

Chapter 1 Solutions

Problem 1.3-1

$$L = 4.25\text{m} \quad q_0 = 175 \frac{\text{N}}{\text{m}} \quad P = 225\text{N} \quad M_0 = 410\text{N}\cdot\text{m}$$

Reactions

$$\Sigma F_x = 0 \quad B_x = \frac{3}{5} \cdot P = 135 \cdot \text{N}$$

$$\Sigma M_A = 0 \quad B_y = \frac{1}{L} \cdot \left[-M_0 + \left(\frac{1}{2} \cdot q_0 \right) \cdot L \cdot \left(\frac{2 \cdot L}{3} \right) + \frac{4}{5} \cdot P \cdot \left(L + \frac{L}{2} \right) \right] = 421.446 \cdot \text{N}$$

$$\Sigma F_y = 0 \quad A_y = \left(\frac{1}{2} \cdot q_0 \right) \cdot L + \frac{4}{5} \cdot P - B_y = 130.429 \cdot \text{N}$$

N, V and M at midspan of AB - LHFB is used below

$$N_{\text{mid}} = 0$$

$$V_{\text{mid}} = A_y - \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} = 37.46 \cdot \text{N}$$

$$M_{\text{mid}} = -M_0 + A_y \cdot \frac{L}{2} - \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \left(\frac{1}{3} \cdot \frac{L}{2} \right) = -198.691 \cdot \text{N}\cdot\text{m}$$

Problem 1.3-2

$$L = 4\text{m} \quad q_0 = 160 \frac{\text{N}}{\text{m}} \quad P = 200 \cdot \text{N} \quad M_0 = 380 \text{N}\cdot\text{m}$$

Reactions

$$\Sigma F_x = 0 \quad B_x = \frac{-3}{5} \cdot P = -120 \text{N}$$

$$\Sigma M_A = 0 \quad B_y = \frac{1}{L} \left[M_0 + \left(\frac{1}{2} \cdot q_0 \right) \cdot L \cdot \left(\frac{L}{3} \right) - \frac{4}{5} \cdot P \cdot \left(L + \frac{L}{2} \right) \right] = -38.333 \cdot \text{N}$$

$$\Sigma F_y = 0 \quad A_y = \left(\frac{1}{2} \cdot q_0 \right) \cdot L - \frac{4}{5} \cdot P - B_y = 198.333 \cdot \text{N}$$

N, V and M at midspan of AB - LHFB is used below

$$N_{\text{mid}} = 0$$

$$V_{\text{mid}} = A_y - \frac{1}{2} \cdot \left(\frac{q_0}{2} + q_0 \right) \cdot \frac{L}{2} = -41.667 \cdot \text{N}$$

$$M_{\text{mid}} = M_0 + A_y \cdot \frac{L}{2} - \frac{1}{2} \cdot q_0 \cdot \frac{L}{2} \cdot \left(\frac{2}{3} \cdot \frac{L}{2} \right) - \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \left(\frac{1}{3} \cdot \frac{L}{2} \right) = 510 \cdot \text{N}\cdot\text{m}$$

Check using RHFB

$$N_{\text{mid}} = B_x + \frac{3}{5} \cdot P = 0 \text{N} \quad V_{\text{mid}} = \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} - B_y - \frac{4}{5} \cdot P = -41.667 \text{N}$$

$$M_{\text{mid}} = \frac{-1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \left(\frac{1}{3} \cdot \frac{L}{2} \right) + B_y \cdot \frac{L}{2} + \frac{4}{5} \cdot P \cdot \left(\frac{L}{2} + \frac{L}{2} \right) = 510 \cdot \text{N}\cdot\text{m}$$

Problem 1.3-3

(a) APPLY LAWS OF STATICS

$$\Sigma F_x = 0 \quad C_x = N_A - N_2 = 220 \text{ N}$$

$$\text{FBD of BC:} \quad \Sigma M_B = 0 \quad C_y = \frac{1}{L_1}(0) = 0$$

$$\text{Entire FBD:} \quad \Sigma M_A = 0 \quad B_y = \frac{1}{L_2}(-M_B) = -22.667 \text{ N}$$

$$\Sigma F_y = 0 \quad A_y = -B_y = 22.667 \text{ N}$$

$$\text{Reactions are } \boxed{A_y = 22.7 \text{ N}} \quad \boxed{B_y = -22.7 \text{ N}} \quad \boxed{C_x = 220 \text{ N}} \quad \boxed{C_y = 0}$$

(b) INTERNAL STRESS RESULTANTS N , V , AND M AT $x = 4.5 \text{ m}$

Use FBD of segment from A to $x = 4.5 \text{ m}$.

$$\Sigma F_x = 0 \quad \boxed{N_x = N_A - N_2 = 220 \text{ N}}$$

$$\Sigma F_y = 0 \quad \boxed{V_x = A_y = 22.7 \text{ N}}$$

$$\Sigma M = 0 \quad \boxed{M_x = A_y(4.5 \text{ m}) = 102 \text{ N}\cdot\text{m}}$$

Problem 1.3-4

(a) APPLY LAWS OF STATICS

$$\Sigma F_x = 0 \quad A_x = 0$$

$$\text{FBD of } AB \quad \Sigma M_B = 0 \quad M_A = 0$$

$$\text{Entire FBD} \quad \Sigma M_C = 0 \quad D_y = \frac{1}{3} \text{ m} \left[200 \text{ N}\cdot\text{m} - \frac{1}{2} (80 \text{ N/m}) 4 \text{ m} \left(\frac{2}{3} \right) 4 \text{ m} \right] = -75.556 \text{ N}$$

$$\Sigma F_y = 0 \quad C_y = \frac{1}{2} (80 \text{ N/m}) 4 \text{ m} - D_y = 235.556 \text{ N}$$

$$\text{Reactions are} \quad \boxed{M_A = 0} \quad A_x = 0 \quad \boxed{C_y = 236 \text{ N}} \quad \boxed{D_y = -75.6 \text{ N}}$$

(b) INTERNAL STRESS RESULTANTS N , V , AND M AT $x = 5 \text{ m}$

Use FBD of segment from A to $x = 5 \text{ m}$; ordinate on triangular load at $x = 5 \text{ m}$ is $\frac{3}{4} (80 \text{ N/m}) = 60 \text{ N/m}$.

$$\Sigma F_x = 0 \quad N_x = -A_x = 0$$

$$\Sigma F_y = 0 \quad V = \frac{-1}{2} [(80 \text{ N/m} + 60 \text{ N/m}) 1 \text{ m}] = -70 \text{ N} \quad \boxed{V = -70 \text{ N}} \quad \text{Upward}$$

$$\Sigma M = 0 \quad M = -M_A - \frac{1}{2} (80 \text{ N/m}) 1 \text{ m} \left(\frac{2}{3} 1 \text{ m} \right) - \frac{1}{2} (60 \text{ N/m}) 1 \text{ m} \left(\frac{1}{3} 1 \text{ m} \right) = -36.667 \text{ N}\cdot\text{m}$$

(break trapezoidal load into two triangular loads in moment expression)

$$\boxed{M = -36.7 \text{ N}\cdot\text{m}} \quad \text{CW}$$

(c) REPLACE ROLLER SUPPORT AT C WITH SPRING SUPPORT

Structure remains statically determinate so all results above in (a) and (b) are unchanged.

