Chapter 11

11.1

$$\textbf{(a)}\ m = \frac{6.8\ \text{N}}{1.62\ \text{m/s}^2} = 4.198\ \text{kg} \blacktriangleleft$$

$$(\textbf{b})\ W = mg = 4.198(9.81) = 41.2\ \text{kg} \blacktriangleleft$$

11.2

$$W = \frac{1}{3}\pi R^2 h\rho g = \frac{\pi}{3}(0.075^2)(0.125)(2700)(9.81) = 19.503\ \text{N} \blacktriangleleft$$

11.3

$$KE = \frac{1}{2}mv^2 + \frac{1}{2}mk^2\omega^2$$

Since the dimensions of each term must be the same, we have

$$[KE] = [M]\left[\frac{L^2}{T^2}\right] = [M][k^2]\left[\frac{1}{T^2}\right]$$

Therefore,

$$[k] = [L]$$

(a) In the SI system

$$[KE] = [M]\left[\frac{L^2}{T^2}\right] = \frac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \blacktriangleleft \qquad [k] = \text{m} \blacktriangleleft$$

11.4

$$[g][k][x]\left[\frac{1}{W}\right] = \left[\frac{L}{T^2}\right]\left[\frac{F}{L}\right][L]\left[\frac{1}{F}\right] = \left[\frac{L}{T^2}\right] = [a]\ \text{Q.E.D}$$

11.5

(a)

$$[\delta] = \left[\frac{PL}{EA}\right] \qquad [L] = \left[\frac{1}{E}\right]\left[\frac{FL}{L^2}\right] \qquad [E] = \left[\frac{F}{L^2}\right] \blacktriangleleft$$

(b) Substituting $[F] = [ML/T^2]$ into the result of part (a):

$$[E] = \left[\frac{ML}{T^2}\right]\left[\frac{1}{L^2}\right] = \left[\frac{M}{T^2 L}\right] \blacktriangleleft$$

11.6 (a) $[mv^2] = \left[\frac{FT^2}{L}\right]\left[\frac{L^2}{T^2}\right] = [FL] \blacktriangleleft$

(b) $[mv] = \left[\frac{FT^2}{L}\right]\left[\frac{L}{T}\right] = [FT] \blacktriangleleft$

(c) $[ma] = \left[\frac{FT^2}{L}\right]\left[\frac{L}{T^2}\right] = [F] \blacktriangleleft$

11.7 Rewrite the equation as $y = 1.0\ x^2$

$$[y] = [1.0][x^2] \qquad [L] = [1.0][L^2] \qquad [1.0] = \left[\frac{1}{L}\right]$$

$y = x^2$ can be dimensionally correct only if the units of the implied constant 1.0 are cm^{-1}. $\blacktriangleleft$

11.8 (a) $[I] = [mR^2] = \left[\frac{FT^2}{L}\right][L^2] = [FLT^2] \blacktriangleleft$

(b) $[I] = [mR^2] = [ML^2] \blacktriangleleft$

11.9 (a) $[v^3] = [A][x^2] + [B][v][t^2] \qquad \left[\frac{L^3}{T^3}\right] = [A][L^2] + [B]\left[\frac{L}{T}\right][T^2]$

$$[A] = \left[\frac{L}{T^3}\right] \blacktriangleleft \qquad [B] = \left[\frac{L^2}{T^4}\right] \blacktriangleleft$$

(b) $[x^2] = [A][t^2]\left[e^{[B][t^2]}\right] \qquad [L^2] = [A][T^2][1] \quad [B][T^2] = [1]$

$$[A] = \left[\frac{L^2}{T^2}\right] \blacktriangleleft \qquad [B] = \left[\frac{1}{T^2}\right] \blacktriangleleft$$

11.10

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = P_0 \sin \omega t$$

$$[m]\left[\frac{d^x}{dt^2}\right] = \left[\frac{FT^2}{L}\right]\left[\frac{L}{T^2}\right] = [F]$$

Therefore, the dimension of each term in the expression is $[F]$.

$$[c]\left[\frac{dx}{dt}\right] = [c]\left[\frac{L}{T}\right] = [F] \qquad [c] = \left[\frac{FT}{L}\right] \blacktriangleleft$$

$$[k][x] = [k][L] = [F] \qquad [k] = \left[\frac{F}{L}\right] \blacktriangleleft$$

$$[P_0][\sin\omega t] = [P_0][1] = [F] \qquad [P_0] = [F] \blacktriangleleft$$

$$[\omega][t] = [\omega][T] = [1] \qquad [\omega] = \left[\frac{1}{T}\right] \blacktriangleleft$$

11.11

$$F = G\frac{m_A m_B}{R^2} \quad G = \frac{FR^2}{m_A m_b} \quad [G] = \frac{[F][L^2]}{[M^2]}$$

(a) $[G] = \dfrac{[F][L^2]}{[FT^2/L]^2} = \left[\dfrac{L^4}{FT^4}\right] \blacktriangleleft$

(b) $[G] = \dfrac{[ML/T^2][L^2]}{[M^2]} = \left[\dfrac{L^3}{MT^2}\right] \blacktriangleleft$

11.12 Using the base dimensions of an absolute [MLT] system:

$$[F] = [C][\rho][v^2][A] \qquad \left[\frac{ML}{T^2}\right] = [C]\left[\frac{M}{L^3}\right]\left[\frac{L^2}{T^2}\right][L^2] \qquad [C] = [1] \text{ Q.E.D} \blacktriangleleft$$

11.13

$$F = G\frac{m^2}{R^2} = (6.67\times10^{-11})\frac{8^2}{0.4^2} = 2.668\times10^{-8}\text{ N}$$

$$W = mg = 8(9.81) = 78.48\text{ N}$$

$$\frac{F}{W}\times100\% = \frac{2.668\times10^{-8}}{78.48}\times100\% = 3.40\times10^{-8}\% \blacktriangleleft$$

11.14

$$F = G\frac{m^2}{R^2} = \left(6.67\times10^{-11}\right)\frac{(0.907)^2}{(0.406)^2} = 3.33\times10^{-10}\text{ N} \blacktriangleleft$$

11.15

$$m = \frac{WR^2}{GM_e} = \frac{(3000)\,(6378+1600)^2\times10^6}{(6.67\times10^{-11})\,(5.9742\times10^{24})} = 479\text{ kg} \blacktriangleleft$$

