CHAPTER 3

AEROTHERMODYNAMICS OF TURBOMACHINES AND DESIGN-RELATED TOPICS

PROBLEM # 1

With a constant density, the continuity equation becomes one of conserving the volumetric flow rate (Q), as follows:

$$Q_{in} = V_s r b d\theta + V_\theta ds(b + \frac{db}{2})$$

$$Q_{out} = [(V_s + dV_s)(r + dr)(b + db)d\theta] + (V_\theta + dV_\theta)ds(b + \frac{db}{2})$$

Expanding the preceding two expressions, ignoring higher-order terms and dividing by ($rbdsd\theta$), the equality of Q_{in} and Q_{out} yields:

$$\frac{1}{b}\frac{\partial}{\partial s}(rbV_s) + \frac{\partial V_\theta}{\partial \theta} = 0$$

PROBLEM # 2

Applying the continuity-equation version 3.40, we obtain:

$$\dot{m} = 7.07 \text{ kg/s}$$

As the stator reaches the choking status ($M_{cr,1} = 1$), we now reapply the same equation at the stator exit station. This step yields:

$$(\alpha_1)_{new} = 68.8^o$$

Thus:

$$\Delta\alpha = 1.8^o$$

PROBLEM # 3

With the friction coefficient (f) being 0.11, application of eqn 3.76 leads to the following non-linear equation:

$$0.876[ln(1 + 0.165{M_2}^2)] - \frac{0.75}{{M_2}^2} = -0.769$$

Iterative solution of this equation yields:

$$M_2 \approx 0.92$$

Applying eqn 3.75, we get the following:

$$p_{t2} = 6.97 \text{ bars}$$

Now, application of the continuity equation yields:

$$h_2 = 0.02188 \text{ m}$$

Finally:

$$\frac{\Delta h}{h_1} = \frac{h_1 - h_2}{h_1} = 0.53\%$$

PROBLEM # 4

Part 1:

The two velocity components V_{r1} & $V_{\theta 1}$ can be computed as follows:

$$V_{r1} = V_1 \cos \alpha_1 = 23.5 \text{ m/s}$$

$$V_{\theta 1} = V_1 \sin \alpha_1 = 72.3 \text{ m/s}$$

Now, applying the continuity at station 2, and utilizing the free-vortex flow condition across the radial gap, we get:

$$V_{r2} = 28.1 \text{ m/s}$$

$$V_{\theta 2} = 86.5 \text{ m/s}$$

Finally, the magnitude of V_2 is:

$$V_2 = 90.9 \text{ m/s}$$

Part 2:

With the flow process being adiabatic, we have:

$$T_{t2} = T_{t1} = T_1 + \frac{V_1{}^2}{2c_p} = T_2 + \frac{V_2{}^2}{2c_p}$$

which, upon substitution, yields:

$$T_{t2} = 1224.5 \text{ K}$$
$$T_2 = 1220.9 \text{ K}$$

Part c:

Applying the continuity equation at station 1, we get:

$$\dot{m} = (2\pi r_1 b)(\frac{p_1}{RT_1})V_{r1} = 0.80 \text{ kg/s}$$

Note that low mass-flow rate is typically a feature of a radial-flow turbomachine.

PROBLEM # 5

Part a:

$$T_{t1} = T_{t0} = 446 \text{ K} = T_1 + \frac{V_1{}^2}{2c_p}$$

which gives:

$$V_1 = 268.9 \text{ m/s}$$

We can now calculate M_{cr1} as follows:

$$V_{cr1} = \sqrt{\frac{2\gamma}{\gamma+1}RT_{t1}} = 386.4 \text{ m/s}$$
$$M_{cr1} = \frac{V_1}{V_{cr1}} = 0.696$$

The flow angle β_1 can be computed as follows:

$$\beta_1 = \beta_1{}' - \mid i_R \mid = 56^o$$

The process leading to the shaft speed is as follows:

$$V_{z1} = V_1 = 268.9 \text{ m/s} = W_1 \cos\beta_1$$

$$W_1 = 480.9 \text{ m/s}$$

$$U_m = W_1 \sin\beta_1 = 398.7 \text{ m/s}$$

$$\omega = \frac{U_m}{r_m} = 3{,}986.6 \text{ rad/s}$$

$$N = 38{,}069 \text{ rpm}$$

Part b:

Calculation of the inlet total pressure p_{t0} follows:

$$p_{t1} = \frac{p_1}{[1 - \frac{\gamma-1}{\gamma+1} M_{cr1}{}^2]} = 4.70 \text{ bars}$$

$$p_{t0} = p_{t1} + \Delta p_t = 5.0 \text{ bars}$$

PROBLEM # 6

Part Ia:

