10^{-3}\,\frac{N}{m^3}\right)\left(10^9\,m^3\right) = 1.962\times10^6\,N$$

$$= \left(1.962\times10^6\,N\right)\left(2.248\times10^{-1}\,\frac{lb}{N}\right) = \underline{\underline{4.41\times10^5\,lb}}$$

1.12

1.12 An important dimensionless parameter in certain types of fluid flow problems is the *Froude number* defined as $V/\sqrt{g\ell}$, where V is a velocity, g the acceleration of gravity, and ℓ a length. Determine the value of the Froude number for $V = 10\ \text{ft/s}$, $g = 32.2\ \text{ft/s}^2$, and $\ell = 2\ \text{ft}$. Recalculate the Froude number using SI units for V, g, and ℓ. Explain the significance of the results of these calculations.

In BG units,

$$\frac{V}{\sqrt{g\ell}} = \frac{10\ \frac{ft}{s}}{\sqrt{\left(32.2\ \frac{ft}{s^2}\right)(2\ ft)}} = \underline{\underline{1.25}}$$

In SI units:

$$V = \left(10\ \frac{ft}{s}\right)\left(0.3048\ \frac{m}{ft}\right) = 3.05\ \frac{m}{s}$$

$$g = 9.81\ \frac{m}{s^2}$$

$$\ell = (2\ ft)\left(0.3048\ \frac{m}{ft}\right) = 0.610\ m$$

Thus,

$$\frac{V}{\sqrt{g\ell}} = \frac{3.05\ \frac{m}{s}}{\sqrt{\left(9.81\ \frac{m}{s^2}\right)(0.610\ m)}} = \underline{\underline{1.25}}$$

The value of a dimensionless parameter is independent of the unit system.

1.13

1.13 The specific weight of a certain liquid is 85.3 lb/ft^3. Determine its density and specific gravity.

$$\rho = \frac{\gamma}{g} = \frac{85.3 \frac{lb}{ft^3}}{32.2 \frac{ft}{s^2}} = \underline{\underline{2.65 \frac{slugs}{ft^3}}}$$

$$SG = \frac{\rho}{\rho_{H_2O} @ 4°C} = \frac{2.65 \frac{slugs}{ft^3}}{1.94 \frac{slugs}{ft^3}} = \underline{\underline{1.37}}$$

1.14

1.14 A *hydrometer* is used to measure the specific gravity of liquids. (See **Video V2.6.**) For a certain liquid a hydrometer reading indicates a specific gravity of 1.15. What is the liquid's density and specific weight? Express your answer in SI units.

$$SG = \frac{\rho}{\rho_{H_2O} @ 4°C}$$

$$1.15 = \frac{\rho}{1000 \frac{kg}{m^3}}$$

$$\rho = (1.15)\left(1000 \frac{kg}{m^3}\right) = \underline{\underline{1150 \frac{kg}{m^3}}}$$

$$\gamma = \rho g = \left(1150 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right) = \underline{\underline{11.3 \frac{kN}{m^3}}}$$

1.16

1.16 When poured into a graduated cylinder, a liquid is found to weigh 6 N when occupying a volume of 500 ml (milliliters). Determine its specific weight, density, and specific gravity.

$$\gamma = \frac{\text{weight}}{\text{volume}} = \frac{6\,N}{(0.500\,\ell)\left(10^{-3}\,\frac{m^3}{\ell}\right)} = \underline{\underline{12.0\,\frac{kN}{m^3}}}$$

$$\rho = \frac{\gamma}{g} = \frac{12.0\times10^{3}\,\frac{N}{m^3}}{9.81\,\frac{m}{s^2}} = \underline{\underline{1.22\times10^{3}\,\frac{kg}{m^3}}}$$

$$SG = \frac{\rho}{\rho_{H_2O}\,@\,4^\circ C} = \frac{1.22\times10^{3}\,\frac{kg}{m^3}}{10^{3}\,\frac{kg}{m^3}} = \underline{\underline{1.22}}$$

1.17 *

***1.17** The variation in the density of water, ρ, with temperature, T, in the range of 20°C ≤ T ≤ 60°C, is given in the following table.

Density (kg/m³)	998.2	997.1	995.7	994.1	992.2	990.2	998.1
Temperature (°C)	20	25	30	35	40	45	50

Use these data to determine an empirical equation of the form $\rho = c_1 + c_2T + c_3T^2$, which can be used to predict the density over the range indicated. Compare the predicted values with the data given. What is the density of water at 42.1 °C?

Fit the data to a second order polynomial using a standard curve-fitting program such as found in EXCEL. Thus,

$$\rho = 1001 - 0.0533\,T - 0.0041\,T^2 \qquad (1)$$

As shown in the table below, ρ (predicted) from Eq. (1) is in good agreement with ρ (given).

