Chapter 1

1.1 Since $pV = mRT$, $\quad \dfrac{V_1}{V_1} = \dfrac{T_1 p_2}{T_2 p_1}$

$$\therefore V_1 = \frac{\pi}{6}(20\ \text{m})^3 \frac{288.15}{233.15}\frac{1.1}{101.3} = \mathbf{56.2\ m^3}$$

1.2
$$\varrho = \frac{p}{RT} = \frac{1.4 \times 10^5\ \text{N}\cdot\text{m}^{-2}}{287\ \text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1} \times 323.15\ \text{K}} = \mathbf{1.51\ kg\cdot m^{-3}}$$

1.3 $K = \varrho \dfrac{\partial p}{\partial \varrho}$ Assume K constant. Then $\ln(\varrho/\varrho_0) = \dfrac{p - p_0}{K}$

$$\therefore \varrho = \varrho_0 \exp\left(\frac{p-p_0}{K}\right) = 1025\ \text{kg}\cdot\text{m}^{-3} \exp\left(\frac{81.7 \times 10^6}{2.34 \times 10^9}\right)$$

$$= \mathbf{1061\ kg\cdot m^{-3}}$$

1.4
$$\varrho = \frac{\mu}{\nu} = \frac{2 \times 10^{-5}\ \text{N}\cdot\text{s}\cdot\text{m}^{-2}}{15 \times 10^{-6}\ \text{m}^2\cdot\text{s}^{-1}} = 1.333\ \text{kg}\cdot\text{m}^{-3}$$

$$R = \frac{p}{\varrho T} = \frac{1.013 \times 10^5\ \text{N}\cdot\text{m}^{-2}}{1.333\ \text{kg}\cdot\text{m}^{-3} \times 293.15\ \text{K}} = 259.2\ \text{J}\cdot\text{kg}^{-1}\cdot\text{K}^{-1}$$

$$\therefore M = \frac{8310}{259.2} = \mathbf{32.06}$$

1.5
$$\mu = \nu\varrho = 400 \times 10^{-6}\ \text{m}^2\cdot\text{s}^{-1} \times 850\ \text{kg}\cdot\text{m}^{-3} = 0.34\ \text{Pa}\cdot\text{s}$$

$$\text{Velocity gradient} = \frac{0.12\ \text{m}\cdot\text{s}^{-1}}{0.1 \times 10^{-3}\ \text{m}} = 1200\ \text{s}^{-1}$$

$$\text{Area} = \pi 0.2 \times 1.2\ \text{m}^2 = 0.754\ \text{m}^2$$

$$\text{Force} = 0.754\ \text{m}^2 \times 0.34\ \text{Pa}\cdot\text{s} \times 1200\ \text{s}^{-1} = \mathbf{307.6\ N}$$

1.6 $$\text{Total force on plate} = \text{Area} \times \mu \left\{ \left(\frac{\partial u}{\partial y}\right)_{\text{side A}} + \left(\frac{\partial u}{\partial y}\right)_{\text{side B}} \right\}$$

$$= (0.25\ \text{m})^2 \times 0.7\ \text{Pa}\cdot\text{s} \left\{ \frac{0.15\ \text{m}\cdot\text{s}^{-1}}{0.006\ \text{m}} + \frac{0.15\ \text{m}\cdot\text{s}^{-1}}{0.019\ \text{m}} \right\}$$

$$= \mathbf{1.439\ N}$$

1.7 For annulus, radius r, width δr

$$\text{Force} = \text{Area} \times \mu \times \frac{\text{Velocity}}{\text{Clearance}} = 2\pi r \delta r \mu \frac{\omega r}{c}$$

$$\therefore \text{Torque} = \text{Force} \times r = 2\pi r^3 \delta r \frac{\mu\omega}{c}$$

$$\text{Total torque} = \int_0^R 2\pi r^3 \frac{\mu\omega}{c} \mathrm{d}r = \frac{\pi R^4 \mu\omega}{2c}$$

$$= \frac{\pi (0.1\ \text{m})^4 0.14\ \text{Pa}\cdot\text{s} \times 2\pi \times 7\ \text{rad}\cdot\text{s}^{-1}}{2 \times 0.00013\ \text{m}} = \mathbf{7.44\ N\cdot m}$$

