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Chapter 1.1, Problem 2E (5 Bookmarks) Show all steps: ON

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Problem

Determine which of the following statements are true for all sets *A*, *B*, *C*, and *D*. If a double implication fails, determine whether one or the other of the possible implications holds. If an equality fails, determine whether the statement becomes true if the "equals" symbol is replaced by one or the other of the inclusion symbols ⊂ or ⊃.

(a) $A \subset B$ and $A \subset C \Leftrightarrow A \subset (B \cup C)$.

(b) $A \subset B$ or $A \subset C \Leftrightarrow A \subset (B \cup C)$.

(c) $A \subset B$ and $A \subset C \Leftrightarrow A \subset (B \cap C)$.

(d) $A \subset B$ or $A \subset C \Leftrightarrow A \subset (B \cap C)$.

(e) $A - (A - B) = B$.

(f) $A - (B - A) = A - B$.

(g) $A \cap (B - C) = (A \cap B) - (A \cap C)$.

(h) $A \cup (B - C) = (A \cup B) - (A \cup C)$.

(i) $(A \cap B) \cup (A - B) = A$.

(i) $A \subset C$ and $B \subset D \Rightarrow (A \times B) \subset (C \times D)$.

(k) The converse of (j).

(l) The converse of (j), assuming that *A* and *B* are nonempty.

(m) $(A \times B) \cup (C \times D) = (A \cup C) \times (B \cup D)$.

(n) $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$.

(o) $A \times (B - C) = (A \times B) - (A \times C)$.

(p) $(A - B) \times (C - D) = (A \times C - B \times C) - A \times D$.

(q) $(A \times B) - (C \times D) = (A - C) \times (B \times D)$.

Step-by-step solution

Step 1 of 40

(a)

$A \subset B$ and $A \subset C \Leftrightarrow A \subset (B \cap C)$
The double implication is false.
But, $A \subset B$ and $A \subset C \Rightarrow A \subset (B \cup C)$ is true
$A \subset B$ and $A \subset C \Rightarrow A \subset (B \cap C)$
$\Rightarrow A \subset (B \cup C) \quad (\because B \cap C \subset B \cup C)$

Comments (1)

Step 2 of 40

Here,
$A \subset (B \cup C) \Rightarrow A \subset B$ and $A \subset C$ is false
Example:
$A = \{1,2\}$
$B = \{1\}$
$C = \{2,3\}$
$B \cup C = \{1,2,3\}$
$A = \{1,2\} \subset \{1,2,3\} = B \cup C$
But,
$A \not\subset B \quad (\because 2 \in A$ and $2 \notin B)$
$A \not\subset C \quad (\because 1 \in A$ and $1 \notin C)$

Comment

Step 3 of 40

(b)

$A \subset B$ or $A \subset C \Leftrightarrow A \subset (B \cup C)$
The double implication is false.
But, $A \subset B$ or $A \subset C \Rightarrow A \subset (B \cup C)$ is true
$\neg[A \cup (B \cup C)] \Rightarrow A \not\subset (B \cup C)$
$\Rightarrow \exists x \in A \; St \; x \notin B \cup C$
$\Rightarrow \exists x \in A \; St \; \neg(x \in B \cup C)$
$\Rightarrow \exists x \in A \; St \; \neg(x \in B$ or $x \in C)$
$\Rightarrow \exists x \in A \; St \; \neg(x \in B)$ and $\neg(x \in C)$
$\Rightarrow \exists x \in A \; St \; x \notin B$ and $x \notin C$
$\Rightarrow A \not\subset B$ and $A \not\subset C$
$\Rightarrow \neg(A \subset B)$ and $\neg(A \subset C)$

Comments (2)

Step 4 of 40

Thus,
$\neg[A \subset (B \cup C)] \Rightarrow \neg[(A \subset B)$ or $(A \subset C)]$
Equivalently,
$(A \subset B)$ or $(A \subset C) \Rightarrow A \subset (B \cup C)$
Therefore,
$p \Rightarrow q$ and its contra positive $\neg q \Rightarrow \neg p$ are equivalent
That is, whenever $p \Rightarrow q$ is true
$\neg q \Rightarrow \neg p$ is true and viseversa.