11.16

$$g_m = \frac{GM_m}{R_m^2} \qquad g_e = \frac{GM_e}{R_e^2}$$

$$\frac{g_m}{g_e} = \frac{M_m}{M_e}\left(\frac{R_e}{R_m}\right)^2 = \frac{0.073483}{5.9742}\left(\frac{6378}{1738}\right)^2 = 0.1656 \approx \frac{1}{6} \text{ Q.E.D}$$

11.17

$$M_e = 5.9742 \times 10^{24} \text{ kg}$$
$$R_e = 6378 \times 10^3 \text{ m}$$
$$W = G\frac{M_e m}{(2R_e)^2} = 3.44 \times 10^{-8}\frac{(5.9742 \times 10^{24})(667/9.81)}{(2 \times 6378 \times 10^3)^2} = 166.6 \text{ N} \blacktriangleleft$$

11.18

$$F = G\frac{M_s m}{R^2} = 6.67 \times 10^{-11}\frac{(1.9891 \times 10^{30})(1.0)}{(149.6 \times 10^9)^2} = 0.00593 \text{ N} \blacktriangleleft$$

11.19

$$G\frac{M_e m}{r^2} = G\frac{M_s m}{(R-r)^2} \qquad \frac{M_e}{r^2} = \frac{M_s}{(R-r)^2}$$
$$\frac{M_e}{M_s} = \frac{r^2}{R^2 - 2Rr + r^2}$$
$$\frac{5.9742 \times 10^{24}}{1.9891 \times 10^{30}} = \frac{r^2}{(149.6 \times 10^9)^2 - 2(149.6 \times 10^9)r + r^2}$$
$$0 = 2.238 \times 10^{22} - 2.992 \times 10^{11} r - 3.3294 \times 10^5 r^2$$

$$r = 259 \times 10^6 \text{ m} = 259 \times 10^3 \text{ km} \blacktriangleleft$$

Chapter 12

12.1

$$y = -0.16t^4 + 4.9t^3 + 0.14t^2 \text{ m}$$
$$v = \dot{y} = -0.64t^3 + 14.7t^2 + 0.28t \text{ m/s}$$
$$a = \dot{v} = -1.92t^2 + 29.4t + 0.28 \text{ m/s}^2$$

At maximum velocity ($a = 0$):

$$-1.92t^2 + 29.4t + 0.28 = 0 \qquad t = 15.322 \text{ s}$$

$$v_{\max} = -0.64(15.322^3) + 14.7(15.322^2) + 0.28(15.322)$$
$$= 1153 \text{ m/s} \blacktriangleleft$$
$$y = -0.16(15.322^4) + 4.9(15.322^3) + 0.14(15.322^2)$$
$$= 8840 \text{ m} \blacktriangleleft$$

12.2

(a) $x = -\frac{1}{2}gt^2 + v_0t \quad \therefore\ v = \dot{x} = -gt + v_0 \ \blacklozenge \quad \therefore\ a = \ddot{x} = -g \ \blacklozenge$

When $t = 0$, then $x = 0$ and $v = v_0$. Hence v_0 is the initial velocity.

Since gravity is the only source of acceleration in this problem, g must be the gravitation acceleration.

(b) When $x = x_{\max'}$ then $v = 0$. $\quad \therefore -gt + v_0 = 0 \quad \therefore\ t = \frac{v_0}{g}$

$$\therefore\ x_{\max} = -\frac{1}{2}g\left(\frac{v_0}{g}\right)^2 + v_0\left(\frac{v_0}{g}\right) = \frac{v_0^2}{2g} \ \blacklozenge$$

At the end of flight $x = 0$. $\quad \therefore -\frac{1}{2}gt^2 + v_0t = 0 \ \therefore t = \frac{2v_0}{g} \ \blacklozenge$

(c) $\therefore x_{\max} = \frac{(26.8)^2}{2(9.81)} = 36.6 \text{ m} \ \blacklozenge \quad \therefore t = \frac{2(26.8)}{9.81} = 5.46 \text{ s} \ \blacklozenge$

12.3

$$x = 6\left(1 - e^{-t/2}\right) \text{ m}$$
$$v = \dot{x} = 6\left(\frac{1}{2}e^{-t/2}\right) = 3e^{-t/2} \text{ m/s}$$
$$a = \dot{v} = -3\left(\frac{1}{2}e^{-t/2}\right) = -1.5e^{-t/2} \text{ m/s}^2$$

(a) $x_{max} = 6$ m at $t = \infty$ ◀
$v_{max} = 3$ m/s and $|a|_{max} = 1.5$ m/s^2 both occurring at $t = 0$ ◀

(b) When $x = 3$m : $3 = 6(1 - e^{-t/2})$ $\quad e^{-t/2} = 0.5$
$t = -2\ln(0.5) = 1.3863$ s ◀
$v = 3(0.5) = 1.5$ m/s ◀ $\quad a = -1.5(0.5) = -0.75$ m/s^2 ◀

12.4

$$x = t^3 - 6t^2 - 32t \text{ m}$$
$$v = \dot{x} = 3t^2 - 12t - 32 \text{ m/s}$$
$$a = \dot{v} = 6t - 12 \text{ m/s}^2$$

At $t = 10$ s:

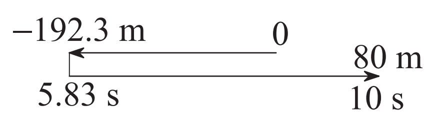
$$x = 10^3 - 6(10^2) - 32(10) = 80 \text{ m} \blacktriangleleft$$
$$v = 3(10^2) - 12(10) - 32 = 148 \text{ m/s} \blacktriangleleft$$
$$a = 6(10) - 12 = 48 \text{ m/s}^2 \blacktriangleleft$$

Reversal of velocity occurs when $v = 0$ $(t \neq 0)$:

$$v = 3t^2 - 12t - 32 = 0 \qquad t = 5.830 \text{ s}$$
$$x = 5.830^3 - 6(5.830^2) - 32(5.830) = -192.3 \text{ m}$$

At $t = 10$ s the distance travelled is

$$s = 192.3 + (193.3 + 80) = 466 \text{ m} \blacktriangleleft$$



12.5 (a) $x = t^2 - \dfrac{t^3}{90}$ m $\qquad v = \dot{x} = 2t - \dfrac{t^2}{30}$ m/s $\qquad v = 0$ when $t = 60$ s

$$x_{max} = 60^2 - \frac{60^3}{90} = 1200 \text{ m} \blacktriangleleft$$

(b) $a = \dot{v} = 2 - \dfrac{t}{15}$ m/s^2 $\qquad a = 0$ when $t = 30$ s

$$v_{max} = 2(30) - \frac{30^2}{30} = 30 \text{ m/s} \blacktriangleleft$$

12.6

(a) $x = v_0(t - t_0 + t_0 e^{-t/t_0})$ $\quad \therefore \quad v = \dot{x} = v_0(1 - e^{-t/t_0})$ ◆

Since $v \to v_0$ as $t \to \infty$, v_0 is the limiting or <u>terminal velocity</u>.