Problem 1.3-5

(a) STATICS

FBD of AB (cut through beam at pin): $\sum M_B = 0 \quad A_y = \frac{1}{3 \text{ m}}(-M_A) = -\frac{200 \text{ N}}{3} = -66.667 \text{ N}$

Entire FBD: $\sum M_D = 0 \quad L_1 = 3 \text{ m}$

$$C_y = \frac{1}{3 \text{ m}} \left[\frac{4}{5} P (1.5 \text{ m}) + \frac{1}{2} q_2 L_1 \left(L_1 + \frac{L_1}{3} \right) + \frac{1}{2} q_1 L_1 \left(L_1 + \frac{2}{3} L_1 \right) - M_A - A_y (3L_1) \right] = 464.333 \text{ N}$$

$$\sum F_y = 0 \quad D_y = \frac{4}{5} P + \frac{1}{2} (q_1 + q_2) L_1 - A_y - C_y = -87.167 \text{ N} \quad \text{so} \quad D_x = \frac{-D_y}{\tan(60^\circ)} = 50.326 \text{ N}$$

$$\sum F_x = 0 \quad A_x = \frac{3}{5} P - D_x = 57.674 \text{ N}$$

CHECK

$$A_x = 57.7 \text{ N} \quad A_y = -66.7 \text{ N} \quad C_y = 464.3 \text{ N} \quad A_x + D_x = 108 \text{ N} \quad P \frac{3}{5} = 108 \text{ N}$$

$$D_x = 50.3 \text{ N} \quad D_y = -87.2 \text{ N} \quad A_y + C_y + D_y = 310.5 \text{ N} \quad \frac{4}{5} P + \frac{1}{2} (q_1 + q_2) L_1 = 310.5 \text{ N}$$

(b) USE FBD OF AB ONLY; MOMENT AT PIN IS ZERO

$$F_{Bx} = -A_x \quad F_{Bx} = -57.674 \text{ N} \quad F_{By} = -A_y \quad F_{By} = 66.667 \text{ N} \quad \boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 88.2 \text{ N}}$$

(c) ADD ROTATIONAL SPRING AT A & REMOVE ROLLER AT C; APPLY EQUATIONS OF STATICAL EQUILIBRIUM

Use FBD of BCD: $\sum M_B = 0 \quad D_y = \frac{1}{2L_1} \left[\frac{1}{2} q_2 L_1 \left(\frac{2}{3} L_1 \right) + \frac{1}{2} q_1 L_1 \left(\frac{1}{3} L_1 \right) + \frac{4}{5} P \left(\frac{3}{2} L_1 \right) \right] = 145 \text{ N}$

so $D_x = \frac{-D_y}{\tan(60^\circ)} = -83.716 \text{ N}$

Use entire FBD: $\sum F_y = 0 \quad A_y = \frac{1}{2} (q_1 + q_2) L_1 + \frac{4}{5} P - D_y = 165.5 \text{ N}$

$$\sum F_x = 0 \quad A_x = \frac{3}{5} P - D_x = 191.716 \text{ N}$$

Use FBD of AB: $\sum M_B = 0 \quad M_{Az} = M_A + A_y L_1 = 696.5 \text{ N}\cdot\text{m}$

SO REACTIONS ARE $\boxed{A_x = 191.7 \text{ N}} \quad \boxed{A_y = 165.5 \text{ N}} \quad \boxed{M_{Az} = 697 \text{ N}\cdot\text{m}} \quad \boxed{D_x = -83.72 \text{ N}} \quad \boxed{D_y = 145 \text{ N}}$

RESULTANT FORCE IN PIN CONNECTION AT B

$$F_{Bx} = -A_x \quad F_{By} = -A_y \quad \boxed{\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 253 \text{ N}}$$

Problem 1.3-6

(a) STATICS

$$\Sigma F_y = 0 \quad R_{3y} = 20 \text{ N} - 45 \text{ N} = -25 \text{ N}$$

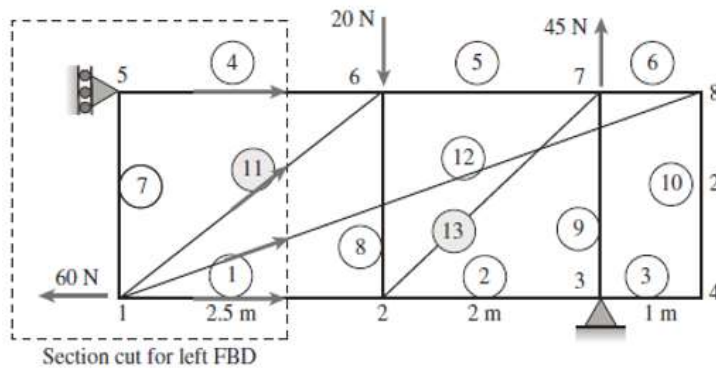
$$\Sigma M_3 = 0 \quad R_{5x} = \frac{1}{2 \text{ m}} (20 \text{ N} \times 2 \text{ m}) = 20 \text{ N}$$

$$\Sigma F_x = 0 \quad R_{3x} = -R_{5x} + 60 \text{ N} = 40 \text{ N}$$

(b) MEMBER FORCES IN MEMBERS 11 and 13

Number of unknowns: $m = 13 \quad r = 3 \quad m + r = 16$

Number of equations: $j = 8 \quad 2j = 16 \quad \text{So statically determinate}$



TRUSS ANALYSIS

- (1) $\Sigma F_V = 0$ at joint 4 so $F_{10} = 0$
- (2) $\Sigma F_V = 0$ at joint 8 so $F_{12} = 0$
- (3) $\Sigma F_H = 0$ at joint 5 so $F_4 = -R_{5x} = -20 \text{ N}$
- (4) Cut vertically through 4, 11, 12, and 1; use left FBD; sum moments about joint 2

$$F_{11V} = \frac{1}{2.5 \text{ m}} (R_{5x} - F_4) \quad \text{so} \quad \boxed{F_{11} = 0}$$
- (5) Sum vertical forces at joint 3; $F_9 = -R_{3y}$

$$F_9 = 25 \text{ N}$$

(6) Sum vertical forces at joint 7 $F_{13V} = 45 \text{ N} - F_9 = 20 \text{ N} \quad \boxed{F_{13} = \sqrt{2} F_{13V} = 28.3 \text{ N}}$

Problem 1.3-7

(a) STATICS $P = 4.5 \text{ kN}$ $L_1 = 3 \text{ m}$

$$\Sigma F_x = 0 \quad A_x = 0$$

$$\Sigma M_A = 0 \quad E_y = \frac{1}{6 \text{ m}} [3P(3 \text{ m}) + 2P(6 \text{ m}) + P(9 \text{ m})] = 22.5 \text{ kN}$$

$$\Sigma F_y = 0 \quad A_y = 3P + 2P + P - E_y = 4.5 \text{ kN}$$

(b) MEMBER FORCE IN MEMBER FE

Number of unknowns: $m = 11$ $r = 3$ $m + r = 14$

Number of equations: $j = 7$ $2j = 14$ so statically determinate

TRUSS ANALYSIS

(1) Cut vertically through AB, GC, and GF; use left FBD; sum moments about C.

$$F_{GFx}(4.5 \text{ m}) - F_{GFy}(6 \text{ m}) = A_y(6 \text{ m}) = 27 \text{ kN}\cdot\text{m} \quad F_{GFx} = F_{GF} \frac{3 \text{ m}}{\sqrt{(0.5 \text{ m})^2 + (3 \text{ m})^2}}$$

$$F_{GFy} = F_{GF} \frac{0.5 \text{ m}}{\sqrt{(0.5 \text{ m})^2 + (3 \text{ m})^2}}$$

$$\text{so } F_{GF} = \frac{A_y(6 \text{ m})}{4.5 \text{ m} \frac{3 \text{ m}}{\sqrt{(0.5 \text{ m})^2 + (3 \text{ m})^2}} - 6 \text{ m} \frac{0.5 \text{ m}}{\sqrt{(0.5 \text{ m})^2 + (3 \text{ m})^2}}} = 7.821 \text{ kN} \quad \text{and}$$

$$F_{GFx} = F_{GF} \frac{3 \text{ m}}{\sqrt{(0.5 \text{ m})^2 + (3 \text{ m})^2}} = 7.714 \text{ kN}$$

(2) Sum horizontal forces at joint F: $F_{FEx} = F_{GFx} = 7.714 \text{ kN}$ $F_{FE} = \frac{\sqrt{(3 \text{ m})^2 + (1 \text{ m})^2}}{3 \text{ m}} F_{FEx} = 8.132 \text{ kN}$

$$\boxed{F_{FE} = 8.13 \text{ kN}}$$

Problem 1.3-8

(a) STATICS

$$\Sigma F_x = 0 \quad F_x = 0$$

$$\Sigma M_F = 0 \quad D_y = \frac{1}{6 \text{ m}} [3 \text{ kN}(6 \text{ m}) + 6 \text{ kN}(3 \text{ m})] = 6 \text{ kN}$$

$$\Sigma F_y = 0 \quad F_y = 9 \text{ kN} + 6 \text{ kN} + 3 \text{ kN} - D_y = 12 \text{ kN}$$