T, °C	ρ, kg/m^3	ρ, Predicted
20	998.2	998.3
25	997.1	997.1
30	995.7	995.7
35	994.1	994.1
40	992.2	992.3
45	990.2	990.3
50	988.1	988.1

At $T = 42.1\ °C$

$$\rho = 1001 - 0.0533\,(42.1\,°C) - 0.0041\,(42.1\,°C)^2 = \underline{\underline{991.5\ \frac{kg}{m^3}}}$$

1.19

1.19 Some experiments are being conducted in a laboratory in which the air temperature is 27 °C and the atmospheric pressure is 14.3 psia. Determine the density of the air. Express your answers in slugs/ft³ and in kg/m³.

$$p = \rho R T$$

$$\text{Temperature} = 27°C = [1.8(27) + 32°]\ °F = 80.6\ °F$$

$$\rho = \frac{p}{RT} = \frac{\left(14.3\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{\left(1716\ \frac{ft \cdot lb}{slug \cdot °R}\right)\left[(80.6°F + 460)°R\right]}$$

$$= \underline{\underline{0.00222\ \frac{slugs}{ft^3}}}$$

$$\rho = \left(0.00222\ \frac{slugs}{ft^3}\right)\left(5.154 \times 10^2\ \frac{\frac{kg}{m^3}}{\frac{slugs}{ft^3}}\right) = \underline{\underline{1.14\ \frac{kg}{m^3}}}$$

1.20

1.20 A closed tank having a volume of 2 ft³ is filled with 0.30 lb of a gas. A pressure gage attached to the tank reads 12 psi when the gas temperature is 80 °F. There is some question as to whether the gas in the tank is oxygen or helium. Which do you think it is? Explain how you arrived at your answer.

Density of gas in tank $\rho = \dfrac{\text{weight}}{g \times \text{volume}} = \dfrac{0.30\ \text{lb}}{\left(32.2\ \frac{\text{ft}}{\text{s}^2}\right)(2\ \text{ft}^3)}$

$= 4.66 \times 10^{-3}\ \dfrac{\text{slugs}}{\text{ft}^3}$

Since $\rho = \dfrac{p}{RT}$ with $p = (12 + 14.7)$ psia

(atmospheric pressure assumed to be $\approx$ 14.7 psia)

and with $T = (80°F + 460)\ °R$ it follows that

$$\rho = \frac{\left(26.7\ \frac{\text{lb}}{\text{in.}^2}\right)\left(144\ \frac{\text{in.}^2}{\text{ft}^2}\right)}{R\ (540\ °R)} = \frac{7.12}{R}\ \frac{\text{slugs}}{\text{ft}^3} \quad (1)$$

From Table 1.6 $R = 1.554 \times 10^3$ for oxygen

and $R = 1.242 \times 10^4\ \dfrac{\text{ft} \cdot \text{lb}}{\text{slug} \cdot °R}$ for helium.

Thus, from Eq. (1) if the gas is oxygen

$$\rho = \frac{7.12}{1.554 \times 10^3}\ \frac{\text{slugs}}{\text{ft}^3} = 4.58 \times 10^{-3}\ \frac{\text{slugs}}{\text{ft}^3}$$

and for helium

$$\rho = \frac{7.12}{1.242 \times 10^4} = 5.73 \times 10^{-4}\ \frac{\text{slugs}}{\text{ft}^3}$$

A comparison of these values with the actual density of the gas in the tank indicates that the gas must be <u>oxygen</u>.

1.22

1.22 A rigid tank contains air at a pressure of 90 psia and a temperature of 60 °F. By how much will the pressure increase as the temperature is increased to 110 °F?

$$p = \rho R T \qquad \text{(Eq. 1.7)}$$

For a rigid closed tank the air mass and volume are constant so ρ = constant. Thus, from Eq. 1.7 (with R constant)

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad (1)$$

where $p_1 = 90$ psia, $T_1 = 60°F + 460 = 520°R$, and $T_2 = 110°F + 460 = 570°R$. From Eq. (1)

$$p_2 = \frac{T_2}{T_1}\, p_1 = \left(\frac{570°R}{520°R}\right)(90\ \text{psia}) = \underline{\underline{98.7\ \text{psia}}}$$

1.23

1.23 Determine the ratio of the dynamic viscosity of water to air at a temperature of 70 °C. Compare this value with the corresponding ratio of kinematic viscosities. Assume the air is at standard atmospheric pressure.

From Table B.2 in Appendix B:

(for water at 70°C) $\mu = 4.042 \times 10^{-4}\ \frac{N\cdot s}{m^2}$; $\nu = 4.134 \times 10^{-7}\ \frac{m^2}{s}$

From Table B.4 in Appendix B:

(for air at 70°C) $\mu = 2.03 \times 10^{-5}\ \frac{N\cdot s}{m^2}$; $\nu = 1.97 \times 10^{-5}\ \frac{m^2}{s}$

Thus,

$$\frac{\mu_{H_2O}}{\mu_{air}} = \frac{4.042 \times 10^{-4}}{2.03 \times 10^{-5}} = \underline{\underline{19.9}}$$

$$\frac{\nu_{H_2O}}{\nu_{air}} = \frac{4.134 \times 10^{-7}}{1.97 \times 10^{-5}} = \underline{\underline{2.10 \times 10^{-2}}}$$