(b) $a = \dot{v} = \dfrac{v_0}{t_0} e^{-t/t_0}$ ◆ $\quad$ But from part (a): $v_0 - v = v_0 e^{-t/t_0}$ $\quad \therefore a = \dfrac{v_0 - v}{t_0}$ ◆

12.7

$$x = 3t^2 - 12t \text{ m} \qquad v = \dot{x} = 6t - 12 \text{ m/s}$$

(a) The bead leaves the wire when $x = 40$ m

$$3t^2 - 12t = 40 \qquad t = 6.16 \text{ s} \quad ◀$$

(b) Reversal of velocity occurs when $v = 0$ $\;(t \neq 0)$:

$$v = 6t - 10 \qquad t = 2.0 \text{ s}$$
$$x = 3(2.0^2) - 12(2.0) = -12.0 \text{ m}$$

The distance travelled is

$$s = 2(12) + 40 = 64.0 \text{ m} \quad ◀$$

12.8

$$x = 4t^2 - 2 \text{ mm}$$
$$y = \frac{x^2}{12} = \frac{16t^4 - 16t^2 + 4}{12} = \frac{4t^4 - 4t^2 + 1}{3} \text{ mm}$$

When $t = 2$ s:

$$v_x = \dot{x} = 8t = 8(2) = 16 \text{ mm/s}$$
$$v_y = \dot{y} = \frac{16t^3 - 8t}{3} = \frac{16(2)^3 - 8(2)}{3} = 37.33 \text{ mm/s}$$
$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{16^2 + 37.33^2} = 40.6 \text{ mm/s} \quad ◀$$

$$a_x = \dot{v}_x = 8 \text{ mm/s}^2$$
$$a_y = \dot{v}_y = \frac{48t^2 - 8}{3} = \frac{48(2)^2 - 8}{3} = 61.33 \text{ mm/s}^2$$
$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{8^2 + 61.33^2} = 61.9 \text{ mm/s}^2 \quad ◀$$

12.9

(a) $x = R\left(1+\frac{1}{2}\cos\omega t\right) \quad \therefore v = \dot{x} = -\frac{1}{2}R\omega\sin\omega t$ ♦ $\quad \therefore a = \dot{v} = -\frac{1}{2}R\omega^2\cos\omega t$ ♦

(b) $|v|_{max} = \frac{1}{2}R\omega \quad |a|_{max} = \frac{1}{2}R\omega^2$

$\therefore$ Doubling ω would double $|v|_{max}$ and quadruple $|a|_{max}$.

12.10

$$y = 50 - 2t \text{ s} \qquad v_y = \dot{y} = -2 \text{ m/s} \qquad a_y = \dot{v}_y = 0$$

$$x = \frac{6}{y} = \frac{6}{50-2t} \text{ m} \qquad v_x = \dot{x} = \frac{3}{(25-t)^2} \text{ m/s} \qquad a_x = v_x = \frac{6}{(25-t)^3}$$

At $t = 20$ s:

$$v_x = \frac{3}{(25-20)^2} = 0.12 \text{ m/s} \qquad v_y = -2 \text{ m/s} \qquad \mathbf{v} = 0.12\mathbf{i} - 2\mathbf{j} \text{ m/s} \blacktriangleleft$$

$$a_x = \frac{6}{(25-20)^3} = 0.048 \text{ m/s}^2 \qquad a_y = 0 \qquad \mathbf{a} = 0.048\mathbf{i} \text{ m/s}^2 \blacktriangleleft$$

12.11

(a) $v^2 = 2gr_0(r_0/r - 1) + v_0^2$

Differentiate with respect to time: $2v\dot{v} = 2gr_0(-r_0/r^2)\dot{r}$ or $2va = -2g(r_0/r)^2 v$

$\therefore a = -g(r_0/r)^2$ ♦

(b) v_0 is the escape velocity if $v \to 0$ when $r \to \infty$.

$$\therefore 0 = \lim_{r\to\infty}[2gr_0(r_0/r - 1) + v_0^2] \quad \therefore 0 = 2gr_0(0-1) + v_0^2 \quad \therefore v_0 = \sqrt{2gr_0} \blacklozenge$$

(c) For earth: $v_0 = \sqrt{2(9.81)(6340 \times 1000)} = 11153$ m/s ♦

12.12

$$x = 15 - 2t^2 \text{ m} \qquad v_x = \dot{x} = -4t \text{ m/s} \qquad a_x = \dot{v}_x = -4 \text{ m/s}^2$$
$$y = 15 - 10t + t^2 \text{ m} \qquad v_y = \dot{y} = -10 + 2t \text{ m/s} \qquad a_y = \dot{v}_y = 2 \text{ m/s}^2$$

(a) At $t = 0$: $\quad \mathbf{v} = -10\mathbf{j}$ m/s ◀ $\quad \mathbf{a} = -4\mathbf{i} + 2\mathbf{j}$ m/s^2 ◀

(b) At $t = 5$ s: $\quad \mathbf{v} = -20\mathbf{i}$ m/s ◀ $\quad \mathbf{a} = -4\mathbf{i} + 2\mathbf{j}$ m/s^2 ◀

12.13

$$x = 58t \text{ m} \qquad v_x = \dot{x} = 58 \text{ m/s} \qquad a_x = \dot{v}_x = 0$$
$$y = 78t - 4.91t^2 \text{ m} \qquad v_y = \dot{y} = 78 - 9.82t \text{ m/s} \qquad a_y = \dot{v}_y = -9.82 \text{ m/s}^2$$

(a) $\mathbf{a} = -9.82\mathbf{j} \text{ m/s}^2$ ◀

(b) $\mathbf{v}|_{t=0} = 58\mathbf{i} + 78\mathbf{j} \text{ m/s}$ ◀

(c) $y = h$ when $v_y = 0$:

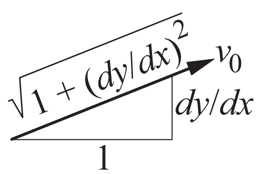
$$v_y = 78 - 9.82t = 0 \qquad t = 7.943 \text{ s}$$
$$h = 78(7.943) - 4.91(7.943^2) = 310 \text{ m} \quad ◀$$

