Chapter 1: Systems of Linear Equations and Matrices

Multiple Choice Questions

1. Which of the following equations is linear?

(A) $2x_1^2 + 3x_2^3 + 4x_3^4 = 5$

(B) $\sqrt{3}x_1 - \sqrt{2}x_2 + x_3 = 5$

(C) $\sqrt{5}x_1 + 5\sqrt{x_2} - x_3 = 1$

(D) $2^2x_1 + \cos(x_2) + 4x_3 = 7$

2. Which system corresponds to the following augmented matrix?

$$\begin{bmatrix} 1 & 11 & 6 & 3 \\ 9 & 4 & 0 & -2 \end{bmatrix}$$

(A) $\begin{aligned} x_1 + 11x_2 &= -3 \\ 9x_1 + 4x_2 &= -2 \end{aligned}$

(B) $\begin{aligned} x_1 + 11x_2 + 6x_3 &= 3 \\ 9x_1 + 4x_2 &= -2 \end{aligned}$

(C) $\begin{aligned} x_1 + 11x_2 + 6x_3 + 3x_4 &= 0 \\ 9x_1 + 4x_2 - 2x_4 &= 0 \end{aligned}$

(D) $\begin{aligned} x_1 + 9x_2 &= 0 \\ 11x_1 + 4x_2 &= 0 \\ 6x_1 &= 0 \\ 3x_1 - 2x_2 &= 0 \end{aligned}$

3. Which of the following statements best describes the following augmented matrix?

$$A = \begin{bmatrix} 1 & 2 & 6 & 5 \\ -1 & 1 & -2 & 3 \\ 1 & -4 & -2 & 1 \end{bmatrix}$$

(A) A is consistent with a unique solution.

(B) A is consistent with infinitely many solutions.

(C) A is inconsistent.

(D) none of the above.

4. Which of the following matrices is in *reduced* row echelon form?

(A) $\begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 3 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 2 & 5 \\ 0 & 1 & -7 & 5 \\ 0 & 0 & 1 & 14 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 & 11 & -3 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{bmatrix}$

5. If the matrix A is 4×2, B is 3×4, C is 2×4, D is 4×3, and E is 2×5, which of the following expressions is *not* defined?

(A) $A^TD + CB^T$ (B) $(B + D^T)A$ (C) $CA + CB^T$ (D) $DBAE$

6. What is the second row of the product AB?

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 5 & 4 & 8 \\ 9 & 7 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 1 & 7 \\ 6 & 3 & 2 \\ 2 & 9 & 7 \end{bmatrix}$$

(A) $\begin{bmatrix} 18 & 33 & 25 \end{bmatrix}$ (B) $\begin{bmatrix} 64 & 48 & 91 \end{bmatrix}$ (C) $\begin{bmatrix} 50 & 89 & 99 \end{bmatrix}$ (D) $\begin{bmatrix} 48 & 89 & 33 \end{bmatrix}$

7. Which of the following is the determinant of the 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$?

(A) $ad - bc$ (B) $bc - ad$ (C) $\frac{1}{bc-ad}$ (D) $\frac{1}{ad-bc}$

8. Which of the following matrices is *not* invertible?

(A) $\begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ (B) $\begin{bmatrix} 7 & 7 \\ 2 & 3 \end{bmatrix}$ (C) $\begin{bmatrix} 9 & 0 \\ 4 & 4 \end{bmatrix}$ (D) $\begin{bmatrix} 9 & 3 \\ 6 & 5 \end{bmatrix}$

9. Which of the following matrices is *not* an elementary matrix?

(A) $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$ (C) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

10. For which elementary matrix E will the equation $EA = B$ hold?

$$A = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 2 & 10 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix}$$

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

11. Which matrix will be used as the inverted coefficient matrix when solving the following system?

$$\begin{aligned} 3x_1 + \ x_2 &= 4 \\ 5x_1 + 2x_2 &= 7 \end{aligned}$$

(A) $\begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$ (B) $\begin{bmatrix} -2 & 1 \\ 5 & -3 \end{bmatrix}$ (C) $\begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$ (D) $\begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix}$

12. What value of b makes the following system consistent?

$$\begin{aligned} 4x_1 + 2x_2 &= b \\ 2x_1 + \ x_2 &= 0 \end{aligned}$$

(A) $b = -1$ (B) $b = 0$ (C) $b = 1$ (D) $b = 2$

13. If A is a 3×3 diagonal matrix, which of the following matrices is *not* a possible value of A^k for some integer k?

(A) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 9 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 16 & 0 \\ 4 & 0 & 25 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

14. The matrix $\begin{bmatrix} 3 & 0 & 0 \\ 0 & -7 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is:

(A) upper triangular.