1.8 $$p = \frac{2\gamma}{d} = \frac{2 \times 0.073\ \text{N}\cdot\text{m}^{-1}}{0.004\ \text{m}} = \mathbf{36.5\ Pa}$$

1.9 $$h = \frac{4\gamma\cos\theta}{\varrho g d} = \frac{4 \times 0.073\ \text{N}\cdot\text{m}^{-1} \times 1}{1000\ \text{kg}\cdot\text{m}^{-3} \times 9.81\ \text{N}\cdot\text{kg}^{-1} \times 0.005\ \text{m}}$$

$$= 0.00595\ \text{m} = \mathbf{5.95\ mm}$$

1.10 $$h = \frac{4 \times 0.377\ \text{N}\cdot\text{m}^{-1} \times \cos 140°}{(13.56 - 1)1000\ \text{kg}\cdot\text{m}^{-3} \times 9.81\ \text{N}\cdot\text{kg}^{-1} \times 0.006\ \text{m}}$$

$$= \mathbf{-1.563\ mm}$$

1.11 $$Re = \frac{ud\varrho}{\mu} = \frac{4Q\varrho}{\pi d\mu} = \frac{4 \times 0.0025\ \text{m}^3\cdot\text{s}^{-1} \times 900\ \text{kg}\cdot\text{m}^{-3}}{\pi 0.05\ \text{m} \times 0.038\ \text{N}\cdot\text{s}\cdot\text{m}^{-2}} = \mathbf{1508}$$

$$u = \frac{2000\mu}{d\varrho} = \frac{2000 \times 0.038\ \text{N}\cdot\text{s}\cdot\text{m}^{-2}}{0.05\ \text{m} \times 900\ \text{kg}\cdot\text{m}^{-3}} = \mathbf{1.689\ m\cdot s^{-1}}$$

1.12 $$Re = \frac{4Q\varrho}{\pi d\mu} = \frac{4 \times 0.01\ \text{m}^3\cdot\text{s}^{-1}}{\pi 0.08\ \text{m} \times 370 \times 10^{-6}\ \text{m}^2\cdot\text{s}^{-1}} = \mathbf{430} \ \therefore \textbf{Laminar}$$

Chapter 2

2.1 $h = \dfrac{p}{\varrho g} = \dfrac{200 \times 10^3\ \mathrm{N \cdot m^{-2}}}{1590\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}}} = \mathbf{12.82\ m}$

2.2 Pressure depends only on depth below free surface.

(a) $p = \varrho g h = (820\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}})(3 - 0.15)\ \mathrm{m}$
$= 22\,930\ \mathrm{N \cdot m^{-2}} = \mathbf{22.93\ kPa}$

(b) $p = 820 \times 9.81\ \mathrm{N \cdot m^{-3}} \times (3 + 2)\ \mathrm{m} = \mathbf{40.2\ kPa}$

(c) $p = 820 \times 9.81\ \mathrm{N \cdot m^{-3}} \times \{3 + 2 - (1.2 \sin 30^\circ + 0.6)\}\ \mathrm{m}$
$= 820 \times 9.81 \times 3.8\ \mathrm{N \cdot m^{-2}} = \mathbf{30.57\ kPa}$

(d) Load = Pressure × Area
$= 820 \times 9.81 \times 3\ \mathrm{N \cdot m^{-2}} \times (3.5 \times 2.5)\ \mathrm{m^2} = \mathbf{211.2\ kN}$

2.3 $h_{\text{air}} = \dfrac{p}{\varrho_{\text{air}} g} = \dfrac{\varrho_{\text{water}} g h_{\text{water}}}{\varrho_{\text{air}} g} = \dfrac{\varrho_{\text{water}}}{\varrho_{\text{air}}} h_{\text{water}}$

$$= \frac{1000\ \mathrm{kg \cdot m^{-3}} \times 287\ \mathrm{J \cdot kg^{-1} \cdot K^{-1}} \times 288.15\ \mathrm{K}}{1.013 \times 10^5\ \mathrm{N \cdot m^{-2}}} 0.075\ \mathrm{m}$$