(d) $x = L$ when $y = -140$ m:

$$y = 78t - 4.91t^2 = -140 \qquad t = 17.514 \text{ s}$$
$$L = 58(17.514) = 1016 \text{ m} \quad ◀$$

12.14

$$y = \frac{x^2}{1000} \qquad \frac{dy}{dx} = \frac{x}{500}$$



$$v_x = v_0 \frac{1}{\sqrt{1 + (dy/dx)^2}} = \frac{v_0}{\sqrt{1 + (x/500)^2}} = \frac{500v_0}{\sqrt{500^2 + x^2}}$$
$$v_y = v_0 \frac{dy/dx}{\sqrt{1 + (dy/dx)^2}} = \frac{x/500}{\sqrt{1 + (x/500)^2}} = \frac{xv_0}{\sqrt{500^2 + x^2}}$$

When $x = 100$ m:

$$v_x = \frac{500(6)}{\sqrt{500^2 + 100^2}} = 5.883 \text{ m/s}$$
$$v_y = \frac{100(6)}{\sqrt{500^2 + 100^2}} = 1.1767 \text{ m/s}$$
$$\mathbf{v} = 5.88\mathbf{i} + 1.177\mathbf{j} \text{ m/s} \quad ◀$$

$$a_x = \dot{v}_x = \frac{dv_x}{dx}\frac{dx}{dt} = \frac{dv_x}{dx}v_x = -\frac{500xv_0}{(500^2 + x^2)^{3/2}}v_x$$
$$a_y = \dot{v}_y = \frac{dv_y}{dx}\frac{dx}{dt} = \frac{dv_y}{dx}v_x = \frac{500^2 v_0}{(500^2 + x^2)^{3/2}}v_x$$

When $x = 100$ m:

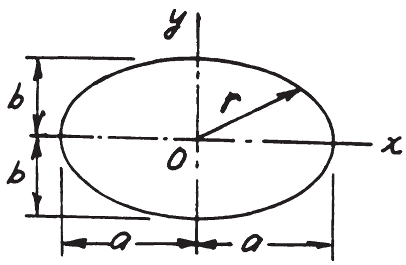
$$a_x = -\frac{500(100)(6)}{(500^2 + 100^2)^{3/2}}(5.883) = -0.013\ 31\ \text{m/s}^2$$
$$a_y = \frac{500^2(6)}{(500^2 + 100^2)^{3/2}}(5.883) = 0.0666\ \text{m/s}^2$$
$$\mathbf{a} = -0.013\ 32\mathbf{i} + 0.666\mathbf{j}\ \text{m/s}^2 \quad ◀$$

12.15

(a) $x = a\cos\omega t \qquad y = b\sin\omega t$

$\therefore \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \cos^2\omega t + \sin^2\omega t = 1$

which is the equation of ellipse $\therefore$ Q.E.D.

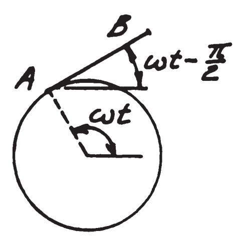


(b) $v_x = \dot{x} = -a\omega\sin\omega t \qquad a_x = \ddot{x} = -a\omega^2\cos\omega t = -\omega^2 x$

$v_y = \dot{y} = -b\omega\cos\omega t \qquad a_y = \ddot{y} = -b\omega^2\sin\omega t = -\omega^2 y$

$\therefore \mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} = -\omega^2(x\mathbf{i} + y\mathbf{j}) = -\omega^2\mathbf{r}$ $\quad\therefore$ **a** and **r** are <u>collinear</u> $\quad\therefore$ Q.E.D.

12.16



$x = R\cos\omega t + R\omega t\sin\omega t \qquad y = R\sin\omega t - R\omega t\cos\omega t$

$\therefore v_x = \dot{x} = -R\omega\sin\omega t + R\omega\sin\omega t + R\omega^2 t\cos\omega t = R\omega^2 t\cos\omega t$

$\therefore v_y = \dot{y} = R\omega\cos\omega t - R\omega\cos\omega t + R\omega^2 t\sin\omega t = R\omega^2 t\sin\omega t$

$\therefore \mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = R\omega^2 t(\mathbf{i}\cos\omega t + \mathbf{j}\sin\omega t) \qquad \therefore v = R\omega^2 t$ ◆

$\overrightarrow{AB} = \overline{AB}\left[\mathbf{i}\cos(\omega t - \pi/2) + \mathbf{j}\sin(\omega t - \pi/2)\right]$

$\quad = \overline{AB}(\mathbf{i}\sin\omega t - \mathbf{j}\cos\omega t)$

By inspection: $\mathbf{v}\cdot\overrightarrow{AB} = 0 \qquad \therefore$ **v** is perpendicular to AB $\quad\therefore$ Q.E.D.

12.17

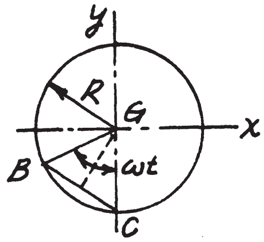
(a) $x = R(\omega t - \sin \omega t) \quad \therefore\ v_x = \dot{x} = R\omega(1 - \cos \omega t)$

$y = R(1 - \cos \omega t) \quad \therefore\ v_y = \dot{y} = R\omega \sin \omega t$

$$\therefore\ v^2 = v_x^2 + v_y^2 = R^2\omega^2\left[(1 - \cos \omega t)^2 + \sin^2 \omega t\right] = 2R^2\omega^2(1 - \cos \omega t) = 4R^2\omega^2 \sin^2 \frac{\omega t}{2}$$

$$\therefore\ v = 2R\omega \sin \frac{\omega t}{2}$$

But $\overline{BC} = 2R \sin \dfrac{\omega t}{2} \quad \therefore\ v = \omega\, \overline{BC} \quad \therefore$ Q.E.D.



(b) $a_x = \ddot{x} = R\omega^2 \sin \omega t \qquad a_y = \ddot{y} = R\omega^2 \cos \omega t$

$$\therefore\ \mathbf{a} = a_x \mathbf{i} + a_y \mathbf{j} = (R\omega^2 \sin \omega t)\mathbf{i} + (R\omega^2 \cos \omega t)\mathbf{j}$$

$\overrightarrow{BG} = (R \sin \omega t)\mathbf{i} + (R \cos \omega t)\mathbf{j} = \dfrac{1}{\omega^2}\mathbf{a} \quad \therefore\ \overrightarrow{BG}$ and $\mathbf{a}$ are parallel $\quad \therefore$ Q.E.D.