Comment

Step 5 of 40

As in (a) $A \subset B \cup C \not\Leftrightarrow A \subset B$ or $A \subset C$
$A = \{1,2\}$
$B = \{1\}$
$C = \{2,3\}$
$A \subset B \cup C$ but $A \not\subset B$ and $A \not\subset C$
$p \Rightarrow q$ is false only when p is true and q is false.
Here $A \subset B \cup C$ is true but "$A \subset B$ or $A \subset C$" is false
Since, "$A \not\subset B$ and $A \not\subset C$" is true.

Comment

Step 6 of 40

(c)

$A \subset B$ and $A \subset C \Leftrightarrow A \subset (B \cap C)$
$\neg[A \subset (B \cap C)] \Leftrightarrow A \not\subset (B \cap C)$
$\Leftrightarrow \exists x \in A \; St \; x \notin (B \cap C)$
$\Leftrightarrow \exists x \in A \; St \; \neg(x \in (B \cap C))$
$\Leftrightarrow \exists x \in A \; St \; \neg(x \in B$ and $x \in C)$
$\Leftrightarrow \exists x \in A \; St \; \neg(x \in B)$ or $\neg(x \in C)$
$\Leftrightarrow \exists x \in A \; St \; x \notin B$ or $x \notin C$
$\Leftrightarrow A \not\subset B$ or $A \not\subset C$
$\Leftrightarrow \neg(A \subset B$ and $A \subset C)$

Comment

Step 7 of 40

Therefore,
$\neg[A \subset (B \cap C)] \Rightarrow \neg(A \subset B$ and $A \subset C)$
Therefore,
$A \subset B$ and $A \subset C \Rightarrow A \subset (B \cap C)$
And,
$\neg(A \subset B$ and $A \subset C) \Rightarrow \neg[A \subset (B \cap C)]$
Therefore,
$A \subset (B \cap C) \Rightarrow A \subset B$ and $A \subset C$
Hence the double implication is true.

Comment

Step 8 of 40

(d)

$A \subset B$ or $A \subset C \Leftrightarrow A \subset (B \cap C)$
The double implication is false
But,
$A \subset (B \cap C) \Rightarrow A \subset B$ or $A \subset C$ is true
$\neg[A \subset B$ or $A \subset C] \Rightarrow \neg(A \subset B)$ and $\neg(A \subset C)$
$\Rightarrow A \not\subset B$ and $A \not\subset C$
$\Rightarrow \exists x \in A \; St \; x \notin B$ and $\exists y \in A \; St \; y \notin C$
$\Rightarrow \exists x \in A \; St \; x \notin B \cap C$ and $\exists y \in A \; St \; y \notin B \cap C$
$\Rightarrow A \not\subset (B \cap C)$
$\Rightarrow \neg[A \subset (B \cap C)]$

Comment

Step 9 of 40

$\neg[A \subset B$ or $A \subset C] \Rightarrow \neg[A \subset (B \cap C)]$
Equivalently,
$A \subset (B \cap C) \Rightarrow A \subset B$ or $A \subset C$
But,
$A \subset B$ or $A \subset C \not\Rightarrow A \subset (B \cap C)$
$A = \{1\}$
$B\{2,1\}$
$C = \{2,3\}$

Comment

Step 10 of 40

Here, $A \subset B$
Therefore,
"$A \subset B$ or $A \subset C$" is true.
But,
$B \cap C = \{2\}$
And $A \not\subset B \cap C$
That is, $\neg(A \subset B \cap C)$ is true
That is, $A \subset B \cap C$ is false.
$A \subset B$ or $A \subset C \Rightarrow A \subset (B \cap C)$ is false.

Comment

Step 11 of 40

If we can produce at least one example of sets A, B, C such that the above implication is false.
That is where "$A \subset B$ or $A \subset C$" is true, but $A \subset (B \cap C)$ is false
One can produce an example such as,
$A = \{1\}$
$B = \{1,2\}$
$C = \{1,3\}$
Where, $A \subset B$ and $A \subset C$
Therefore, $A \subset B$ and $A \subset C$ is true in this case.
And, $A \subset B \cap C = \{1\}$ is also true

Comment

Step 12 of 40

But for the implication,
$A \subset B$ or $A \subset C \Rightarrow A \subset (B \cap C)$ to be true
Every choice of sets A, B, C must satisfy it.
Even if one choice of A, B and C fails to satisfy the implication
Then the implication cannot be accepted as a true statement.