$= \mathbf{61.2\ m}$

2.4 $pV = \text{constant}$

$$\therefore \left(\frac{d}{4\ \mathrm{mm}}\right)^3 = \frac{101.3 \times 10^3\ \mathrm{Pa} + 1000\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}} \times 9\ \mathrm{m}}{101.3 \times 10^3\ \mathrm{Pa}}$$

whence $d = \mathbf{4.93\ mm}$

2.5 $\Delta p = 820\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}} \times 2\ \mathrm{m} + (13.56 - 0.82)$
$\times 1000\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}} \times 0.225\ \mathrm{m} = \mathbf{44.2\ kPa}$

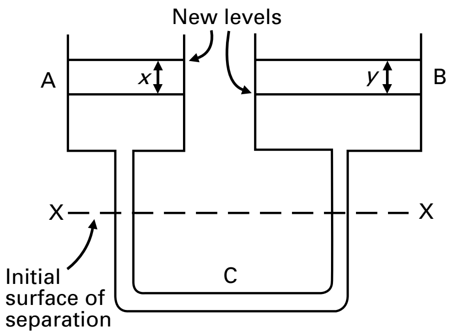
$$\frac{\Delta p^*}{\varrho g} = \Delta h = \frac{0.225\ \text{m}(13.56 - 0.82)1000\ \text{kg}\cdot\text{m}^{-3} \times 9.81\ \text{N}\cdot\text{kg}^{-1}}{820\ \text{kg}\cdot\text{m}^{-3} \times 9.81\ \text{N}\cdot\text{kg}^{-1}}$$

$$= \mathbf{3.496\ m}$$

$$44\,200\ \text{N}\cdot\text{m}^{-2} = 820 \times 9.81 \times 2\ \text{N}\cdot\text{m}^{-2} + x(0.82 - 0.74)1000 \times 9.81\ \text{N}\cdot\text{m}^{-3}$$

whence $\boldsymbol{x} = \mathbf{35.83\ m}$

2.6



Movement of fluid in

$$\text{C} = 60\ \text{mm} \times 70\ \text{mm}^2$$
$$= (500\ \text{mm}^2)x$$
$$= (800\ \text{mm}^2)y$$
$$\therefore x = 8.4\ \text{mm};$$
$$y = 5.25\ \text{mm}$$

Measuring above XX: Initially $0.8h_\text{A} = 0.9h_\text{B}$

Later: $800 \times 9.81(\text{Old } h_\text{A} - 60 + 8.4)10^{-3}$ Pa

$= p + 900 \times 9.81(\text{Old } h_\text{B} - 60 + 5.25)10^{-3}$ Pa

$\therefore\ p = 9.81 \times 10^{-3}(-800 \times 51.6 + 900 \times 65.25)\text{Pa} = \mathbf{171.1\ Pa}$

2.7 From eqn 2.7 $p = p_0\left(1 - \frac{\lambda z}{T_0}\right)^{g/R\lambda}$

$$= 101.5\ \text{Pa}\left(1 - \frac{0.0065 \times 7500}{288.15}\right)^{9.81/287\times0.0065}$$

$$= \mathbf{38.3\ kPa}$$

2.8 $$\frac{p}{p_0} = \left(\frac{T_0 - \lambda z}{T_0}\right)^{g/R\lambda} = \left(\frac{T_\text{top}}{T_\text{top} + \lambda z}\right)^{g/R\lambda}$$

$$\therefore z = \frac{T_\text{top}}{\lambda}\left\{\left(\frac{p_0}{p}\right)^{R\lambda/g} - 1\right\}$$

$$= \frac{268.15}{0.0065}\ \text{m}\left\{\left(\frac{749}{566}\right)^{287\times0.0065/9.81} - 1\right\}$$

$$= \mathbf{2257\ m}$$

2.9 $F = (1.2 \times 1.8)\ \mathrm{m}^2 \times 1000\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}}$
$\times (x + 0.9 \sin 30°)\ \mathrm{m}$