12.18

$x = R \cos \omega t \qquad y = R \sin \omega t \qquad z = -\dfrac{h}{2\pi}\omega t$

$\therefore\ v_x = \dot{x} = -R\omega \sin \omega t \qquad v_y = \dot{y} = R\omega \cos \omega t \qquad v_z = \dot{z} = -\dfrac{h\omega}{2\pi}$

$$v^2 = v_x^2 + v_y^2 + v_z^2 = R^2\omega^2 \sin^2 \omega t + R^2\omega^2 \cos^2 \omega t + \left(\frac{h\omega}{2\pi}\right)^2 = (R\omega)^2 + \left(\frac{h\omega}{2\pi}\right)^2$$

$\therefore\ v = R\omega\sqrt{1 + \left[h/(2\pi R)\right]^2} = \text{constant} \quad \therefore$ Q.E.D.

$a_x = \ddot{x} = -R\omega^2 \cos \omega t \qquad a_y = \ddot{y} = -R\omega^2 \sin \omega t \qquad a_z = \ddot{z} = 0$

$a^2 = a_x^2 + a_y^2 + a_z^2 = R^2\omega^4(\cos^2 \omega t + \sin^2 \omega t) = R^2\omega^4 \qquad \therefore\ a = R\omega^2 = \text{constant} \quad \therefore$ Q.E.D.

Using the given data:

$v = (1.2)(4\pi)\sqrt{1 + \left[\dfrac{0.75}{2\pi(1.2)}\right]^2} = 15.15 \text{ m/s}$ ◆ $\qquad a = (1.2)(4\pi)^2 = 189.5 \text{ m/s}^2$ ◆

12.19

$x = 0.8v_0t \qquad y = 0.6v_0t \qquad z = -0.04v_0^2t^2$

$\therefore v_x = \dot{x} = 0.8v_0 \qquad v_y = \dot{y} = 0.6v_0 \qquad v_z = \dot{z} = -0.08v_0^2t$

$\therefore a_x = \ddot{x} = 0 \qquad a_y = \ddot{y} = 0 \qquad a_z = \ddot{z} = -0.08v_0^2$

(a) <u>At point B</u>: $x = 4$ in $\quad \therefore 4 = 0.8v_0t \quad \therefore t = 5/v_0 \quad \therefore v_z = -0.4v_0$

$$\therefore v = \sqrt{v_x^2 + v_y^2 + v_z^2} = v_0\sqrt{0.8^2 + 0.6^2 + 0.4^2} = 1.0770v_0 \ \blacklozenge$$

$$a = |a_z| = 0.08v_0^2 \ \blacklozenge$$

(b) Let θ be the angle between the path and the z-axis at point B.

$$\therefore \cos\theta = \frac{\mathbf{v}\bullet\mathbf{k}}{|\mathbf{v}|} = \frac{-0.4v_0}{1.0770v_0} = -0.3714 \qquad \therefore \theta = 111.8°$$

∴ The angle between the path and the x-y plane is: $\theta - 90° = 21.8°$ ♦

12.20

(a) $\mathbf{r} = (3t^2 + 4t)\mathbf{i} + (-4t^2 + 3t)\mathbf{j} + (-6t + 9)\mathbf{k}$ m

$\therefore \mathbf{v} = \dot{\mathbf{r}} = (6t + 4)\mathbf{i} + (-8t + 3)\mathbf{j} - 6\mathbf{k}$ m/s ♦

$\therefore \mathbf{a} = \dot{\mathbf{v}} = 6\mathbf{i} - 8\mathbf{j}$ m/s^2 ♦

(b) The vector normal to the plane formed by **v** and **a** (the instantaneous plane of motion) is

$$\mathbf{v} \times \mathbf{a} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6t + 4 & -8t + 3 & -6 \\ 6 & -8 & 0 \end{vmatrix} = -48\mathbf{i} - 36\mathbf{j} - 50\mathbf{k}$$

and the corresponding unit vector is

$$\mathbf{n} = \pm\frac{48\mathbf{i} + 36\mathbf{j} + 50\mathbf{k}}{\sqrt{48^2 + 36^2 + 50^2}} = \pm(0.615\mathbf{i} + 0.461\mathbf{j} + 0.640\mathbf{k})$$

Since this vector is independent of t, the orientation of the plane does not vary with the location of the particle. Thus the particle is in plane motion on an inclined plane. Q.E.D.

12.21

$$x = R\cos\omega t \qquad y = R\sin\omega t \qquad z = \frac{R}{2}\sin 2\omega t$$

$$\therefore\ v_x = \dot{x} = -R\omega\sin\omega t \qquad v_y = \dot{y} = R\omega\cos\omega t \qquad v_z = \dot{z} = R\omega\cos 2\omega t$$

$$\therefore\ v = \sqrt{v_x^2 + v_y^2 + v_z^2} = R\omega\sqrt{\cos^2\omega t + \sin^2\omega t + \cos^2 2\omega t} = R\omega\sqrt{1 + \cos^2 2\omega t}$$

$$\therefore\ v_{max} = \sqrt{2}\,R\omega \ \blacklozenge$$

$$a_x = \ddot{x} = -R\omega^2\cos\omega t \qquad a_y = \ddot{y} = -R\omega^2\sin\omega t \qquad a_z = \ddot{z} = -2R\omega^2\sin 2\omega t$$

$$\therefore\ a = \sqrt{a_x^2 + a_y^2 + a_z^2} = R\omega^2\sqrt{\cos^2\omega t + \sin^2\omega t + 4\sin^2 2\omega t} = R\omega^2\sqrt{1 + 4\sin^2 2\omega t}$$

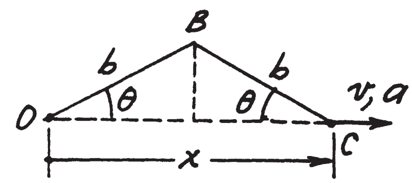
$$\therefore\ a_{max} = \sqrt{5}\,R\omega^2 \ \blacklozenge$$

12.22

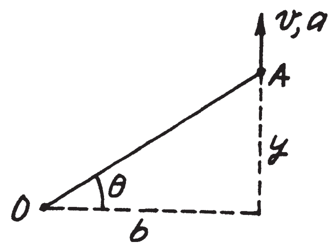
(a) From geometry: $x = 2b\cos\theta$

$$\therefore\ v = \dot{x} = -2b\dot{\theta}\sin\theta \rightarrow \ \blacklozenge$$

(b) $\therefore\ a = \dot{v} = -2b(\ddot{\theta}\sin\theta + \dot{\theta}^2\cos\theta) \rightarrow \ \blacklozenge$