Comment

Step 13 of 40

(d)

$A - (A - B) = B$
$A - (A - B) = \{x \mid x \in A$ and $x \notin A - B\}$
$= \{x \mid x \in A$ and $\neg(x \in A - B)\}$
$= \{x \mid x \in A$ and $\neg(x \in A$ and $x \notin B)\}$
$= \{x \mid x \in A$ and $\neg(x \in A$ and $\neg(x \in B))\}$
$= \{x \mid x \in A$ and $(\neg(x \in A)$ or $\neg\neg(x \in B))\}$

Comments (1)

Step 14 of 40

$= \{x \mid x \in A$ and $(\neg(x \in A)$ or $(x \in B))\}$
$= \{x \mid (x \in A$ and $\neg(x \in A))$ or $((x \in A)$ and $(x \in B))\}$
$= \{x \mid$ False or $((x \in A)$ and $(x \in B))\}$
$= \{x \mid x \in A$ and $x \in B\}$
$= \{x \mid x \in A \cap B\}$
Therefore,
$A - (A - B) = A \cap B$

Comment

Step 15 of 40

Now,
$A \cap B = B \Leftrightarrow A \supset B$
$A \cap B = B \Leftrightarrow A \cap B \subset B$ and $B \subset A \cap B$
$\Leftrightarrow A \cap B \subset B$ and $B \subset A$ and $B \subset B$
$\Leftrightarrow A \cap B \subset B$ and $B \subset A$
$\Leftrightarrow B \subset A$
Therefore,
$A - (A - B) = B$ is true only when $B \subset A$

Comment

Step 16 of 40

(f)

$A - (B - A) = A - B$
$A - (B - A) = \{x \mid x \in A$ and $x \notin (B - A)\}$
$= \{x \mid x \in A$ and $\neg(x \in B - A)\}$
$= \{x \mid x \in A$ and $\neg(x \in B$ and $x \notin A)\}$
$= \{x \mid x \in A$ and $\neg(x \in B$ and $\neg x \in A)\}$
$= \{x \mid x \in A$ and $(\neg(x \in B)$ or $\neg\neg(x \in A))\}$

Comment

Step 17 of 40

$= \{x \mid x \in A$ and $(\neg(x \in B)$ or $x \in A)\}$
$= \{x \mid (x \in A$ and $(\neg(x \in B)))$ or $(x \in A$ and $x \in A)\}$
$= \{x \mid x \in A - B$ or $x \in A\}$
$= \{x \mid x \in A \cup A - B\}$
$= \{x \mid x \in A\} = \{x \mid x \in A\} \quad \because A - B \subset A$
$A \subset A - B$ only if $B = \phi$
Therefore,
The statement of equality is false.

Comment

Step 18 of 40

(g)

$A \cap (B - C) = (A \cap B) - (A \cap C)$
$(A \cap B) - (A \cap C) = \{x \mid x \in [(A \cap B) - (A \cap C)]\}$
$= \{x \mid x \in A \cap B$ and $x \notin A \cap C\}$
$= \{x \mid x \in A \cap B$ and $\neg(x \in A \cap C)\}$
$= \{x \mid x \in A \cap B$ and $\neg(x \in A$ and $x \in C)\}$
$= \{x \mid x \in A \cap B$ and $(\neg(x \in A)$ or $\neg(x \in C))\}$
$= \{x \mid x \in A \cap B$ and $(x \notin A$ or $x \notin C)\}$

Comment

Step 19 of 40

$= \{x \mid (x \in A \cap B$ and $x \notin A)$ or $(x \in A \cap B$ and $x \notin C)\}$
$= \{x \mid$ False or $(x \in A \cap B$ and $x \notin C)\}$
$= \{x \mid x \in A \cap B$ and $x \notin C\}$
$= \{x \mid x \in A$ and $x \in B$ and $x \notin C\}$
$= \{x \mid x \in A$ and $(x \in B$ and $x \notin C)\}$
$= \{x \mid x \in A$ and $x \in B - C\}$
$= \{x \mid x \in A \cap (B - C)\}$
$= A \cap (B - C)$
Therefore, the statement,
$A \cap (B - C) = (A \cap B) - (A \cap C)$ is true.