(b) MEMBER FORCE IN MEMBER FE

$$\text{Number of unknowns:} \quad m = 11 \quad r = 3 \quad m + r = 14$$

$$\text{Number of equations:} \quad j = 7 \quad 2j = 14 \quad \text{So statically determinate}$$

TRUSS ANALYSIS

(1) Cut vertically through AB , GD , and GF ; use left FBD; sum moments about D to get $F_{GF} = 0$

(2) Sum horizontal forces at joint F $F_{FE_x} = -F_x = 0$ so $\boxed{F_{FE} = 0}$

Problem 1.3-9

$$c = 2.5m \quad P = 90kN$$

$$a = \frac{\sin(60deg)}{\sin(80deg)} \cdot c = 2.198 \cdot m \quad b = \frac{\sin(40deg)}{\sin(80deg)} \cdot c = 1.632 \cdot m$$

$$\Sigma M_A = P \cdot \frac{c}{2} - P \cdot b \cdot \cos(60deg) - 2P \cdot b \cdot \sin(60deg) + B_y \cdot c = 0$$

$$B_y = \frac{P \cdot b \cdot \cos(60deg) + 2P \cdot b \cdot \sin(60deg) - P \cdot \frac{c}{2}}{c} = 86.118 \cdot kN \quad A_y = -B_y = -86.118 \cdot kN$$

$$A_x = 0$$

Joint A

$$F_{AC} = \frac{-A_y}{\sin(60deg)} = 99.441 \cdot kN \quad F_{AD} = -F_{AC} \cdot \cos(60deg) - A_x = -49.72 \cdot kN$$

Joint B

$$F_{BC} = \frac{-B_y}{\sin(40deg)} = -133.976 \cdot kN$$

$$F_{BD} = -F_{BC} \cdot \cos(40deg) = 102.632 \cdot kN$$

$$CD = \sqrt{b^2 + \left(\frac{c}{2}\right)^2 - 2 \cdot b \cdot \frac{c}{2} \cdot \cos(60deg)} = 1.478 \cdot m$$

Joint D

$$F_{DC} = \frac{-P}{\cos(90deg - 72.923deg)} = -94.151 \cdot kN$$

$$ACD = \arcsin\left(\frac{\sin(60deg)}{CD} \cdot \frac{c}{2}\right) = 47.077 \cdot deg$$

$$180deg - 60deg - ACD = 72.923 \cdot deg$$

$$BCD = \arcsin\left(\frac{\sin(40deg)}{CD} \cdot \frac{c}{2}\right) = 32.923 \cdot deg$$

$$ACD + BCD = 80 \cdot deg$$

$$\frac{a \cdot \sin(BCD)}{\sin(ACD)} = 1.632 \cdot m$$

Problem 1.3-10

Geometry $b = 3\text{ m}$ $P = 80\text{ kN}$

$$a = \sin(60\text{deg}) \cdot \left(\frac{b}{\sin(40\text{deg})} \right) = 4.042\text{ m}$$

$$L_{AB} = \sin(80\text{deg}) \cdot \left(\frac{b}{\sin(40\text{deg})} \right) = 4.596\text{ m}$$

$$L_{DB} = \sqrt{\left(\frac{b}{2} \right)^2 + L_{AB}^2 - 2 \cdot \left(\frac{b}{2} \right) \cdot (L_{AB}) \cdot \cos(60\text{deg})} = 4.06\text{ m}$$

$$\frac{L_{DB}}{\sin(60\text{deg})} = \frac{\frac{b}{2}}{\sin(\text{DBA})} \quad \text{so} \quad \text{DBA} = \text{asin} \left(\frac{\frac{b}{2}}{L_{DB}} \cdot \sin(60\text{deg}) \right) = 18.662\text{ deg}$$

$$\text{and} \quad \text{CBD} = 40\text{deg} - \text{DBA} = 21.338\text{ deg} \quad \text{ADB} = 180\text{deg} - 60\text{deg} - \text{DBA} = 101.338\text{ deg}$$

$$\text{CDB} = 180\text{deg} - \text{ADB} = 78.662\text{ deg}$$

Reactions

$$\Sigma F_x = 0 \quad A_x = -2 \cdot P + 2 \cdot P = 0\text{ N}$$

$$\Sigma M_A = 0 \quad B_y = \frac{1}{L_{AB}} \cdot \left[-2 \cdot P \cdot \left(\frac{b}{2} \cdot \sin(60\text{deg}) \right) + P \cdot (b \cdot \cos(60\text{deg})) + 2 \cdot P \cdot (b \cdot \sin(60\text{deg})) \right] = 71.329\text{ kN}$$

$$\Sigma F_y = 0 \quad A_y = P - B_y = 8.671\text{ kN}$$

MoJ to find member forces

$$\text{Joint A} \quad AD = \frac{-A_y}{\sin(60\text{deg})} = -10.013\text{ kN} \quad AB = -A_x - AD \cdot \cos(60\text{deg}) = 5.006\text{ kN}$$

$$\text{Joint D - sum forces normal to \& along line ADC} \quad DB = \frac{2 \cdot P \cdot \sin(60\text{deg})}{\cos(90\text{deg} - \text{CDB})} = 141.322\text{ kN}$$

$$DC = AD + 2 \cdot P \cdot (\cos(60\text{deg})) - DB \cdot \cos(\text{CDB}) = 42.204\text{ kN}$$

$$\text{Joint C} \quad CB = \frac{1}{\cos(40\text{deg})} \cdot (-2 \cdot P + DC \cdot \cos(60\text{deg})) = -181.319\text{ kN}$$

$$\text{Joint B} \quad \text{check} \quad -AB - DB \cdot \cos(\text{DBA}) - CB \cdot \cos(40\text{deg}) = 0\text{ N}$$

$$DB \cdot \sin(\text{DBA}) + CB \cdot \sin(40\text{deg}) + B_y = 0\text{ N}$$

Problem 1.3-11

Reactions $c = 2.5\text{m}$ $P = 90\text{kN}$

$$a = \frac{\sin(60\text{deg})}{\sin(80\text{deg})} \cdot c = 2.198 \cdot \text{m} \qquad b = \frac{\sin(40\text{deg})}{\sin(80\text{deg})} \cdot c = 1.632 \cdot \text{m}$$

$$\Sigma M_A = P \cdot \frac{c}{2} - P \cdot b \cdot \cos(60\text{deg}) - 2P \cdot b \cdot \sin(60\text{deg}) + B_y \cdot c = 0$$

$$B_y = \frac{P \cdot b \cdot \cos(60\text{deg}) + 2P \cdot b \cdot \sin(60\text{deg}) - P \cdot \frac{c}{2}}{c} = 86.118 \cdot \text{kN} \qquad A_y = -B_y = -86.118 \cdot \text{kN}$$

AC: MoS - cut through AC and AD, use LHFB

$$\Sigma M_D = 0 \qquad -A_y \cdot \frac{c}{2} - AC \cdot \sin(60\text{deg}) \cdot \frac{c}{2} = 0$$

$$AC = \frac{-A_y}{\sin(60\text{deg})} = 99.441 \cdot \text{kN}$$

BD: MoS - cut through BC andf BD, use RHFB

$$b = \frac{\sin(40\text{deg})}{\sin(80\text{deg})} \cdot c = 1.632 \cdot \text{m}$$

$$\Sigma M_C = 0 \qquad B_y \cdot (c - b \cdot \cos(60\text{deg})) - BD \cdot (b \cdot \sin(60\text{deg})) = 0$$

$$BD = \frac{B_y \cdot (c - b \cdot \cos(60\text{deg}))}{b \cdot \sin(60\text{deg})} = 102.632 \cdot \text{kN}$$

Problem 1.3-12

Reactions $b = 3\text{m}$ $P = 80\text{kN}$

$$A_x = 0 \quad A_y = 8.671\text{kN} \quad B_y = 71.329\text{kN}$$

AB: MoS - cut through AD and AB, use LHFB

$$\Sigma M_D = 0 \quad AB \cdot \frac{b}{2} \cdot \sin(60\text{deg}) + A_x \cdot \frac{b}{2} \cdot \sin(60\text{deg}) - A_y \cdot \frac{b}{2} \cdot \cos(60\text{deg}) = 0$$

$$AB = \frac{-\left(A_x \cdot \frac{b}{2} \cdot \sin(60\text{deg}) - A_y \cdot \frac{b}{2} \cdot \cos(60\text{deg})\right)}{\left(\frac{b}{2} \cdot \sin(60\text{deg})\right)} = 5.006\text{kN}$$