12.23



(a) Geometry: $y = b\tan\theta \qquad \therefore\ v = \dot{y} = b\dot{\theta}\sec^2\theta \uparrow \ \blacklozenge$

(b) $\therefore\ a = \dot{v} = b\left[\ddot{\theta}\sec^2\theta + 2\dot{\theta}\sec\theta(\sec\theta\tan\theta\,\dot{\theta})\right]$

$$= b\sec^2\theta(\ddot{\theta} + 2\dot{\theta}^2\tan\theta) \uparrow \ \blacklozenge$$

12.24

$$x = R\cos\theta \qquad v_x = \dot{x} = (-R\sin\theta)\,\dot{\theta}$$

$$y = R\sin\theta \qquad v_y = \dot{y} = (R\cos\theta)\dot{\theta}$$

$$v_y = v_0 \quad \text{yields} \quad \dot{\theta} = \frac{v_0}{R\cos\theta}$$

$$\therefore\ v_x = (-R\sin\theta)\,\frac{v_0}{R\cos\theta} = -v_0\tan\theta$$

$$a_x = \dot{v}_x = \left(-v_0\sec^2\theta\right)\dot{\theta} = \left(-v_0\sec^2\theta\right)\frac{v_0}{R\cos\theta} = -\frac{v_0^2}{R}\sec^3\theta$$

With $R = 6$ m, $v_0 = 2.5$ m/s and $\theta = 60^\circ$ we get

$$v_y = 2.5 \text{ m/s} \qquad v_x = -2.5\tan 60^\circ = -4.330 \text{ m/s}$$

$$\mathbf{v} = -4.33\mathbf{i} + 2.5\mathbf{j} \text{ m/s} \ \blacktriangleleft$$

$$a_y = 0 \qquad a_x = -\frac{2.5^2}{6}\sec^3 60^\circ = -8.333 \text{ m/s}^2$$

$$\mathbf{a} = -8.33\mathbf{i} \text{ m/s}^2 \ \blacktriangleleft$$

12.25

$$\dot{\theta} = \frac{1200 \text{ rev}}{1.0 \text{ min}} \times \frac{2\pi \text{ rad}}{1.0 \text{ rev}} \times \frac{1.0 \text{ min}}{60 \text{ s}} = 125.66 \text{ rad/s}$$

$$r = 55 + 10\cos\theta + 5\cos 2\theta \text{ mm}$$

$$v = \dot{r} = \frac{dr}{d\theta}\dot{\theta} = (-10\sin\theta - 10\sin 2\theta)(125.66) \text{ mm/s}$$

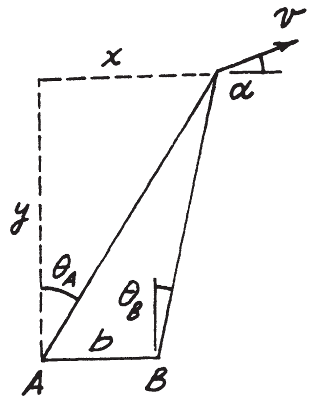
$$a = \dot{v} = \frac{dv}{d\theta}\dot{\theta} = (-10\cos\theta - 20\cos 2\theta)(125.66)^2 \text{ mm/s}^2$$

$$|a|_{\max} = 30(125.66)^2 = 474\ 000 \text{ mm/s}^2 = 474 \text{ m/s}^2 \text{ (at } \theta = 0) \blacktriangleleft$$

***12.26**

(a) Geometry: $x = y\tan\theta_A = y\tan\theta_B + b$ .... (a)

$$\therefore y = \frac{b}{\tan\theta_A - \tan\theta_B} = \frac{1000}{\tan 30° - \tan 22°} = 5770 \text{ m} \ \blacklozenge$$



(b) Differentiate Eq. (a):

$$\dot{x} = y\dot{\theta}_A \sec^2\theta_A + \dot{y}\tan\theta_A = y\dot{\theta}_B \sec^2\theta_B + \dot{y}\tan\theta_B \quad \text{(b)}$$

$$\therefore \dot{y} = -y\frac{\dot{\theta}_A \sec^2\theta_A - \dot{\theta}_B \sec^2\theta_B}{\tan\theta_A - \tan\theta_B}$$

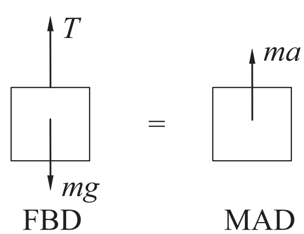
$$= -(5770)\frac{(0.026)\sec^2 30° - (0.032)\sec^2 22°}{\tan 30° - \tan 22°} = 85.11 \text{ m/s}$$

From Eq. (b): $\dot{x} = (5770)(0.026)\sec^2 30° + (85.11)\tan 30° = 249.2$ m/s

$$\therefore v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{249.2^2 + 85.11^2} = 263 \text{ m/s} \ \blacklozenge$$

(c) $\therefore \alpha = \tan^{-1}(\dot{y}/\dot{x}) = \tan^{-1}\dfrac{85.11}{249.2} = 18.86° \ \blacklozenge$

12.27

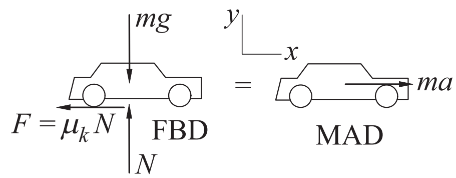


$$v = 4t \text{ m/s} \qquad a = \dot{v} = 4 \text{ m/s}^2$$

$$\Sigma F = ma \quad +\uparrow \qquad T - mg = ma$$

$$T = m(g + a) = 50(9.81 + 4) = 691 \text{ N} \ \blacktriangleleft$$

12.28



$$v_0 = 100 \text{ km/h} = 100 \text{ km/h}\left(\frac{1000 \text{ m/km}}{3600 \text{ s/h}}\right) = 27.78 \text{ m/s}$$

$$\Sigma F_y = 0 \quad +\uparrow \qquad N - mg = 0 \qquad \therefore N = mg$$

$$\Sigma F_x = ma \quad \overset{+}{\rightarrow} \qquad -\mu_k N = ma \qquad \therefore a = -\frac{\mu_k N}{m} = -\mu_k g$$

$$v = \int a\ dt = -\mu_k g t + C_1$$

$$x = \int v\ dt = -\frac{1}{2}\mu_k g t^2 + C_1 t + C_2$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \quad v = v_0 \quad \therefore C_1 = v_0$$