Comment

Step 20 of 40

(h)

$A \cup (B - C) = (A \cup B) - (A \cup C)$
This is a false statement
Let, $A = \{1,2,3,4\}$
$B = \{5,2,3,6\}$
$C = \{7,6,3,4\}$
$B - C = \{5,2\}$
$A \cup (B - C) = \{1,2,3,4,5\} \quad \rightarrow$ (I)

Comment

Step 21 of 40

$A \cup B = \{1,2,3,4,5,6\}$
$A \cup C = \{1,2,3,4,7,6\}$
$(A \cup B) - (A \cup C) = \{5\} \quad \rightarrow$ (II)
Therefore, from (I) and (II),
$A \cup (B - C) \neq (A \cup B) - (A \cup C)$
Also,
$A \cup (B - C) \not\subset (A \cup B) - (A \cup C)$

Comment

Step 22 of 40

But we can show that $(A \cup B) - (A \cup C) \subset A \cup (B - C)$
$x \in (A \cup B) - (A \cup C) \Rightarrow x \in (A \cup B)$ and $x \notin (A \cup C)$
$\Rightarrow x \in (A \cup B)$ and $\neg(x \in A \cup C)$
$\Rightarrow (x \in A$ or $x \in B)$ and $\neg(x \in A$ or $x \in C)$
$\Rightarrow (x \in A$ or $x \in B)$ and $(\neg x \in A$ or $\neg x \in C)$
$\Rightarrow (x \in A$ or $x \in B)$ and $((x \notin A)$ or $(x \notin C))$
$\Rightarrow (x \in A$ or $x \in B)$ and $x \notin C$ or $x \notin A$
$\Rightarrow ((x \in A$ or $x \notin C)$ or $(x \in B$ and $x \in C))$ and $x \notin A$
$\Rightarrow (x \in A - C$ or $x \in B - C)$ and $x \notin A$

Comment

Step 23 of 40

$\Rightarrow (x \in A - C$ and $x \notin A)$ or $(x \in B$ and $x \notin C$ and $x \notin A)$
$\Rightarrow (x \in A$ and $x \notin C$ and $x \notin A)$
or $(x \in B$ and $\neg x \in C$ and $\neg x \in A)$
$\Rightarrow (x \in A$ and $x \notin A$ and $x \notin C)$
or $(x \in B$ and $\neg(x \in A$ or $x \in C))$
$\Rightarrow$ (False and $x \notin C)$ or $(x \in B$ and $\neg(x \in A \cup C))$
$\Rightarrow$ False or $(x \in B - (A \cup C))$
$\Rightarrow x \in B - (A \cup C)$
$\Rightarrow x \in B$ and $x \notin C \quad \because C \subset A \cup C$
$\Rightarrow x \in B - C$
$\Rightarrow x \in A \cup (B - C)$
Therefore,
$x \in (A \cup B) - (A \cup C) \Rightarrow x \in A \cup (B - C)$
Therefore,
$(A \cup B) - (A \cup C) \subset A \cup (B - C)$

Comment

Step 24 of 40

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$$x\in(A\cap B)\cup(A-B)\Leftrightarrow x\in A\cap B \text{ or } x\in A-B$$
$$\Leftrightarrow (x\in A \text{ and } x\in B) \text{ or } (x\in A \text{ and } x\notin B)$$
$$\Leftrightarrow (x\in A) \text{ and } (x\in B \text{ or } x\notin B)$$
$$\Leftrightarrow (x\in A) \text{ and True}$$
$$\Leftrightarrow x\in A$$

Therefore,

$x(A\cap B)\cup(A-B)\Leftrightarrow x\in A$

Therefore,

$(A\cap B)\cup(A-B)=A$

Comment

Step 25 of 40

(j)