DC: MoS - cut through DC and CB, use upper FBD $a = \sin(60\text{deg}) \cdot \left(\frac{b}{\sin(40\text{deg})}\right) = 4.042\text{m}$

$$DC_x = DC \cdot \cos(60\text{deg}) \quad DC_y = DC \cdot \sin(60\text{deg})$$

$$\Sigma M_B = 0 \quad -(-DC_x + 2 \cdot P) \cdot (a \cdot \sin(40\text{deg})) + (DC_y + P) \cdot (a \cdot \cos(40\text{deg})) = 0$$

$$-(-DC \cdot \cos(60\text{deg}) + 2 \cdot P) \cdot (a \cdot \sin(40\text{deg})) + (DC \cdot \sin(60\text{deg}) + P) \cdot (a \cdot \cos(40\text{deg})) = 0$$

Collect and simplify, solve for DC

$$DC = \frac{1.0 \cdot (80.0 \cdot \text{kN} \cdot \cos(40.0 \cdot \text{deg}) - 160.0 \cdot \text{kN} \cdot \sin(40.0 \cdot \text{deg}))}{\cos(60.0 \cdot \text{deg}) \cdot \sin(40.0 \cdot \text{deg}) + \sin(60.0 \cdot \text{deg}) \cdot \cos(40.0 \cdot \text{deg})} = 42.204 \cdot \text{kN}$$

Problem 1.3-13

(a) FIND REACTIONS USING STATICS $m = 3$ $r = 9$ $m + r = 12$ $j = 4$ $3j = 12$
 $m + r = 3j$ so truss is statically determinate

$$r_{AQ} = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} \quad r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \quad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.8 \\ -0.6 \\ 0 \end{pmatrix} \quad P_A = P e_{AQ} = \begin{pmatrix} 0.8P \\ -0.6P \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$\Sigma M = 0$

$$M_O = r_{OA} \times P_A + r_{OC} \times \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} + r_{OB} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 4C_z + 3.0P \\ 4.0P - 2B_z \\ 2B_y - 4C_x \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad C_z = \frac{-3}{4}P$$

$$\Sigma M_y = 0 \quad \text{gives} \quad \boxed{B_z = 2P}$$

$\Sigma F = 0$

$$R_O = P_A + \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} B_x + C_x + O_x + 0.8P \\ B_y + C_y + O_y + -0.6P \\ O_z + \frac{5P}{4} \end{pmatrix} \quad \text{so} \quad \Sigma M_z = 0 \quad \text{gives} \quad \boxed{O_z = \frac{-5}{4}P}$$

METHOD OF JOINTS

Joint O	$\Sigma F_x = 0$	$O_x = 0$	$\Sigma F_y = 0$	$O_y = 0$
Joint B	$\Sigma F_y = 0$	$B_y = 0$		
Joint C	$\Sigma F_x = 0$	$C_x = 0$		

For entire structure: $\Sigma F_x = 0$ gives $\boxed{B_x = -0.8P}$ $\Sigma F_y = 0$ $C_y = 0.6P - B_y = O_y$ $C_y = 0.6P$

(b) FORCE IN MEMBER AC

$$\Sigma F_z = 0 \quad \text{at joint C:} \quad F_{AC} = \frac{\sqrt{4^2 + 5^2}}{5} |C_z| = \frac{3\sqrt{41}}{20} |P| \quad \boxed{F_{AC} = \frac{3\sqrt{41}}{20} P} \quad \text{tension} \quad \frac{3\sqrt{41}}{20} = 0.96$$

Problem 1.3-14

(a) FIND REACTIONS USING STATICS $m = 4$ $r = 8$ $m + r = 12$ $j = 4$ $3j = 12$
 $m + r = 3j$ so truss is statically determinate

$$r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 0.8L \end{pmatrix} \quad r_{OB} = \begin{pmatrix} L \\ 0 \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0.6L \\ 0 \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ P \end{pmatrix} \quad F_B = \begin{pmatrix} 0 \\ B_y \\ B_z \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ -2P \\ 0 \end{pmatrix} \quad F_O = \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix}$$

$$\Sigma M = 0$$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} -0.8A_yL \\ 0.8A_xL - B_zL \\ B_yL - 0.6C_xL \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad A_y = 0$$

$$\Sigma F = 0$$

Resultant force at O

$$R_O = F_O + F_A + F_B + F_C = \begin{pmatrix} A_x + C_x + O_x \\ A_y + B_y + O_y - 2P \\ B_z + O_z + P \end{pmatrix}$$

METHOD OF JOINTS Joint O $\Sigma F_z = 0$ $O_z = 0$

so from $\Sigma F_z = 0$ $B_z = -P$ and $\Sigma M_y = 0$ $A_x = \frac{B_z}{0.8} = -1.25P$

Joint B $\Sigma F_y = 0$ $B_y = 0$

Joint C $\Sigma F_x = 0$ $C_x = 0$

(b) FORCE IN MEMBER AB

$$\Sigma F_z = 0 \quad \text{at joint } B \quad F_{AB} = \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L} |B_z| \quad |B_z| = |P| \quad \frac{\sqrt{(0.8L)^2 + L^2}}{0.8L} = 1.601$$

$$F_{AB} = 1.601P \quad \text{tension}$$

Problem 1.3-15

(a) FIND REACTIONS USING STATICS $m = 3$ $r = 6$ $m + r = 9$ $j = 3$ $3j = 9$

$m + r = 3j$ So truss is statically determinate

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 2L \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} -2P \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ B_y \\ 3P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ P \end{pmatrix}$$

$$\Sigma M = 0$$

Resultant moment at O

$$M_O = r_{OA} \times F_A + r_{OB} \times F_B + r_{OC} \times F_C = \begin{pmatrix} 14LP - 4C_yL \\ 4C_xL - 3A_zL \\ 3A_yL - 4B_xL - 2C_xL \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \quad \text{gives} \quad C_y = \frac{14}{4}P$$

$$\Sigma F = 0$$

Resultant force at O

$$R_O = F_A + F_B + F_C = \begin{pmatrix} B_x + C_x - 2P \\ A_y + B_y + C_y \\ A_z + 4P \end{pmatrix} \quad \text{so} \quad \Sigma F_z = 0 \quad \text{gives} \quad \boxed{A_z = -4.0P}$$

METHOD OF JOINTS

$$\text{Joint A} \quad \Sigma F_z = 0 \quad F_{ACz} = -A_z = 4.0P \quad \text{so} \quad F_{ACy} = \frac{2}{4}F_{ACz} = 2.0P \quad F_{ACx} = \frac{3}{4}F_{ACz} = 3.0P$$

$$\Sigma F_x = 0 \quad F_{ABx} = -2P - F_{ACx} = -3.0P - 2P \quad \text{so} \quad F_{ABy} = \frac{4}{3}F_{ABx} = -4.0P - \frac{8P}{3}$$

$$\Sigma F_y = 0 \quad A_y = -(F_{ABy} + F_{ACy}) = \frac{8P}{3} + 4.0P + -2.0P \quad \boxed{A_y = 4.67P}$$

(b) FORCE IN MEMBER AB

$$F_{AB} = \sqrt{F_{ABx}^2 + F_{ABy}^2} \quad F_{AB} = -\sqrt{5^2 + \left(\frac{20}{3}\right)^2}P = -\frac{25P}{3} \quad \frac{25}{3} = 8.33$$

$$\boxed{F_{AB} = -8.33P} \quad \text{compression}$$

Problem 1.3-16

(a) FIND REACTIONS USING STATICS $m = 3$ $r = 6$ $m + r = 9$ $j = 3$ $3j = 9$
 $m + r = 3j$ so truss is statically determinate