$$\therefore x = -\frac{1}{2}\mu_k g t^2 + v_0 t \quad v = -\mu_k g t + v_0$$

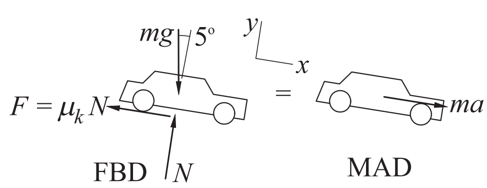
When $v = 0$:

$$-\mu_k g t + v_0 \qquad \therefore t = \frac{v_0}{\mu_k g}$$

$$x = -\frac{1}{2}\mu_k g\left(\frac{v_0}{\mu_k g}\right)^2 + v_0\left(\frac{v_0}{\mu_k g}\right) = \frac{v_0^2}{2\mu_k g}$$

$$= \frac{27.78^2}{2(0.65)(9.81)} = 60.5 \text{ m} \quad \blacktriangleleft$$

12.29



$$v_0 = 100 \text{ km/h} = 27.78 \text{ m/s}$$

$$\Sigma F_y = 0 \quad +\uparrow \qquad N - mg\cos 5^\circ = 0 \qquad \therefore N = mg\cos 5^\circ$$
$$\Sigma F_x = ma \quad \xrightarrow{+} \qquad -\mu_k N + mg\sin 5^\circ = ma$$
$$\therefore a = -\frac{\mu_k N}{m} + g\sin 5^\circ = (\sin 5^\circ - \mu_k \cos 5^\circ)g$$
$$= (\sin 5^\circ - 0.65\cos 5^\circ)9.81 = -5.497 \text{ m/s}^2$$

$$v = \int a\ dt = -5.497t + C_1$$
$$x = \int v\ dt = -2.749t^2 + C_1 t + C_2$$

When $t = 0$ (initial conditions):

$$x = 0 \quad \therefore C_2 = 0 \quad v = v_0 \quad \therefore C_1 = v_0 = 27.78 \text{ m/s}$$

$$x = -2.749t^2 + 27.78t \text{ m}$$
$$v = -5.497t + 27.78 \text{ m/s}$$

When $v = 0$:

$$-5.497t + 27.78 = 0 \qquad \therefore t = 5.054 \text{ s}$$
$$x = -2.749(5.054)^2 + 27.78(5.054) = 70.2 \text{ m} \quad \blacktriangleleft$$

12.30

$$a = \frac{F}{m} = \frac{-1.2t}{0.1} = -12t \text{ m/s}^2$$
$$v = \int a_x dt = -6t^2 + C_1 \text{ m/s}$$
$$x = \int v_x dt = -2t^3 + C_1 t + C_2 \text{ m}$$

When $t = 0$ (initial conditions):

$$x = 0 \qquad \therefore C_2 = 0 \qquad v = 64 \text{ m/s} \qquad \therefore C_1 = 64 \text{ m/s}$$
$$\therefore x = -2t^3 + 64t \text{ m} \qquad v = -6t^2 + 64 \text{ m/s}$$

When $t = 4$ s:

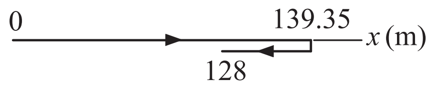
$$x = -2(4)^3 + 64(4) = 128 \text{ m}$$

When $v = 0$:

$$-6t^2 + 64 = 0 \qquad t = 3.266 \text{ s}$$
$$x = -2(3.266)^3 + 64(3.266) = 139.35 \text{ m}$$

Distance traveled:

$$d = 2(139.35) - 128 = 150.7 \text{ m} \blacktriangleleft$$



12.31

$$a = \frac{F}{m} = \frac{0.06\sqrt{v}}{0.012} = 5\sqrt{v} \text{ m/s}^2$$
$$dt = \frac{dv}{a} = \frac{dv}{5\sqrt{v}} \qquad t = \int \frac{dv}{5\sqrt{v}} = \frac{2}{5}\sqrt{v} + C_1$$

Given $v = 0.25$ m/s when $t = 0.8$ s:

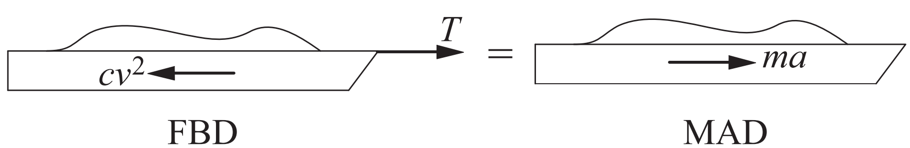
$$0.8 = \frac{2}{5}\sqrt{0.25} + C_1 \qquad C_1 = 0.6 \text{ s}$$

$$t = \frac{2}{5}\sqrt{v} + 0.6 \text{ s}$$
$$v = (2.5t - 1.5)^2 = 6.25t^2 - 7.5t + 2.25 \text{ m/s}$$
$$x = \int v\ dt = \frac{6.25}{3}t^3 - \frac{7.5}{2}t^2 + 2.25t + C_2$$

Initial condition: $x = 0$ when $t = 0$ $\qquad \therefore C_2 = 0$

$$x|_{t=1.2\text{s}} = \frac{6.25}{3}(1.2)^3 - \frac{7.5}{2}\left(1.2^2\right) + 2.25(1.2) = 0.90 \text{ m} \blacktriangleleft$$

12.32



$$ma = T - F_D = T - cv^2 \qquad a = \frac{T - cv^2}{m}$$
$$x = \int \frac{v}{a}dv = m\int \frac{v}{T - cv^2}dv + C = -\frac{m}{2c}\ln\left(T - cv^2\right) + C$$

Initial condition: $v = 0$ at $x = 0$:

$$0 = -\frac{m}{2c}\ln(T) + C \qquad C = \frac{m}{2c}\ln(T)$$
$$\therefore x = -\frac{m}{2c}\ln(T - cv^2) + \frac{m}{2c}\ln(T) = \frac{m}{2c}\ln\frac{T}{(T - cv^2)}$$

Solve for v:

$$\frac{T}{(T - cv^2)} = \exp\left(\frac{2c}{m}x\right) \qquad T - cv^2 = T\exp\left(-\frac{2c}{m}x\right)$$

$$v^2 = \frac{T}{c}\left[1 - \exp\left(-\frac{2c}{m}x\right)\right] \qquad v = \sqrt{\frac{T}{c}\left[1 - \exp\left(-\frac{2c}{m}x\right)\right]} \blacktriangleleft$$