$$A\subset C \text{ and } B\subset D \Rightarrow (A\times B)\subset(C\times D)$$
$$\neg[(A\times B)\subset(C\times D)] \Rightarrow x\in(A\times B) \text{ and } x\notin(C\times D)$$
$$\Rightarrow \exists a\in A \text{ and } \exists b\in B \text{ S.t } x=(a,b) \text{ and } (a,b)\notin(C\times D)$$
$$\Rightarrow (\exists a\in A \text{ and } \exists b\in B) \text{ and } (a,b)\notin C\times D$$
$$\Rightarrow (\exists a\in A \text{ and } \exists b\in B) \text{ and } (a\notin C \text{ or } b\notin D)$$
$$\Rightarrow (\exists a\in A \text{ and } \exists a\notin C) \text{ or } (\exists b\in B \text{ and } b\in D)$$
$$\Rightarrow A\not\subset C \text{ or } B\not\subset D$$
$$\Rightarrow \neg A\subset C \text{ or } \neg B\subset D$$
$$\Rightarrow \neg[A\subset C \text{ and } B\subset D]$$

Comment

Step 26 of 40

Therefore,

$\neg[(A\times B)\subset(C\times D)] \Rightarrow \neg[A\subset C \text{ and } B\subset D]$

Equivalently, $A\subset C$ and $B\subset D \Rightarrow A\times B\subset C\times D$

Therefore. the statement is true.

Comment

Step 27 of 40

(k)

$$(A\times B)\subset(C\times D) \Rightarrow A\subset C \text{ and } B\subset D$$
$$\neg[A\subset C \text{ and } B\subset D] \Rightarrow \neg(A\subset C) \text{ or } \neg(B\subset D)$$
$$\Rightarrow A\not\subset C \text{ or } B\not\subset D$$
$$\Rightarrow (\exists a\in A \text{ and } a\notin C) \text{ or } (\exists b, b\in B \text{ and } b\notin D)$$
$$\Rightarrow \exists a((a\in A \text{ and } a\notin C) \text{ or } b\in B)$$
$$\text{and } ((a\in A \text{ and } a\notin C) \text{ or } b\notin D)$$
$$\Rightarrow \exists a, a\in A, \exists b, b\in B \text{ S.t } (a\notin C \text{ or } b\notin D)$$
$$\Rightarrow \exists(a,b)\in A\times B \text{ S.t } (a,b)\notin C\times D$$

Note: Here we assume $A\neq\phi$, $B=\phi$

Comment

Step 28 of 40

(l)

$(A\times B)\subset(C\times D) \Rightarrow A\subset C$ and $B\subset D$

Case (i):

$A=\phi \quad (B\neq\phi)$

In this case the implication is false.

$A+B=\{1\}$

$D=\{2\}$

$A=\phi$

$A\times B=\phi$

Comment

Step 29 of 40

Since, if $A\times B\neq\phi$ then $\exists x\in A\times B$

But any $x\in A\times B$ is of the form (a, b) where $a\in A$ and $b\in B$ but this is not possible since $A=\phi$

Since $A\times B=\phi$, $A\times B\subset C\times D$ is true

But $B\subset D$ is not true.

$p\Rightarrow q$ is false, when and only when p is true and q is false.

Comment

Step 30 of 40

Case (ii)

$A=\phi,\ B\neq\phi$

In this case the implication is vacuously true. Since $A\subset C$ and $B\subset D$ is always true in this case.

Comment

Step 31 of 40

(m)

$$(A\times B)\cup(C\times D)=(A\cup C)\times(B\cup D)$$
$$x\in(A\times B)\cup(C\times D) \Rightarrow x\in A\times B \text{ or } x\in C\times D$$
$$\Rightarrow x=(x_1,x_2)\in A\times B \text{ or } (x_1,x_2)\in C\times D$$
$$\Rightarrow (x_1\in A \text{ and } x_2\in B) \text{ or } (x_1\in C \text{ and } x_2\in D)$$
$$\Rightarrow ((x_1\in A \text{ and } x_2\in B) \text{ or } x_1\in C)$$
$$\text{or } ((x_1\in A \text{ and } x_2\in B) \text{ or } x_1\in D)$$
$$\Rightarrow (x_1\in A \text{ or } x_1\in C) \text{ and } (x_2\in B \text{ or } x_1\in C)$$
$$\text{and } (x_1\in A \text{ or } x_2\in D) \text{ and } (x_2\in B \text{ or } x_2\in D)$$
$$\Rightarrow (x_1\in A \text{ or } x_1\in C) \text{ and } (x_2\in B \text{ or } x_2\in D) \quad (\because p\wedge q\Rightarrow p)$$
$$\Rightarrow (x_1\in A\cup C) \text{ and } (x_2\in B\cup D)\Rightarrow (x_1,x_2)\in(A\cup C)\times(B\cup D)$$