$L = 2 \text{ m}$ $P = 5 \text{ kN}$

$$r_{OA} = \begin{pmatrix} 3L \\ 0 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 0 \\ 4L \\ 2L \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 0 \\ 4L \end{pmatrix} \quad F_A = \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} \quad F_B = \begin{pmatrix} B_x \\ 0 \\ P \end{pmatrix} \quad F_C = \begin{pmatrix} C_x \\ C_y \\ -P \end{pmatrix}$$

$\Sigma F = 0$

Resultant force at O $R_O = F_A + F_B + F_C = \begin{pmatrix} A_x + B_x + C_x \\ A_y + C_y \\ A_z \end{pmatrix}$ so $\Sigma F_z = 0$ gives $A_z = 0$

RESULTANT MOMENT AT A

$$r_{AC} = \begin{pmatrix} -3L \\ 0 \\ 4L \end{pmatrix} \quad e_{AC} = \frac{r_{AC}}{|r_{AC}|} = \begin{pmatrix} -0.6 \\ 0 \\ 0.8 \end{pmatrix} \quad r_{AB} = \begin{pmatrix} -3L \\ 4L \\ 2L \end{pmatrix}$$

$$M_A = r_{AB} \times F_B + r_{AC} \times F_C = \begin{pmatrix} 40 \text{ kN}\cdot\text{m} - 8 \text{ C}_y \cdot \text{m} \\ 4 \text{ B}_x \cdot \text{m} + 8 \text{ C}_x \cdot \text{m} \\ -8 \text{ B}_x \cdot \text{m} - 6 \text{ C}_y \cdot \text{m} \end{pmatrix} \quad M_A e_{AC} = -6.4 \text{ B}_x - 24.0 \text{ kN} \quad \text{so} \quad \boxed{B_x = \frac{-24}{6.4} \text{ kN} = -3.75 \text{ kN}}$$

(b) FORCE IN MEMBER AB

Method of joints at B $\Sigma F_x = 0$ $F_{ABx} = -B_x$ $\boxed{F_{AB} = \frac{\sqrt{29}}{3} F_{ABx} = 6.73 \text{ kN}}$

Problem 1.3-17

(a) APPLY LAWS OF STATICS

$$\Sigma M_x = 0 \quad \boxed{T_A = T_1 - T_2 = 1270 \text{ N}\cdot\text{m}}$$

(b) INTERNAL STRESS RESULTANT T AT TWO LOCATIONS

Cut shaft at midpoint between A and B at $x = L_1/2$
(use left FBD).

$$\Sigma M_x = 0 \quad \boxed{T_{AB} = -T_A = -1270 \text{ N}\cdot\text{m}}$$

Cut shaft at midpoint between B and C at $x = L_1 + L_2/2$
(use right FBD).

$$\Sigma M_x = 0 \quad \boxed{T_{BC} = T_2 = 1130 \text{ N}\cdot\text{m}}$$

Problem 1.3-18

(a) REACTION TORQUE AT A $L_1 = 0.75 \text{ m}$ $L_2 = 0.75 \text{ m}$ $t_1 = 3100 \text{ N}\cdot\text{m}/\text{m}$ $T_2 = 1100 \text{ N}\cdot\text{m}$

Statics $\Sigma M_x = 0$ $T_A = -t_1 L_1 + T_2 = -1225 \text{ N}\cdot\text{m}$ $T_A = -1225 \text{ N}\cdot\text{m}$

(b) INTERNAL TORSIONAL MOMENTS AT TWO LOCATIONS

Cut shaft between A and B
(use left FBD) $T_1(x) = -T_A - t_1 x$ $T_1\left(\frac{L_1}{2}\right) = 62.5 \text{ N}\cdot\text{m}$

Cut shaft between B and C
(use left FBD) $T_2(x) = -T_A - t_1 L_1$ $T_2\left(L_1 + \frac{L_2}{2}\right) = -1100 \text{ N}\cdot\text{m}$

Problem 1.3-19

(a) STATICS

$$\Sigma F_H = 0 \quad A_x = \frac{-1}{2} \left(1300 \frac{\text{N}}{\text{m}} \right) 3.70 \text{ m} = -2405 \text{ N}$$

$$\Sigma F_V = 0 \quad A_y + C_y = 0$$

$$\Sigma M_{FBDBC} = 0 \quad C_y = \frac{680 \text{ N}\cdot\text{m}}{2.75 \text{ m}} = 247 \text{ N} \quad A_y = -C_y = -247 \text{ N}$$

$$\Sigma M_A = 0 \quad M_A = 680 \text{ N}\cdot\text{m} + \frac{1}{2} \left(1300 \frac{\text{N}}{\text{m}} \right) 3.70 \text{ m} \left(\frac{2}{3} 3.70 \text{ m} \right) - C_y 2.75 \text{ m} = 5932 \text{ N}\cdot\text{m}$$

$$A_x = -2405 \text{ N} \quad A_y = -247 \text{ N} \quad M_A = 5932 \text{ N}\cdot\text{m} \quad C_y = 247 \text{ N} \quad \leftarrow$$

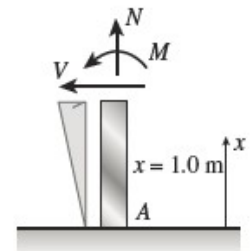
(b) INTERNAL STRESS RESULTANTS

$$N_x = -A_y = 247 \text{ N}$$

$$V_x = -A_x - \frac{1}{2} \left(\frac{1.0}{3.70} 1300 \frac{\text{N}}{\text{m}} \right) 1.0 \text{ m} = 2229 \text{ N}$$

$$M_x = -M_A - A_x 1.0 \text{ m} - \frac{1}{2} \left(\frac{1.0}{3.70} 1300 \frac{\text{N}}{\text{m}} \right) 1.0 \text{ m} \left(\frac{1}{3} 1.0 \text{ m} \right) = -3586 \text{ N}\cdot\text{m}$$

$$N_x = 247 \text{ N} \quad V_x = 2229 \text{ N} \quad M_x = -3586 \text{ N}\cdot\text{m} \quad \leftarrow$$



Problem 1.3-20

(a) STATICS

$$\Sigma F_x = 0 \quad A_x = \frac{3}{5}(200 \text{ N}) + \frac{1}{2}(80 \text{ N/m}) 4 \text{ m} = 280 \text{ N}$$

$$\Sigma M_{BRHFB} = 0 \quad D_y = \frac{1}{3 \text{ m}} \left[\frac{4}{5}(200 \text{ N})(1.5 \text{ m}) + \frac{1}{2}(80 \text{ N/m}) 4 \text{ m} \left(\frac{1}{3} 4 \text{ m} \right) \right]$$

$$= 151.1 \text{ N} < \text{ use right hand FBD (BCD only)}$$

$$\Sigma F_y = 0 \quad A_y = -D_y + \frac{4}{5}(200 \text{ N}) = 8.89 \text{ N}$$

$$\Sigma M_A = 0 \quad M_A = \frac{4}{5}(200 \text{ N})(1.5 \text{ m}) - \frac{3}{5}(200 \text{ N})(4 \text{ m}) - D_y 3 \text{ m} - \frac{1}{2}(80 \text{ N/m}) 4 \text{ m} \left(\frac{2}{3} 4 \text{ m} \right) = -1120 \text{ N}\cdot\text{m}$$

(b) RESULTANT FORCE IN PIN AT B

LEFT HAND FBD (SEE FIGURE)

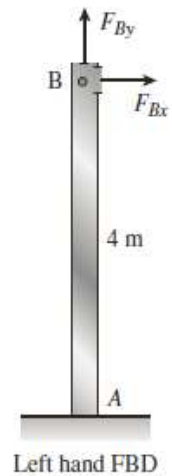
$$F_{Bx} = -A_x = -280 \text{ N} \quad F_{By} = -A_y = -8.89 \text{ N}$$