Terminal velocity:

$$v_\infty = \lim_{x\to\infty} v(x) = \sqrt{\frac{T}{c}} \blacktriangleleft$$

12.33

$$a = \frac{F}{m} = \frac{4t - 4}{4} = t - 1 \text{ m/s}^2$$

$$v = \int a\ dt = \frac{1}{2}t^2 - t + C_1$$

$$y = \int v\ dt = \frac{1}{6}t^3 - \frac{1}{2}t^2 + C_1 t + C_2$$

When $t = 0$ (initial condition):

$$y = 0 \quad \therefore C_2 = 0 \quad v = -8 \text{ m/s} \quad \therefore C_1 = -8 \text{ m/s}$$

$$\therefore y = \frac{1}{6}t^3 - \frac{1}{2}t^2 - 8t \text{ m} \quad v = \frac{1}{2}t^2 - t - 8 \text{ m/s}$$

When $t = 8$ s:

$$y = \frac{1}{6}(8)^3 - \frac{1}{2}(8)^2 - 8(8) = -10.67 \text{ m}$$

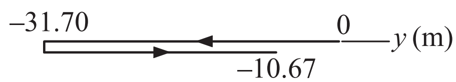
When $v = 0$:

$$\frac{1}{2}t^2 - t - 8 = 0 \quad t = 5.123 \text{ s}$$

$$y = \frac{1}{6}(5.123)^3 - \frac{1}{2}(5.123)^2 - 8(5.123) = -31.70 \text{ m}$$

Distance traveled:

$$d = 2(31.70) - 10.67 = 52.7 \text{ m} \blacktriangleleft$$

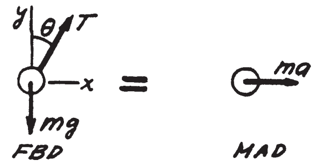


12.34

$\Sigma F_y = ma_y$: $+\uparrow$ $T\cos\theta - mg = 0$

$\Sigma F_x = ma_x$: $\overset{+}{\rightarrow}$ $T\sin\theta = ma$

The solution is: $\theta = \tan^{-1}\dfrac{a}{g}$ ◆



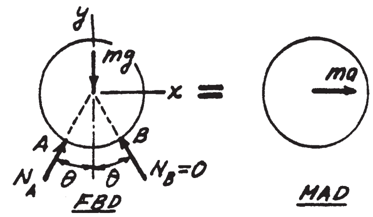
12.35

Assume impending climbing ($N_B = 0$).

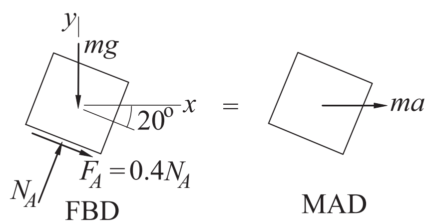
$\Sigma F_y = ma_y$: $+\uparrow$ $N_A\cos\theta - mg = 0$

$\Sigma F_x = ma_x$: $\overset{+}{\rightarrow}$ $N_A\sin\theta = ma$

The solution is: $a = g\tan\theta$ ◆



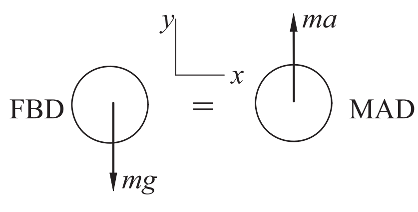
12.36



Assume impending sliding ($F_A = 0.4N_A$)

$$\Sigma F_y = 0 \; +\uparrow \quad N_A\cos 20^\circ - 0.4N_A\sin 20^\circ - mg = 0$$
$$N_A = 1.2455\ mg$$
$$\Sigma F_x = ma_x \; \overset{+}{\rightarrow} \quad N_A\sin 20^\circ + 0.4N_A\cos 20^\circ = ma$$
$$1.2455\ mg(\sin 20^\circ + 0.4\cos 20^\circ) = ma \quad a = 0.894g \;◀$$

12.37 Let y be measured up from the base of the cliff.



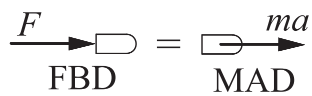
$$\Sigma F_y = ma \; +\uparrow \quad -mg = ma \qquad \therefore a = -g$$
$$a = \frac{dv}{dy}v \quad \therefore v\ dv = -g\ dy \quad \frac{1}{2}v^2 = -gy + C$$

Initial condition: $v = v_0$ when $y = h$. $\therefore C = \dfrac{1}{2}v_0^2 + gh$

$$\therefore \frac{1}{2}\left(v^2 - v_0^2\right) = g(h - y)$$

At impact $y = 0$ $\quad \therefore \dfrac{1}{2}(v^2 - v_0^2) = gh \quad \therefore v = \sqrt{v_0^2 + 2gh}$ ◄

12.38



$$F = ma \qquad a = \frac{F}{m} = \frac{F_0}{m}e^{-x/b} \qquad v\ dv = \frac{F_0}{m}e^{-x/b}dx$$

$$\frac{1}{2}v^2 = \frac{F_0}{m}\int e^{-x/b}\ dx = -\frac{F_0 b}{m}e^{-x/b} + C$$

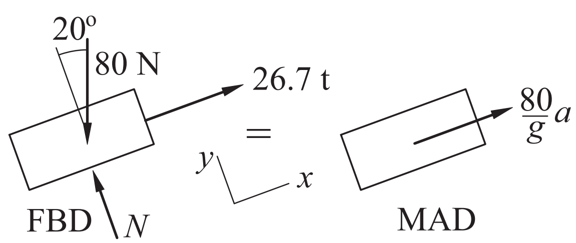
Initial condition : $v = 0$ at $x = 0$ $\qquad \therefore C = \dfrac{F_0 b}{m}$

$$\frac{1}{2}v^2 = \frac{F_0 b}{m}\left(1 - e^{-x/b}\right)$$

$$v = \sqrt{\frac{2F_0 b}{m}\left(1 - e^{-x/b}\right)}$$

$$v|_{x=0.55\ \text{m}} = \sqrt{\frac{2(7010\ \text{N})(2)}{0.0275/9.81}\left(1 - e^{-0.55/0.61}\right)} = 1346\ \text{m/s}$$ ◄

12.39



$$\Sigma F_x = ma \qquad 26.7t - 80\sin 20^\circ = \frac{80}{9.81}a$$

$$a = \frac{9.81}{80}(26.7t - 80\sin 20^\circ) = 3.274t - 3.355\ \text{m/s}^2$$

$$v = \int a\ dt = 1.637t^2 - 3.355t + C_1\ \text{m/s}$$

$$x = \int v\ dt = 0.5457t^3 - 1.678t^2 + C_1 t + C_2\ \text{m}$$