Applying the continuity equation at the stator-inlet station, we get:

$$\dot{m} = 7.83 \text{ kg/s}$$

Part Ib:

The procedure to compute the axial-velocity-component rise includes application of the continuity equation at the stator-exit station, and is as follows:

$$p_{t2} = p_{t1} + 0.06 p_{t1} = 10.62 \text{ bars}$$

$$\alpha_2 = 69^o \text{ . . . . (obtained through the continuity eqn.)}$$

$$V_1 = 276.4 \text{ m/s . . . . (computed in example 8)}$$

$$V_2 = V_{cr2} = 658.1 \text{ m/s . . . . (computed in example 8)}$$

$$V_{z1} = V_1 \cos\alpha_1 = 214.8 \text{ m/s}$$

$$V_{z2} = V_2 \cos\alpha_2 = 235.8 \text{ m/s}$$

$$\frac{\Delta V_z}{V_{z1}} = 9.8\%$$

Part II

First, we calculate the arithmetic average of the swirl angle ($\alpha_{av.}$):

$$\alpha_{av.} = \frac{1}{2}(\alpha_1 + \alpha_2) = \frac{1}{2}(-39.0 + 69.0) = 15.0^o$$

With all of the given and computed flow properties in example 8, we can now substitute in the provided $\bar{e}_s$ expression:

$$\bar{e}_s \equiv 1 - \frac{V_{act.,1}{}^2}{V_{id.,1}{}^2} = 0.122 \; . \; . \; . \; . \; \text{(higher by comparison)}$$

The procedure to convert $\bar{e}_s$ into the pressure loss coefficient $\bar{\omega}$ is as follows:

$$V_{act.,1} = 259.0 \text{ m/s}$$

$$T_{act.,1} = T_{t1} - \frac{V_{act.,1}{}^2}{2c_p} = 1293.0 \text{ K}$$

$$\Delta s = c_p ln(\frac{T_{act.,1}}{T_1}) - Rln(\frac{p_{act.,1}}{p_1}) = 3.58 \text{ J/(kg K)}$$

$$\Delta s = -Rln(\frac{p_{t2}}{p_{t1}}) = 3.58 \text{ J/(kg K)}$$

$$p_{t2} = 0.988 p_{t1} = 11.16 \text{ bars}$$

$$\bar{\omega} \equiv \frac{\Delta p_t}{p_{t1}} = 1.24\%$$

PROBLEM # 9

Part a:

$$V_{cr1} = \sqrt{\frac{2\gamma}{\gamma+1} R T_{t1}} = 673.4 \text{ m/s}$$

$$V_1 = V_{cr1} M_{cr1} = 612.8 \text{ m/s}$$

$$V_{\theta 1} = U_1 = \omega r_1 = 578.1 \text{ m/s}$$

$$\alpha_1 = sin^{-1}(\frac{U_1}{V_1}) = 70.6^o$$

Applying the continuity equation (3.40) at the stator-exit station, we get:

$$\dot{m} = 4.07 \text{ kg/s}$$

Part b:

The procedure leading to the total-relative pressure decline across the rotor, is as follows:

$$W_1 = V_1 \cos\alpha_1 = 203.5 \text{ m/s}$$

$$T_{tr1} = T_{t1} + \left(\frac{W_1{}^2 - V_1{}^2}{2c_p}\right) = 1239.6 \text{ K}$$

$$p_{tr1} = p_{t1}\left(\frac{T_{tr1}}{T_{t1}}\right)^{\frac{\gamma}{\gamma-1}} = 8.21 \text{ bars}$$

$$U_2 = \omega r_2 = 216.8 \text{ m/s}$$

$$p_2 = p_{t1}\{1 - \frac{1}{\eta_{t-s}}[1 - (\frac{T_{t2}}{T_{t1}})]\}^{\frac{\gamma}{\gamma-1}} = 3.45 \text{ bars}$$

$$V_{\theta 2} = \frac{U_1 V_{\theta 1} - c_p(T_{t1} - T_{t2})}{U_2} = \text{-}160.5 \text{ m/s}$$

$$\rho_2 = \left(\frac{p_{t2}}{RT_{t2}}\right)[1 - \frac{\gamma-1}{\gamma+1} M_{cr2}{}^2]^{\frac{1}{\gamma-1}} = 1.164 \text{ kg}/m^3$$