RIGHT HAND FBD

$$F_{Bx} = \frac{3}{5}(200 \text{ N}) + \frac{1}{2}(80 \text{ N/m}) 4 \text{ m} = 280 \text{ N}$$

$$F_{By} = \frac{4}{5}(200 \text{ N}) - D_y = 8.89 \text{ N}$$

$$\text{Resultant}_B = \sqrt{F_{Bx}^2 + F_{By}^2} = 280 \text{ N}$$



Problem 1.3-21

$$L = 4.25\text{m} \quad q_0 = 175 \frac{\text{N}}{\text{m}} \quad P = 225\text{N} \quad M_o = 400\text{N}\cdot\text{m}$$

$$\Sigma M_D = -M_o + \frac{1}{2} \cdot q_0 \cdot L \cdot \frac{L}{3} - \frac{4}{5} \cdot P \cdot \frac{L}{2} + \frac{3}{5} P \cdot L - \frac{1}{2} \cdot q_0 \cdot L \cdot \frac{2L}{3} - A_y \cdot L = 0$$

$$A_y = \frac{-M_o + \frac{1}{2} \cdot q_0 \cdot L \cdot \frac{L}{3} - \frac{4}{5} \cdot P \cdot \frac{L}{2} + \frac{3}{5} P \cdot L - \frac{1}{2} \cdot q_0 \cdot L \cdot \frac{2L}{3}}{L} = -173.076\text{N}$$

$$D_y = -A_y + \frac{1}{2} \cdot q_0 \cdot L + \frac{4}{5} \cdot P = 724.951\text{N}$$

$$D_x = \frac{-1}{2} \cdot q_0 \cdot L + \frac{3}{5} \cdot P = -236.875\text{N}$$

$$V_{\text{midAB}} = A_y - \frac{1}{2} \cdot \frac{L}{2} \cdot \frac{q_0}{2} = -266.045\text{N} \quad N_{\text{mid}} = 0$$

$$N_{\text{midAB}} = 0$$

$$M_{\text{midAB}} = M_o + A_y \cdot \frac{L}{2} - \frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \cdot \frac{\frac{L}{2}}{3} = -33.639\text{N}\cdot\text{m}$$

Problem 1.3-22

$$L = 4\text{m} \quad q_0 = 160 \frac{\text{N}}{\text{m}} \quad P = 200\text{N} \quad M_0 = 380 \cdot \text{N} \cdot \text{m}$$

Reactions

$$\Sigma F_x = 0 \quad A_x = 2 \cdot \left(\frac{3}{5} \cdot P \right) - \frac{1}{2} \cdot q_0 \cdot L = -80\text{N}$$

$$\Sigma M_A = 0 \quad D_y = \frac{1}{L} \left[M_0 + \frac{4}{5} \cdot P \cdot \frac{L}{2} + \frac{4}{5} \cdot P \cdot \frac{3 \cdot L}{2} - \frac{1}{2} \cdot q_0 \cdot L \cdot \left(\frac{L}{3} \right) \right] = 308.333\text{N}$$

$$\Sigma F_y = 0 \quad A_y = -D_y + 2 \cdot \left(\frac{4}{5} \cdot P \right) = 11.667\text{N}$$

Column BD internal forces and moment at mid-height - cut through column, use lower FBD (D on your left)

$$N_{\text{mid}} = -D_y = -308.333\text{N} \quad V_{\text{mid}} = \frac{-1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} = -80\text{N} \quad M_{\text{mid}} = - \left(\frac{1}{2} \cdot \frac{q_0}{2} \cdot \frac{L}{2} \right) \cdot \left(\frac{1}{3} \cdot \frac{L}{2} \right) = -53.333 \cdot \text{N} \cdot \text{m}$$

Problem 1.3-23

(a) STATICS

$$\Sigma M_A = 0 \quad C_y = \frac{1}{L_{AC}} \left(1900 \text{ N} \frac{1}{2} \frac{3}{5} 0.75 \text{ m} \right) = 570 \text{ N} \quad C_x = \frac{-1}{2} C_y = -285 \text{ N} \quad \leftarrow$$

$$\Sigma F_x = 0 \quad A_x = -C_x = 285 \text{ N} \quad \leftarrow \quad (\text{resultant of } C_x \text{ and } C_y \text{ acts along line of strut})$$

$$\Sigma F_y = 0 \quad A_y = 1900 \text{ N} - C_y = 1330 \text{ N} \quad \leftarrow$$

(b) INTERNAL STRESS RESULTANTS N , V , M (SEE FIGURE)

$$\text{Distributed weight of door in } -y \text{ direction: } w = \frac{1900 \text{ N}}{0.75 \text{ m}} = 2533.333 \frac{\text{N}}{\text{m}}$$

Components of w along and perpendicular to door:

$$w_a = \frac{4}{5} w = 2026.667 \frac{\text{N}}{\text{m}} \quad w_p = \frac{3}{5} w = 1520 \frac{\text{N}}{\text{m}}$$

$$N_x = w_a(0.5 \text{ m}) - \frac{3}{5} A_x - \frac{4}{5} A_y = -221.667 \text{ N}$$

$$V_x = -w_p(0.5 \text{ m}) - \frac{4}{5} A_x + \frac{3}{5} A_y = -190 \text{ N}$$

$$M_x = -w_p(0.5 \text{ m}) \frac{0.5 \text{ m}}{2} - \frac{4}{5} A_x(0.5 \text{ m}) + \frac{3}{5} A_y(0.5 \text{ m}) = 95 \text{ N}\cdot\text{m}$$

$$\boxed{N_x = -222 \text{ N}} \quad \boxed{V_x = -190 \text{ N}} \quad \boxed{M_x = 95 \text{ N}\cdot\text{m}}$$

Problem 1.3-24

(a) STATICS

$$\Sigma M_A = 0$$

$$10 \text{ kN}(6 \text{ m}) - 10 \text{ kN}\left(\frac{1}{\sqrt{2}}\right)(6 \text{ m}) + 90 \text{ kN}\cdot\text{m} + E_y(6 \text{ m}) - E_x(3 \text{ m}) = 6E_y \text{ m} - 3E_x \text{ m} + 150 \text{ kN}\cdot\text{m} - 30\sqrt{2} \text{ kN}\cdot\text{m}$$

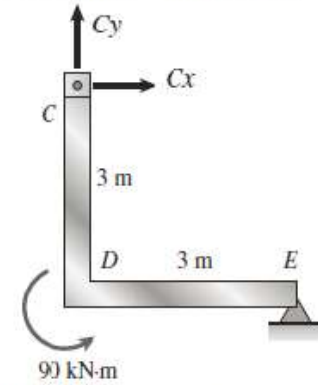
$$\text{so } 6E_y \text{ m} - 3E_x \text{ m} + 150 \text{ kN}\cdot\text{m} - 30\sqrt{2} \text{ kN}\cdot\text{m} = 0$$

$$\text{or } -E_x + 2E_y = \frac{-(150 \text{ kN}\cdot\text{m} - 30\sqrt{2} \text{ kN}\cdot\text{m})}{3 \text{ m}} = -35.858 \text{ kN}$$

$\Sigma M_{\text{CRHFB}} = 0$ < right hand FBD (CDE) - see figure.

$$(E_x + E_y)3 \text{ m} = -90 \text{ kN}\cdot\text{m} \quad E_x + E_y = \frac{-90 \text{ kN}\cdot\text{m}}{3 \text{ m}} = -30 \text{ kN}$$

$$\text{Solving } \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} -35.858 \text{ kN} \\ -30 \text{ kN} \end{pmatrix} = \begin{pmatrix} -8.05 \\ -21.95 \end{pmatrix} \text{ kN}$$



$$E_x = -8.05 \text{ kN}$$

$$E_y = -22 \text{ kN}$$

$$\Sigma F_x = 0 \quad A_x = -E_x + 10 \text{ kN} - 10 \text{ kN}\left(\frac{1}{\sqrt{2}}\right) = 10.98 \text{ kN}$$

$$A_x = 10.98 \text{ kN}$$

$$\Sigma F_y = 0 \quad A_y = -E_y + 10 \text{ kN}\left(\frac{1}{\sqrt{2}}\right) = 29.07 \text{ kN}$$

$$A_y = 29.1 \text{ kN}$$

(b) RIGHT HAND FBD $C_x = -E_x = 8.05 \text{ kN}$ $C_y = -E_y = 22 \text{ kN}$

$$\text{Resultant}_C = \sqrt{C_x^2 + C_y^2} = 23.4 \text{ kN}$$

Problem 1.3-25

(a) STATICS

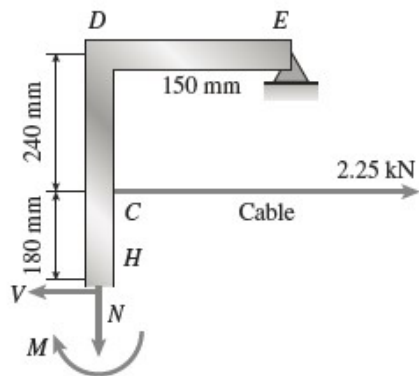
$$\Sigma F_x = 0 \quad E_x = 0$$

$$\Sigma M_E = 0 \quad A_y = \frac{1}{300 \text{ mm}}[-2250 \text{ N}(750 \text{ mm})] = -5625 \text{ N}$$

$$\Sigma F_y = 0 \quad E_y = 2250 \text{ N} - A_y = 7875 \text{ N}$$

$$A_y = -5625 \text{ N} \quad E_x = 0 \quad E_y = 7875 \text{ N} \quad \leftarrow$$