(a) $2160 \times 9.81\ \mathrm{N \cdot m^{-1}} \times 0.45\ \mathrm{m} = \mathbf{9.54\ kN}$
(b) $2160 \times 9.81\ \mathrm{N \cdot m^{-1}} \times 0.95\ \mathrm{m} = \mathbf{20.13\ kN}$
(c) $2160 \times 9.81\ \mathrm{N \cdot m^{-1}} \times 30.45\ \mathrm{m} = \mathbf{645\ kN}$

Centre of pressure is at slant depth $\dfrac{(bd^3/12) + bd(2x+0.9)^2}{bd(2x+0.9)}$

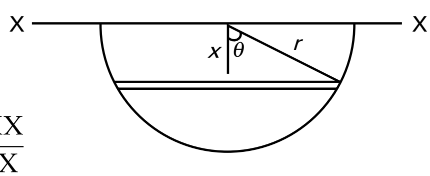
$$= \frac{d^2}{12(2x+0.9)} + 2x + 0.9 (\text{metres})$$

$$= \frac{(1.8\ \mathrm{m})^2}{12(2x+0.9)} + 2x + 0.9\ \mathrm{m}$$

that is $\left\{ \dfrac{1.8^2}{12(2x+0.9)} + 0.9 \right\}$ m from upper edge

= (a) **1.2 m;** (b) **1.042 m;** (c) **0.904 m** from upper edge

2.10 By symmetry, centre of pressure is on vertical centre-line



$$\text{Depth} = \frac{\text{2nd moment about XX}}{\text{1st moment about XX}}$$

$$= \frac{\int_0^r x^2 2(r^2 - x^2)^{1/2} \mathrm{d}x}{\int_0^r x 2(r^2 - x^2)^{1/2} \mathrm{d}x}$$

$$= \frac{\int_{\pi/2}^0 (r\cos\theta)^2 2r\sin\theta(-r\sin\theta \mathrm{d}\theta)}{\int_{\pi/2}^0 r\cos\theta 2r\sin\theta(-r\sin\theta \mathrm{d}\theta)}$$

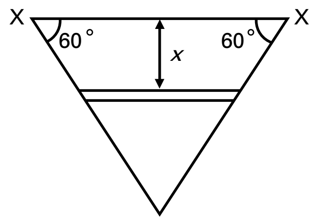
$$= \frac{r\int_0^{\pi/2} \cos^2\theta \sin^2\theta \mathrm{d}\theta}{\int_0^{\pi/2} \sin^2\theta \cos\theta \mathrm{d}\theta}$$

$$= \frac{r\int_0^{\pi} \frac{1}{8} \sin^2 2\theta \mathrm{d}(2\theta)}{\left[\frac{1}{3}\sin^3\theta\right]_0^{\pi/2}}$$

$$= \frac{r/8\,[2\theta/2 - (1/4)\sin 4\theta]_0^{2\theta=\pi}}{1/3}$$

$$= \frac{3}{8} r \frac{\pi}{2} = \frac{\mathbf{3\pi d}}{\mathbf{32}}$$

2.11



Full depth $= (2.5\text{ m})\sin 60^\circ$

Breadth of strip

$$= 2.5\text{ m}\left\{\frac{(2.5\text{ m})\sin 60^\circ - x}{(2.5\text{ m})\sin 60^\circ}\right\}$$

$$= 2.5\text{ m} - x\operatorname{cosec} 60^\circ$$

$\therefore$ Second moment of area about XX

$$= \int_0^{(2.5\text{ m})\sin 60^\circ} (2.5\text{ m} - x\operatorname{cosec} 60^\circ)x^2\mathrm{d}x = \frac{2.5^4}{12}\sin^3 60^\circ\text{ m}^4$$

First moment $= \text{Area} \times \dfrac{\text{Depth}}{3}$

$$= \frac{1}{2}2.5 \times 2.5\sin 60^\circ \times \frac{2.5\sin 60^\circ}{3}\text{ m}^3 = \frac{2.5^3}{6}\sin^2 60^\circ\text{ m}^3$$

$\therefore$ Depth of C.P. $= \dfrac{2.5}{2}\sin 60^\circ\text{ m} = \dfrac{\text{Depth}}{2}$

$\therefore$ Thrust is equally divided between XX and bottom.