$$V_{z2} = \frac{\dot{m}}{\rho_2(2\pi r_2 h_2)} = 309.2 \text{ m/s}$$

$$V_2 = 348.3 \text{ m/s}$$

$$W_{\theta 2} = V_{\theta 2} - U_2 = \text{-}377.3 \text{ m/s}$$

$$W_2 = \sqrt{W_{\theta 2}{}^2 + V_{z2}{}^2} = 487.8 \text{ m/s}$$

$$T_{tr2} = 1115.4 \text{ K}$$

$$p_{tr2} = p_{t2}\left(\frac{T_{tr2}}{T_{t2}}\right) = 4.70 \text{ bars}$$

$$(\Delta p_{t_r})_{rotor} = 3.51 \text{ bars}$$

In order to compute $(\Delta p_{t_r})_{rotor}$ due to the radius change only, we set the total-to-total efficiency (η_{t-t}) to 100%. In the following, the variables which are influenced by this change are re-calculated:

$$V_{\theta 2} = \text{-}344.0 \text{ m/s}$$

$$\rho_2 = 1.202 \text{ kg}/m^3$$

$$V_{z2} = 299.3 \text{ m/s}$$

$$W_{\theta 2} = \text{-}560.8 \text{ m/s}$$

$$V_2 = 456.0 \text{ m/s}$$

$$W_2 = 635.7 \text{ m/s}$$

$$T_{tr2} = 1115.4 \text{ K}$$

$$p_{t_{r2}} = 5.36 \text{ bars}$$

$$(\Delta p_{t_r})_{rad.change} = 2.85 \text{ bars}$$

Finally, we can compute Δp_{t_r} due to the flow-process irreversibility sources only as follows:

$$(\Delta p_{t_r})_{irrev.} = (\Delta p_{t_r})_{tot.} - (\Delta p_{t_r})_{rad.change} = 0.66 \text{ bars}$$

PROBLEM # 10

Part a:

Application of the continuity equation at the stator-exit station yields the local swirl angle, as follows:

$$\alpha_1 = 71.8^o$$

Part b:

Referring to Sovran & Klomp chart (Fig. 3.37), and for an L/h_{in} ratio of 3.0, we get:

$$c_{p_r} \equiv \frac{p_3 - p_2}{\frac{\gamma}{2} p_2 {M_2}^2} = 0.45$$

For the back flow not to occur, the discharge-station pressure (p_3) must, at least, be equal to the ambient pressure, i.e.

$$p_3 = 1.06 \text{ bars}$$

Using the total-to-total efficiency definition, we get:

$$p_{t_2} = 1.17 \text{ bars}$$

Substituting in the c_{p_r}-expression (above), we obtain the following non-linear equation:

$$p_2 = 1.06 - 0.30(p_2 {M_2}^2)$$

The iterative procedure to solve this equation consists of repeatedly assuming M_2, then calculating p_2 towards convergence. The final result is:

$$M_2 = 0.52 \text{ bars}$$

In words, the rotor-exit Mach no. will have to be less than, or equal to, 0.52 bars in order to avoid any diffuser-exit flow reversal.

PROBLEM # 11

Part a:

Application of the continuity equation at the stator-exit station yields:

$$\alpha_1 = +20.3^o$$

Remainder of the procedure, which leads to the rotor-exit relative-critical Mach no., is as follows:

$$V_1 = M_{cr1} V_{cr1} = 173.1 \text{ m/s}$$

$$U_m = \omega r_m = 428.8 \text{ m/s}$$

$$V_{\theta 1} = V_1 \sin\alpha_1 = 60.05 \text{ m/s}$$

$$V_z = V_1 \cos\alpha_1 = 162.3 \text{ m/s}$$

$$W_{\theta 1} = V_{\theta 1} - U_m = \text{-}368.8 \text{ m/s}$$

$$W_1 = 402.9 \text{ m/s}$$

$$T_{tr1} = T_{t1} - \left(\frac{W_1{}^2 - V_1{}^2}{2c_p}\right) = 488.9 \text{ K}$$

$$W_{cr1} = 404.6 \text{ m/s}$$

$$\frac{W_1}{W_{cr1}} = 0.996 \; . \; . \; . \; . \; \text{(high but, nevertheless, subsonic)}$$