(b) USE UPPER (SEE FIGURE BELOW) OR LOWER FBD TO FIND STRESS RESULTANTS N , V , AND M AT H



$$\Sigma F_x = 0 \quad V_x = E_x + 2250 \text{ N} = 2250 \text{ N}$$

$$\Sigma F_y = 0 \quad N_x = E_y = 7875 \text{ N}$$

$$\Sigma M_H = 0$$

$$M_H = -(180 \text{ mm})(2250 \text{ N}) - E_x(180 \text{ mm} + 240 \text{ mm}) + E_y(150 \text{ mm})$$

$$= 776 \text{ N} \cdot \text{m}$$

$$N_x = 7875 \text{ N} \quad V_x = 2250 \text{ N} \quad M_x = 776 \text{ N} \cdot \text{m} \quad \leftarrow$$

Problem 1.3-26

(a) STATICS

$$\sum F_x = 0 \quad A_x = \frac{4}{5}(400 \text{ N}) = 320 \text{ N} \quad \boxed{A_x = 320 \text{ N}}$$

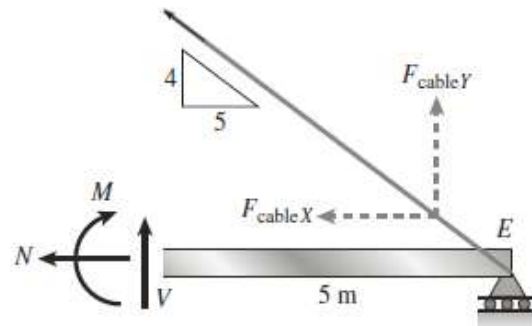
Use left hand FBD (cut through pin just left of C)

$$\sum M_C = 0 \quad A_y = \frac{1}{7 \text{ m}} \left[\left[\frac{-3}{5}(400 \text{ N}) - \frac{4}{5}(400 \text{ N}) \right] (3 \text{ m}) \right] = -240 \text{ N} \quad \boxed{A_y = -240 \text{ N}}$$

Use entire FBD $\sum M_C = 0 \quad E_y = \frac{1}{5 \text{ m}} \left[A_y(7 \text{ m}) + \left(\frac{3}{5} 400 \text{ N} \right) (3 \text{ m}) \right] = -192 \text{ N} \quad \boxed{E_y = -192 \text{ N}}$

$$\sum F_y = 0 \quad C_y = -A_y - E_y - \frac{3}{5}(400 \text{ N}) = 192 \text{ N} \quad \boxed{C_y = 192 \text{ N}}$$

(b) N , V , AND M JUST RIGHT OF C; USE RIGHT HAND FBD $F_{\text{cable}X} = 400 \text{ N} \left(\frac{5}{\sqrt{4^2 + 5^2}} \right) = 312.348 \text{ N}$



$$F_{\text{cable}Y} = \frac{4}{5} F_{\text{cable}X} = 249.878 \text{ N}$$

$$\sum F_x = 0 \quad \boxed{N_x = -F_{\text{cable}X} = -312 \text{ N}}$$

$$\sum F_y = 0 \quad \boxed{V = -F_{\text{cable}Y} - E_y = -57.9 \text{ N}}$$

$$\sum M_C = 0 \quad M = (F_{\text{cable}Y} + E_y)(5 \text{ m}) = \boxed{289 \text{ N}\cdot\text{m}}$$

(c) RESULTANT FORCE IN PIN JUST LEFT OF C; USE LEFT HAND FBD $A_x = 320 \text{ N}$

$$F_{Cx} = -A_x + \left(\frac{4}{5} - \frac{3}{5} \right) 400 \text{ N} = -240 \text{ N} \quad F_{Cy} = -A_y - \left(\frac{3}{5} + \frac{4}{5} \right) 400 \text{ N} = -320 \text{ N}$$

$$\boxed{\text{Res}_C = \sqrt{F_{Cx}^2 + F_{Cy}^2} = 400 \text{ N}}$$

Problem 1.3-27

(a) STATICS $W = 650 \text{ N}$

$$\Sigma M_A = 0 \quad B_x(0.6 \text{ m} + 1.2 \text{ m} \sin(30^\circ)) + W \left[\frac{1.2 \text{ m}}{2} (\cos(30^\circ)) \right] = 0 \text{ solve, } B_x = \frac{650.0 \text{ N} \cos(30.0^\circ)}{2.0 \sin(30.0^\circ) + 1.0}$$

$$\text{so } B_x = -\frac{650.0 \text{ N} \cos(30.0^\circ)}{2.0 \sin(30.0^\circ) + 1.0} = -281.458 \text{ N}$$

$$\Sigma F_X = 0 \quad -A \sin(30^\circ) + B_x + T \cos(30^\circ) + T \cos\left(\arctan\left(\frac{0.9 + 0.6 + 1.2 \sin(30^\circ)}{1.2 \cos(30^\circ)}\right)\right) = 0$$

$$\Sigma F_y = 0 \quad A \cos(30^\circ) + T \sin(30^\circ) + T \sin\left(\arctan\left(\frac{0.9 + 0.6 + 1.2 \sin(30^\circ)}{1.2 \cos(30^\circ)}\right)\right) = W$$

$$\begin{pmatrix} A \\ T \end{pmatrix} = \begin{pmatrix} -\sin(30^\circ) & \cos(30^\circ) + \cos\left(\arctan\left(\frac{0.9 + 0.6 + 1.2 \sin(30^\circ)}{1.2 \cos(30^\circ)}\right)\right) \\ \cos(30^\circ) & \sin(30^\circ) + \sin\left(\arctan\left(\frac{0.9 + 0.6 + 1.2 \sin(30^\circ)}{1.2 \cos(30^\circ)}\right)\right) \end{pmatrix}^{-1} \begin{pmatrix} -B_x \\ W \end{pmatrix}$$

$$\begin{pmatrix} A \\ T \end{pmatrix} = \begin{pmatrix} 250.09 \\ 310.412 \end{pmatrix} \text{ N}$$

SUPPORT REACTIONS

$$\boxed{B_x = -281.5 \text{ N}} \quad \boxed{A = 250.1 \text{ N}}$$

$$A_x = -A \sin(30^\circ) = -125 \text{ N} \quad A_y = A \cos(30^\circ) = 216.6 \text{ N}$$

$$\sqrt{A_x^2 + A_y^2} = 250.09 \text{ N}$$

(b) CABLE FORCE IS T (NEWTONS) FROM ABOVE SOLUTION $\boxed{T = 310.4 \text{ N}}$

Problem 1.3-28

(a) STATICS

RIGHT-HAND FBD

$$\Sigma M_{\text{pin}} = 0 \quad E_y = \frac{1}{6 \text{ m}} \left[\frac{1}{2} (3 \text{ kN/m}) 4 \text{ m} \left(\frac{1}{3} 4 \text{ m} \right) \right] = 1.333 \text{ kN} \quad \boxed{E_y = 1.333 \text{ kN}}$$

