**Chapter 1**

1. Classify the following differential equation by order and linearity. 

A. 2nd order nonlinear

B. 4th order linear

C. 4th order nonlinear

D. 2nd order linear

Ans: A

2. Classify the following differential equation by order and linearity. 

A. 8th order nonlinear

B. 4th order linear

C. 2nd order linear

D. 2nd order nonlinear

Ans: A

3. Classify the following differential equation by order and linearity. 

A. 2nd order linear

B. 2nd order nonlinear

C. 1st order linear

D. 1st order nonlinear

Ans: A

4. Classify the following differential equation by order and linearity.



A. 2nd order linear

B. 2nd order nonlinear

C. 1st order linear

D. 1st order nonlinear

Ans: B

5. For the differential equation, the Existence-Uniqueness Theorem (Theorem 1.2.1) guarantees the existence of a unique solution whose graph passes through

A. 

B. 

C. 

D. 

Ans: A

6. For the DE  the Existence-Uniqueness Theorem (Theorem 1.2.1) guarantees the existence of a unique solution whose graph passes through

A. 

B. 

C. 

D. 

Ans: A

7. For the DE  the Existence-Uniqueness Theorem (Theorem 1.2.1) guarantees the existence of a unique solution whose graph passes through

A. 

B. 

C. 

D. 

Ans: A

8. is a one-parameter family of solutions to the DE  Which of the following is the largest interval of existence for a solution to the IVP 

A. 

B. 

C. 

D. 

Ans: A

9. is a one-parameter family of solutions to the DE  Which of the following is the largest interval of existence for a solution to the IVP  ?

A. 

B. 

C. 

D. 

Ans: A

10. Suppose that the birth rate of a certain population is proportional to the current size of the population with proportionality constant  Suppose that the death rate is also proportional to the current size of the population with proportionality constant  If the change in size of the population depends solely on the births and deaths that occur, which of the following could represent the size of the population  as a function of time 

A. 

B. 

C. 

D. 

Ans: A

11. Suppose two particles *A* and *B* are moving along the -axis. At time  particle A is located at the point on the -axis with coordinate  When, *B* is at the origin. *B*’s velocity is along the positive -axis and is proportional to the distance between *A* and *B* with proportionality constant. The initial value problem that models *B*’s position on the axis as a function of  is

A. 

B. 

C. 

D. 

Ans: A

12.  is a two-parameter family of solutions of the second-order DE . Find a solution of the second-order IVP consisting of this differential equation and the initial conditions , 

A. 

B. 

C. 

D. 

Ans: A

13. A hot cup of coffee is initially at 180° F, sits in a room with a constant air temperature of 72° F, and cools according to Newton’s law of cooling. Write an initial value problems describing the temperature of the coffee at time *t*, .

A. 

B. 

C. 

D. 

Ans: A

True/False

14. The following differential equation is linear: .

Ans: True

15. The following differential equation is linear: 

Ans: False

16. There is a constant solution to the differential equation: 

Ans: True

17. There is a constant solution to the differential equation: 

Ans: False

18.  is a solution to the differential equation .

Ans: True

19. The function  is a solution to the IVP 

Ans: False

20. The function  is a solution to the IVP 

Ans: True

21. Consider a population where the number of births and deaths are proportional to the size of the population with proportionality constants  and  respectively. Suppose also that the population experiences a constant net immigration rate of . Then the differential equation which models the population as a function of time  is 

Ans: False

22.  is a two-parameter family of solutions to the differential equation.

Ans: True

23.  is a solution to the IVP .

Ans: True

24. Every initial value problem has a unique solution.

Ans: False

25. An implicit solution to a differential equation is a relation which defines one or more solutions to the differential equation.

Ans: True

26. A singular solution to a differential equation is a solution that cannot be obtained by specializing any of the parameters in the family of solutions.