Part b:

$$V_{\theta 2} = V_{\theta 1} + \left[\frac{c_p(T_{t2} - T_{t1})}{U_m}\right] = 313.0 \text{ m/s}$$

$$V_2 = 352.6 \text{ m/s}$$

$$V_{cr2} = 421.7 \text{ m/s}$$

$$M_{cr2} = 0.836$$

$$M_2 = 0.812 \ldots . \text{ (obtained by using expression 3.78)}$$

$$p_{t2} = 6.64 \text{ bars (obtained by applying the } \eta_{t-t} \text{ definition)}$$

$$\left(\frac{\Delta p_t}{p_{t2}}\right)_{int.gap} = 3.16\%$$

Part c:

$$\alpha_2 = sin^{-1}\left(\frac{V_{\theta 2}}{V_2}\right) = 62.6^o$$

Substituting in equation 3.76, we get:

$$\frac{fL}{D_h} = 0.062$$

where:

$$L = \frac{\Delta z}{\cos\alpha_2} = 12.2 \text{ cm}$$

$$D_h = 2(\Delta r)_{gap} = 6.4 \text{ cm}$$

With the preceding two variables, the endwall friction coefficient (f) across the interstage gap is:

$$f = 0.033$$

PROBLEM # 12

Part a:

In the following, the intermediate steps leading to the sidewall spacing (b) are summerized:

$$V_{cr1} = 668.8 \text{ m/s}$$

$$V_1 = 628.7 \text{ m/s}$$

$$U_1 = \omega r_1 = 546.6 \text{ m/s}$$

$$V_{\theta 1} = U_1 = 546.6 \text{ m/s} \ldots . \text{ (radial entry to the rotor)}$$

$$\rho_1 = \frac{p_{t1}}{RT_{t1}}\left[1 - \frac{\gamma-1}{\gamma+1}{M_{cr1}}^2\right] = 2.383 \text{ kg/}m^3$$

$$V_{r1} = \sqrt{{V_1}^2 - {V_{\theta 1}}^2} = 310.6 \text{ m/s}$$
$$b_1 = \frac{\dot{m}}{2\pi r_1 \rho_1 V_{r1}} = 1.30 \text{ cm}$$

Part b:

$$W_1 = V_{r1} = 310.6 \text{ m/s} \ldots \text{ (explained above)}$$
$$T_{tr1} = 1235.8 \text{ K}$$
$$p_{tr1} = p_{t1}\left(\frac{T_{tr1}}{T_{t1}}\right)^{\frac{\gamma}{\gamma-1}} = 9.38 \text{ bars}$$
$$p_1 = 8.17 \text{ bars}$$
$$T_1 = 1194.2 \text{ K}$$
$$T_{t2} = \frac{T_2}{[1-\frac{\gamma-1}{\gamma+1}{M_{cr2}}^2]} = 1080.1 \text{ K}$$
$$V_{cr2} = 615.9 \text{ m/s}$$
$$U_2 = 205.0 \text{ m/s}$$
$$V_2 = 357.2 \text{ m/s}$$
$$V_{\theta 2} = \frac{U_1 V_{\theta 1} - c_p(T_{t1} - T_{t2})}{U_2} = \text{-}150.0 \text{ m/s}$$
$$V_{z2} = \sqrt{{V_2}^2 - {V_{\theta 2}}^2} = 324.2 \text{ m/s}$$
$$W_{\theta 2} = V_{\theta 2} - U_2 = \text{-}355.0 \text{ m/s}$$
$$W_2 = 480.8 \text{ m/s}$$
$$T_{tr2} = T_{t2} + \left(\frac{{W_2}^2 - {V_2}^2}{2c_p}\right) = 1124.9 \text{ K}$$
$$p_{tr2} = p_{t2}\left(\frac{T_{tr2}}{T_{t2}}\right)^{\frac{\gamma}{\gamma-1}} = 5.38 \text{ bars}$$
$$p_2 = p_{t2}[1 - \frac{\gamma-1}{\gamma+1}{M_{cr2}}^2]^{\frac{\gamma}{\gamma-1}} = 3.75 \text{ bars}$$

With all of the rotor-exit static, total and total-relative flow properties known, we can proceed to compute the required efficiencies as follows:

$$\eta_{t-s} = \frac{1-(\frac{T_{t2}}{T_{t1}})}{1-(\frac{p_2}{p_{t1}})^{\frac{\gamma-1}{\gamma}}} = 74.9\%$$
$$\eta_{tr-tr} = \frac{1-(\frac{T_{tr2}}{T_{tr1}})}{1-(\frac{p_{tr2}}{p_{tr1}})^{\frac{\gamma-1}{\gamma}}} = 69.7\%$$
$$\eta_{s-s} = \frac{1-(\frac{T_2}{T_1})}{1-(\frac{p_2}{p_1})^{\frac{\gamma-1}{\gamma}}} = 71.1\%$$