Chapter 2: Determinants

Multiple Choice Questions

1. What is M_{23} for the following matrix?

$$\begin{bmatrix} 3 & 4 & 0 \\ -1 & 2 & 7 \\ -2 & -4 & 4 \end{bmatrix}$$

(A) 0 (B) -4 (C) 21 (D) 26

2. Compute.

$$\begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 5 & -6 & 7 \\ 0 & 0 & -8 & 9 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$

(A) -40 (B) -1 (C) 6 (D) 0

3. Let A be a square matrix. If $\det(A) = 5$, what is $\det\left(A^T\right)$?

(A) $-\frac{1}{5}$ (B) $\frac{1}{5}$ (C) -5 (D) 5

4. Evaluate the determinant of the following matrix by using signed elementary products.

$$\begin{bmatrix} 0 & -1 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

(A) -16 (B) 16 (C) -9 (D) 9

5. If A is a 3×3 matrix and $\det(A) = 7$, what is $\det(2A)$?

(A) $\frac{7}{2}$ (B) 14 (C) 42 (D) 56

6. If A is an invertible matrix and $\det(A) = 7$, what is $\det(A^{-1})$?

(A) 7 (B) -7 (C) $\frac{1}{7}$ (D) $-\frac{1}{7}$

Free Response Questions

1. Evaluate.

$$\begin{vmatrix} 1 & 3 & -2 \\ 4 & 2 & 5 \\ 0 & 1 & 1 \end{vmatrix}$$

2. Find all values of λ for which

$$\begin{vmatrix} 1-\lambda & 1 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & -1 & 4-\lambda \end{vmatrix} = 0$$

3. Compute the determinant of the following matrix. Simplify completely.

$$\begin{bmatrix} \sin(\alpha)\cos(\beta) & r\cdot\cos(\alpha)\cos(\beta) & -r\cdot\sin(\alpha)\sin(\beta) \\ \sin(\alpha)\sin(\beta) & r\cdot\cos(\alpha)\sin(\beta) & r\cdot\sin(\alpha)\cos(\beta) \\ \cos(\alpha) & -r\cdot\sin(\alpha) & 0 \end{bmatrix}$$

4. If $\det(A) = -4$, find all possible values of k, where

$$A = \begin{bmatrix} 1 & k \\ k & 3k \end{bmatrix}.$$

5. Justify, without evaluating, that the determinant of the following matrix is zero.

$$\begin{bmatrix} 1 & 0 & 2 & 4 \\ -2 & 3 & 8 & 6 \\ -1 & 3 & 10 & 10 \\ 6 & 6 & -3 & 7 \end{bmatrix}$$

6. Evaluate the determinant of the following matrix by reducing to row-echelon form.

$$\begin{bmatrix} 1 & 2 & 8 \\ 2 & 5 & -3 \\ 8 & 19 & 5 \end{bmatrix}$$

7. Let E be a 3×3 elementary matrix that interchanges two rows of I_3. Prove that $\det(E) = -1$.

8. If the system $A\mathbf{x} = \mathbf{b}$ is inconsistent and A is a square matrix, what is $\det\left(A^T\right)$?

9. Use Cramer's Rule to solve the following system of equations.

$$\begin{aligned} x + y - z &= 2 \\ 3x - y + z &= 5 \\ 3x + 2y + 4z &= 0 \end{aligned}$$

10. Use the determinant to determine whether the following matrix is invertible.

$$\begin{bmatrix} 7 & 1 & -1 \\ 2 & 4 & 3 \\ -14 & -2 & 2 \end{bmatrix}$$

11. Compute the inverse of the following matrix using the adjoint matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 0 \\ 3 & 0 & 3 \end{bmatrix}$$

12. Prove that if $AA^T = I$, then $\det(A) = \pm 1$.

13. Let A, B, and P be $n \times n$ matrices where at least P is invertible. Prove that if $A = P^{-1}BP$, then $\det(A) = \det(B)$.

Answers

Multiple Choice Answers

1. (B)

2. (A)

3. (D)

4. (B)

5. (D)

6. (C)

Free Response Answers

1. -23

2. $\lambda = 1, 2, 4$

3. $r^2 \cdot \sin(\alpha)$

4. $k = 4$ or $k = -1$

5. The third row is a linear combination of the first two rows.

6. -2

8. 0

9. $x = \frac{7}{4}$, $y = -\frac{17}{24}$, $z = -\frac{23}{24}$

10. The determinant equals 0. Not invertible.

11. $A^{-1} = -\frac{1}{6}\begin{bmatrix} -3 & -6 & 3 \\ -6 & -6 & 6 \\ 3 & 6 & -5 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 & -\frac{1}{2} \\ 1 & 1 & -1 \\ -\frac{1}{2} & -1 & \frac{5}{6} \end{bmatrix}$