(B) lower triangular.

(C) both (A) and (B).

(D) neither (A) nor (B).

15. If A is a 4×5 matrix, find the domain and codomain of the transformation $T_A(\mathbf{x}) = A\mathbf{x}$.

(A) Not enough information

(B) Domain: R^4, Codomain: R^5

(C) Domain: R^5, Codomain: R^5

(D) Domain: R^5, Codomain: R^4

16. Which of the following is a matrix transformation?

(A) $T(x, y, z) = (yx^2, yz^2)$

(B) $T(x, y, z, w) = (xy, yz, zw, wx)$

(C) $T(x, y, z) = (x + 1, x + 2, x + z, y + z)$

(D) $T(x, y) = (4x, 5x, -x, 0)$

17. Which matrix represents reflection about the xy-plane?

(A) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

18. Use matrix multiplication to find the image of the vector $(2,\ 1)$ when it is rotated counterclockwise about the origin through an angle $\theta = 45°$.

(A) $\left(\frac{\sqrt{2}}{2},\ \frac{3\sqrt{2}}{2}\right)$ (B) $\left(\frac{3\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right)$ (C) $\left(-\frac{\sqrt{2}}{2},\ \frac{3\sqrt{2}}{2}\right)$ (D) $\left(-\frac{3\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right)$

19. Which of the following pairs of operators $T_1, T_2 : R^2 \to R^2$ commute? (That is, for which pair is it true that $T_1 \circ T_2 = T_2 \circ T_1$?)

(A) T_1 is the reflection about the x-axis.
T_2 is the reflection about line $y = x$.

(B) T_1 is the orthogonal projection onto the x-axis.
T_2 is the reflection about line $y = x$.

(C) T_1 is the counterclockwise rotation about the origin through an angle of π.
T_2 is the projection onto the y-axis.

(D) T_1 is the reflection about the x-axis.
T_2 is the counterclockwise rotation about the origin through an angle of $\pi/2$.

Free Response Questions

1. Find the relationship between a and b such that the following system has infinitely many solutions.

$$-x + 2y = a$$
$$-3x + 6y = b$$

2. Solve the following system and use parametric equations to describe the solution set.

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 11 \\ 2x_1 - x_2 + x_3 &= 2 \\ 3x_1 + x_2 + 4x_3 &= 13 \end{aligned}$$

3. Determine whether the following system has no solution, exactly one solution, or infinitely many solutions.

$$\begin{aligned} 2x_1 + 2x_2 &= 2 \\ x_1 + x_2 &= 4 \end{aligned}$$

4. Find the value of k that makes the system $\begin{bmatrix} 15 & -3 & 6 \\ -10 & k & 9 \end{bmatrix}$ inconsistent.

5. Solve the following system using Gaussian elimination.

$$\begin{aligned} x_1 - x_2 - 5x_3 &= -1 \\ -2x_1 + 2x_2 + 11x_3 &= 1 \\ 3x_1 - x_2 + x_3 &= 3 \end{aligned}$$

6. Solve the following system for x, y, and z.

$$\begin{aligned} \tfrac{1}{x} - \tfrac{1}{y} - \tfrac{1}{z} &= 0 \\ \tfrac{2}{x} + \tfrac{1}{y} + \tfrac{1}{z} &= 3 \\ \tfrac{3}{x} - \tfrac{1}{z} &= 0 \end{aligned}$$

7. The curve $y = ax^3 + bx^2 + x + c$ passes through the points $(0, 0), (1, 1)$, and $(-1, -2)$. Find and solve a system of linear equations to determine the values of a, b, and c.

8. Solve the following system for x and y.

$$\begin{aligned} x^2 + y^2 &= 6 \\ x^2 - y^2 &= 2 \end{aligned}$$

9. Given $C = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$, find CC^T.

10. Express the following matrix equation as a system of linear equations.

$$\begin{bmatrix} -1 & 7 & 0 \\ 0 & 4 & 3 \\ 6 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$