Thrust = Area × Pressure at centroid

$$= \frac{1}{2}2.5^2\sin 60^\circ \times 1000 \times 9.81 \times \frac{2.5\sin 60^\circ}{3}\text{N} = 19\,160\text{ N}$$

$\therefore$ Load at bottom $=$ **9580 N**; at each upper corner **4790 N**

2.12 Let shaft be at depth h below free surface. Then force on disc

$= \pi R^2 \varrho g h.$

By parallel axes theorem, 2nd moment of area about free surface $= \pi R^4/4 + \pi R^2 h^2$.

1st moment of area about free surface $= \pi R^2 h$

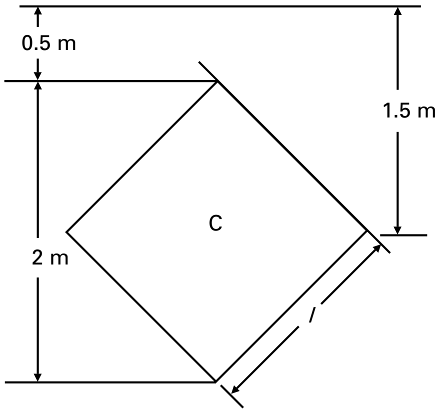
$\therefore$ Depth of C.P. $= \dfrac{R^2}{4h} + h$ below free surface, that is, $R^2/4h$ below shaft

$\therefore$ Turning moment on shaft

$$= \pi R^2 \varrho g h \times \frac{R^2}{4h} = \frac{\pi R^4 \varrho g}{4} \quad [\text{independent of } h]$$

$$= \frac{\pi(0.6\text{ m})^4 1000\text{ kg}\cdot\text{m}^{-3} \times 9.81\text{ N}\cdot\text{kg}^{-1}}{4} = \mathbf{999\ N\cdot m}$$

2.13



Force on plate

$$= 1150\ \text{kg}\cdot\text{m}^{-3} \times 9.81\ \text{N}\cdot\text{kg}^{-1} \times 1.5\ \text{m}(\sqrt{2}\ \text{m})^2$$

$$= \mathbf{33.84\ kN}$$

$$(Ak^2)_{c,\ \perp\ \text{plate}} = \frac{Al^2}{6}$$

$$\therefore\ (Ak^2)_{c,\ \text{diagonal}} = \frac{Al^2}{12}$$

since diagonals are perpendicular

$\therefore$ Depth of C.P. below free surface

$$= \frac{(Al^2/12) + A\bar{y}^2}{A\bar{y}} = \bar{y} + \frac{l^2}{12\bar{y}} = \left\{1.5 + \frac{(\sqrt{2})^2}{12 \times 1.5}\right\}\text{m}$$

$= 1.611$ m, that is, **1.111 m from top of aperture**

$$\therefore\ \text{Total moment about hinge} = 33.84\ \text{kN} \times 1.111/\sqrt{2}\ \text{m}$$

$$= \mathbf{26.59\ kN\cdot m}$$

2.14 Width of gates $= (3\ \text{m})\sec 30^\circ = 3.464$ m

Thrust on 'deep' side of gate

$= (1000 \times 9.81 \times 4.5)(9 \times 3.464)\text{N} = 1.376$ MN

Trust on 'shallow' side of gate

$= (1000 \times 9.81 \times 1.35)(2.7 \times 3.464)\text{N}$

$= \underline{0.124\ \text{MN}}$

Net thrust $= (1.376 - 0.124)$ MN $= 1.252$ MN

$$\therefore\ \text{Force between gates} = \frac{1.252\ \text{MN}}{2\sin 30^\circ} = \mathbf{1.252\ MN}$$

Resultant force F acts at height y given by

$$F_1\frac{h_1}{3} - F_2\frac{h_2}{3} = Fy,\ \text{since } F_1, F_2 \text{ act at } \frac{2}{3}h_1,\ \frac{2}{3}h_2 \text{ below free surfaces}$$

$$\therefore\ y = \frac{1.376 \times 9/3 - 0.124 \times 2.7/3}{1.252}\ \text{m} = 3.208\ \text{m}$$

Total hinge reaction R also acts at this height.