Therefore,

$x\in(A\times B)\cup(C\times D)\Rightarrow x\in(A\cup C)\times(B\cup D)$

Therefore,

$(A\times B)\cup(C\times D)\subset(A\cup C)\times(B\cup D)$

Comment

Step 32 of 40

$$(x,y)\notin(A\times B)\cup(C\times D) \Rightarrow (x,y)\notin(A\times B) \text{ and } (x,y)\notin(C\times D)$$
$$\Rightarrow (x\notin A \text{ or } y\notin B) \text{ and } (x\notin C \text{ or } y\notin D)$$
$$\Rightarrow ((x\notin A \text{ or } y\notin B) \text{ and } x\notin C)$$
$$\text{or } ((x\notin A \text{ or } y\notin B) \text{ and } y\notin D)$$
$$\Rightarrow (x\notin A \text{ or } x\notin C) \text{ or } (y\notin B \text{ or } x\notin C)$$
$$\text{or } (x\notin A \text{ or } y\notin D) \text{ or } (y\notin B \text{ or } y\notin D)$$
$$\Rightarrow (x\notin(A\cup C)) \text{ or } (y\notin B\cup D)$$
$$\text{or } (y\notin B \text{ and } x\notin C) \text{ or } (x\notin A \text{ and } y\notin D)$$
$$\Rightarrow \neg(x\notin(A\cup C) \text{ and } (y\notin B\cup D))$$
$$\text{or } \neg(x\in C \text{ or } y\in B) \text{ or } \neg(x\in A \text{ or } y\in D)$$
$$\Rightarrow \neg\begin{bmatrix}((x,y)\in(A\cup C)\times(B\cup D)) \text{ and} \\ (x,y)\in(C\times B) \text{ and } (x,y)\in A\times D\end{bmatrix}$$
$$\Rightarrow \neg[(x,y)\in(A\cup C)\times(B\cup D) \text{ and } (x,y)\in A\times D]$$

Comments (1)

Step 33 of 40

Since, p *and* $q \Rightarrow q$

Here,

$(x,y)\in A\times D \Rightarrow (x,y)\in(A\cup C)\times(B\cup D)$

$p\wedge q\wedge r \Rightarrow \pi$

Therefore,

$$\neg p \Rightarrow \neg[p\wedge q\wedge r]$$
$$\Rightarrow \neg[(x,y)\in(A\cup C)\times(B\cup D)]$$
$$\Rightarrow (x,y)\notin(A\cup C)\times(B\cup D)$$

Comment

Step 34 of 40

Thus,

$(x,y)\notin(A\times B)\cup(C\times D) \Rightarrow (x,y)\notin(A\cup C)\times(B\cup D)$

That is,

$$(x,y)\notin(A\times B)\cup(C\times D) \Rightarrow (x,y)\notin(A\cup C)\times(B\cup D)$$
$$\Rightarrow (x,y)\in(A\times B)\cup(C\times D)$$

Therefore,

$(A\cup C)\times(B\cup D)\subset(A\times B)\cup(C\times D)$

Thus the equality,

$(A\times B)\cup(C\times D)=(A\cup C)\times(B\cup D)$ is true.

Comment

Step 35 of 40

(n)

$$(A\times B)\cap(C\times D)=(A\cap C)\times(B\cap D)$$
$$(x,y)\in(A\times B)\cap(C\times D)\Leftrightarrow((x,y)\in A\times B) \text{ and } (x,y\in C\times D)$$
$$\Leftrightarrow(x\in A \text{ and } y\in B) \text{ and } (x\in C \text{ and } y\in D)$$
$$\Leftrightarrow(x\in A \text{ and } x\in C) \text{ and } (y\in B \text{ and } y\in D)$$
$$\Leftrightarrow(x\in A\cap C) \text{ and } (y\in B\cap D)$$
$$\Leftrightarrow(x,y)\in(A\cap C)\times(B\cap D)$$

Therefore, the equality,

$(A\times B)\cap(C\times D)=(A\cap C)\times(B\cap D)$ is true.