Ans: True

27. A solution to an initial value problem is always defined on .

Ans: False

Short Answer

28. Find values of so that the function  is a solution of the differential equation 

Ans: 

29. Find values of so that the function  is a solution of the differential equation



Ans: 

30.  is a two-parameter family of solutions of the second-order differential equation  Find a solution of the second-order IVP

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Ans: 

31.  is a two-parameter family of solutions of the second-order differential equation  Find a solution of the second-order differential equation

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Ans: 

32.  is a family of solutions to the differential equation  Find a solution to the IVP 

Ans: 

33. A certain island initially contains 500 inhabitants who communicate only by word of mouth. A boat containing 10 people from another country arrives on the island with news of the outside world. Determine a differential equation for the number of people who have heard the news at time  if the rate at which the news spreads is proportional to the number of interactions between those who have heard the news and those who have not. Treating this as an initial value problem, what is the initial condition? (Assume throughout the problem that the 10 outsiders stay indefinitely and become part of the islander’s population.)

Ans: 

34. Suppose a cylindrical tank with radius 3 meters and height 9 meters is resting on its circular base. Water is leaking from the tank through a circular hole of area  at its bottom. The volume of water leaving the tank per second is equal to  where  is the height of the water in the tank,  is the acceleration due to gravity, and is an empirical constant with. If the tank is full at time  and the radius of the hole is meters, determine a differential equation and initial condition for the height of water at time 

Ans: 

35. Suppose that a large mixing tank initially holds 500 gallons of water in which 75 pounds of salt have been dissolved. Pure water is pumped into the tank at a rate of 6 gal/min, and when the solution is well stirred, it is pumped out at the same rate. Determine a differential equation for the amount  of salt in the tank at time  What is 

Ans: 

36. Suppose that a large mixing tank initially holds 400 gallons of water in which 65 pounds of salt have been dissolved. Another brine solution is pumped into the tank at a rate of 6 gal/min, and when the solution is well stirred, it is pumped out at a slower rate of 5 gal/min. If the concentration of the solution entering is 3 lb/gal, determine a differential equation for the amount  of salt in the tank at time  What is 

Ans: 

37. Suppose that a large mixing tank initially holds 400 gallons of water in which 65 pounds of salt have been dissolved. Another brine solution is pumped into the tank at a rate of 6 gal/min, and when the solution is well stirred, it is pumped out at a faster rate of 8 gal/min. If the concentration of the solution entering is 3 lb/gal, determine a differential equation for the amount  of salt in the tank at time 

Ans: 

38. Determine a differential equation for the velocity  of a falling body of mass  if air resistance is proportional to the square root of the cube of the instantaneous velocity.

Ans: 

39. Suppose water is leaking from a spherical tank with radius 5 meters through a circular hole of area  at its bottom. The volume of water leaving the tank per second is equal to  where  is the height of the water in the tank,  is the acceleration due to gravity, and is an empirical constant with. If the tank is half-full at time  and the radius of the hole is meters, determine a differential equation and initial condition for the height of water at time 

Ans: 

40. Suppose the temperature of a room fluctuates with time so that the temperature at time  is given by . A  F cup of hot tea is placed on a table in the room at time . Determine a differential equation for the temperature of the tea  if the magnitude of the constant of proportionality is .

Ans: 

41. Suppose a dog is chasing a rabbit along the axis, and the coordinate of the rabbit’s position at time  is given by . If the dog’s speed is equal to one half the distance between the two animals, and the dog’s coordinate at  is  determine a differential equation for the position of the dog.

Ans: 

42. Consider a population where the number of births and deaths are proportional to the size of the population with proportionality constants  and  respectively. Suppose also that people immigrate to the country at a constant rate of . If people emigrate from the country at a rate proportional to the population  with proportionality constant , determine a differential equation which models  as a function of time .

Ans: 

43. Find the value of which makes  a two parameter family of solutions of the differential equation .

Ans: 

44. Classify the following differential equation by order and linearity.



Ans: 4th order nonlinear

45. Describe how to identify the order of a differential equation.

Ans: The order of a differential equation is the order of the highest derivative in the equation.

46. Give an example of a linear differential equation and an example of a nonlinear differential equation. Clearly indicate which is which.

Ans: Answers will vary.

47. Water drains out of a tank according to Toricelli’s law. If the upper surface of the water is always 5 times the area of the drain hole and the acceleration due to gravity is 32, then write a differential equation describing , the height of the water in the tank at time *t*.

Ans: 