ENTIRE FBD

$$\Sigma M_A = 0 \quad C_y = \frac{1}{6 \text{ m}} \left[-E_y 12 \text{ m} + (16 \text{ kN}) 4 \text{ m} + (1.5 \text{ kN/m}) 6 \text{ m} (3 \text{ m}) - \frac{1}{2} (3 \text{ kN/m}) 4 \text{ m} \left(\frac{2}{3} 4 \text{ m} \right) \right] = 9.833 \text{ kN}$$

$$\boxed{C_y = 9.83 \text{ kN}}$$

$$\Sigma F_y = 0 \quad A_y = -C_y - E_y + (1.5 \text{ kN/m}) 6 \text{ m} = -2.167 \text{ kN} \quad \boxed{A_y = -2.17 \text{ kN}}$$

$$\Sigma F_x = 0 \quad A_x = -16 \text{ kN} + \frac{1}{2} (3 \text{ kN/m}) 4 \text{ m} = -10 \text{ kN} \quad \boxed{A_x = -10 \text{ kN}}$$

(b) RESULTANT FORCE IN PIN; USE EITHER RIGHT HAND OR LEFT HAND FBD (CUT THROUGH PIN EXPOSING PIN FORCES F_{Dx} AND F_{Dy}) THEN SUM FORCES IN x AND y DIRECTIONS FOR EITHER FBD

LHFB:

$$F_{Dx} = -16 \text{ kN} - A_x = -6 \text{ kN}$$

$$F_{Dy} = -A_y + (1.5 \text{ kN/m}) 6 \text{ m} = 11.167 \text{ kN}$$

$$\text{Resultant}_D = \sqrt{F_{Dx}^2 + F_{Dy}^2} = 12.68 \text{ kN}$$

RHFB:

$$F_{Dx} = \frac{1}{2} (3 \text{ kN/m}) 4 \text{ m} = 6 \text{ kN}$$

$$F_{Dy} = -C_y - E_y = -11.167 \text{ kN}$$

$$\boxed{\text{Resultant}_D = 12.68 \text{ kN}}$$

Problem 1.3-29

(a) STATICS $P_1 = 220 \text{ N}$ $P_2 = 180 \text{ N}$

$$\Sigma F_x = 0 \quad O_x = -P_1 \cos(15^\circ) = -212.5 \text{ N} \quad \Sigma F_y = 0 \quad O_y = P_2 = 180 \text{ N}$$

$$\Sigma F_z = 0 \quad O_z = P_1 \sin(15^\circ) = 56.94 \text{ N}$$

$$\Sigma M_x = 0 \quad M_{O_x} = P_2 (150 \text{ mm}) + P_1 \sin(15^\circ)(178 \text{ mm}) = 37.1 \text{ N}\cdot\text{m}$$

$$\Sigma M_y = 0 \quad M_{O_y} = P_1 \sin(15^\circ)(200 \text{ mm} \sin(15^\circ)) + P_1 \cos(15^\circ)(150 \text{ mm} + 200 \text{ mm} \cos(15^\circ))$$

$$M_{O_y} = 75.9 \text{ N}\cdot\text{m}$$

$$\Sigma M_z = 0 \quad M_{O_z} = P_1 \cos(15^\circ)(178 \text{ mm}) = 37.8 \text{ N}\cdot\text{m}$$

$$O_x = -213 \text{ N} \quad O_y = 180 \text{ N} \quad O_z = 56.9 \text{ N}$$

$$M_{O_x} = 37.1 \text{ N}\cdot\text{m} \quad M_{O_y} = 75.9 \text{ N}\cdot\text{m} \quad M_{O_z} = 37.8 \text{ N}\cdot\text{m} \quad \leftarrow$$

(b) INTERNAL STRESS RESULTANTS AT MIDPOINT OF OA

$$N_x = -O_y = -180 \text{ N}$$

$$V_x = -O_x = 212.5 \text{ N} \quad V_z = -O_z = -56.9 \text{ N} \quad V_{\text{res}} = \sqrt{V_x^2 + V_z^2} = 220 \text{ N}$$

$$T_x = -M_{O_y} = -75.9 \text{ N}\cdot\text{m}$$

$$M_x = -M_{O_x} = -37.1 \text{ N}\cdot\text{m} \quad M_z = -M_{O_z} = -37.8 \text{ N}\cdot\text{m} \quad M_{\text{res}} = \sqrt{M_x^2 + M_z^2} = 53 \text{ N}\cdot\text{m}$$

$$N_x = -180 \text{ N} \quad V_{\text{res}} = 220 \text{ N}$$

$$T_x = -75.9 \text{ N}\cdot\text{m} \quad M_{\text{res}} = 53 \text{ N}\cdot\text{m} \quad \leftarrow$$

Problem 1.3-30

FORCES

$$P_x = 60 \text{ N} \quad P_z = -45 \text{ N} \quad M_y = 120 \text{ N}\cdot\text{m} \quad q_0 = 75 \text{ N/m}$$

$$F_C = \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} = \begin{pmatrix} 60 \\ 0 \\ -45 \end{pmatrix} \text{ N} \quad R_A = \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix}$$

VECTOR ALONG MEMBER CD

$$r_{EC} = \begin{bmatrix} 1.5 - 2.5 \\ 2 - 0 \\ 0 - (-0.5) \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \\ 0.5 \end{bmatrix} \quad |r_{EC}| = 2.291 \quad e_{EC} = \frac{r_{EC}}{|r_{EC}|} = \begin{pmatrix} -0.436 \\ 0.873 \\ 0.218 \end{pmatrix}$$

(a) STATICS (FORCE AND MOMENT EQUILIBRIUM)

$$\Sigma F = 0 \quad \begin{pmatrix} 0 \\ A_y \\ A_z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} + \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = 0 \quad \text{resultant of triangular load: } R_T = \frac{1}{2}q_0(2 \text{ m}) = 75 \text{ N}$$

$$\text{where } \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = D e_{EC}$$

SOLVING ABOVE THREE EQUATIONS:

$$\begin{aligned} \Sigma F_x = 0 \quad D_x = -P_x \text{ so } \quad D &= \frac{-P_x}{e_{EC1}} & D &= 137.477 \text{ N} & \boxed{D_x = -60 \text{ N}} \\ \Sigma F_y = 0 \quad D_y = e_{EC2} D & & \boxed{D_y = 120 \text{ N}} & & \boxed{A_y = -D_y = -120 \text{ N}} \\ \Sigma F_z = 0 \quad D_z = e_{EC3} D & & \boxed{D_z = 30 \text{ N}} & & \sqrt{D_x^2 + D_y^2 + D_z^2} = 137.477 \text{ N} \\ \text{so } A_z = -D_z - R_T - P_z & & \boxed{A_z = -60 \text{ N}} & & \end{aligned}$$

$\Sigma M_A = 0$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} + r_{AE} \times D + r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_y \\ 0 \end{pmatrix} + r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} = 0$$

$$r_{AE} = \begin{pmatrix} 2.5 - 0 \\ 0 - 0 \\ -0.5 - 0 \end{pmatrix} \text{ m} \quad D = \begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} \quad D = \begin{pmatrix} -60 \\ 120 \\ 30 \end{pmatrix} \text{ N} \quad |D| = 137.477 \text{ N} \quad r_{AE} \times D = \begin{pmatrix} 60 \\ -45 \\ 300 \end{pmatrix} \text{ N}\cdot\text{m}$$

$$r_{AC} = \begin{pmatrix} 1.5 - 0 \\ 2 - 0 \\ 0 - 0 \end{pmatrix} \text{ m} \quad r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} = \begin{pmatrix} -90 \\ 67.5 \\ -120 \end{pmatrix} \text{ J} \quad r_{cg} = \begin{pmatrix} 0 \\ \frac{2}{3}(2 \text{ m}) \\ 0 \end{pmatrix} \quad r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \\ 0 \end{pmatrix} \text{ N}\cdot\text{m}$$

$$\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = - \left[r_{AE} \times D + r_{AC} \times \begin{pmatrix} P_x \\ 0 \\ P_z \end{pmatrix} + \begin{pmatrix} 0 \\ M_y \\ 0 \end{pmatrix} + r_{cg} \times \begin{pmatrix} 0 \\ 0 \\ R_T \end{pmatrix} \right] = \begin{pmatrix} -70 \\ -142.5 \\ -180 \end{pmatrix} \text{ N}\cdot\text{m} \quad \boxed{\begin{pmatrix} M_{Ax} \\ M_{Ay} \\ M_{Az} \end{pmatrix} = \begin{pmatrix} -70 \\ -142.5 \\ -180 \end{pmatrix} \text{ N}\cdot\text{m}}$$