Comment

Step 36 of 40

(0)

$$A\times(B-C)=(A\times B)-(A\times C)$$
$$(x,y)\in A\times(B-C)\Rightarrow x\in A \text{ and } y\in B-C$$
$$\Rightarrow x\in A \text{ and } y\in B \text{ and } y\notin C$$
$$\Rightarrow x\in A \text{ and } y\in B \text{ and } y\notin C \text{ or } (x\in A \text{ and } x\notin A)$$
$$\Rightarrow x\in A \text{ and } (x\notin A \text{ or } (y\notin C \text{ and } y\in B \text{ and } y\in A))$$
$$\Rightarrow x\in A \text{ and } \begin{pmatrix}(x\notin A \text{ or } y\notin C) \text{ and } (x\notin A \text{ or } y\in B) \\ \text{and } (x\in A \text{ or } x\notin A)\end{pmatrix}$$
$$\Rightarrow x\in A \text{ and } (x\notin A \text{ or } y\notin C) \text{ and } (x\notin A \text{ or } y\in B) \text{ and True}$$
$$\text{or } [(x\in A \text{ and } y\notin C) \text{ and } (y\in B \text{ and } y\notin C)]$$
$$\Rightarrow x\in A \text{ and } (x\notin A \text{ or } y\notin C) \text{ and } (x\notin A \text{ or } y\in B)$$
$$\Rightarrow x\in A \text{ and } (x\notin A \text{ or } y\in B) \text{ and } (x\notin A \text{ or } y\notin C)$$
$$\Rightarrow (x\in A \text{ and } x\notin A) \text{ or } (x\in A \text{ and } y\in B) \text{ and } (x\notin A \text{ or } y\notin C)$$
$$\Rightarrow \text{False or } (x\in A \text{ and } y\in B) \text{ and } (x\notin A \text{ or } y\notin C)$$
$$\Rightarrow (x\in A \text{ and } y\in B) \text{ and } \neg(x\in A \text{ or } y\in C)$$

Comment

Step 37 of 40

$$\Rightarrow (x,y)\in A\times B \text{ and } (x,y)\notin A\times C$$
$$\Rightarrow (x,y)\in(A\times B)-(A\times C)$$

Therefore,

$(x,y)\in A\times(B-C)\Rightarrow(x,y)\in(A\times B)-(A\times C)$

Therefore,

$A\times(B-C)\subset(A\times B)-(A\times C) \quad \rightarrow \ (I)$

Comment

Step 38 of 40

$$(x,y)\in(A\times B)-(A\times C)\Rightarrow((x,y)\in A\times B \text{ and } |(x,y)\notin A\times C)$$
$$\Rightarrow (x\in A \text{ and } y\in B) \text{ and } \neg((x,y)\in A\times C)$$
$$\Rightarrow (x\in A \text{ and } y\in B) \text{ and } \neg(x\in A \text{ and } y\in C)$$
$$\Rightarrow (x\in A \text{ and } y\in B) \text{ and } (x\notin A \text{ or } y\notin C)$$
$$\Rightarrow [(x\in A) \text{ and } (x\notin A \text{ or } y\notin C)]$$
$$\text{and } [(y\in B) \text{ and } (x\notin A \text{ or } y\notin C)]$$
$$\Rightarrow [\{(x\in A) \text{ and } (x\notin A)\} \text{ or } \{(x\in A) \text{ and } y\notin C\}]$$
$$\text{and } [\{(y\in B \text{ and } x\notin A) \text{ or } \{y\in B \text{ and } y\notin C\}\}]$$
$$\Rightarrow [\text{False or } \{(x\in A) \text{ and } y\notin C\}]$$
$$\text{and } [\{(y\in B \text{ and } x\notin A) \text{ or } \{y\in B \text{ and } y\notin C\}\}]$$

Comment

Step 39 of 40

$$\Rightarrow (x\in A \text{ and } y\in C)$$
$$\text{and } [\{(y\in B \text{ and } x\notin A) \text{ or } \{y\in B \text{ and } y\notin C\}\}]$$
$$\Rightarrow [(x\in A \text{ and } y\in C) \text{ and } (y\in B \text{ and } x\notin A)]$$
$$\text{or } [(x\in A \text{ and } y\notin C) \text{ and } (y\in B \text{ and } y\notin C)]$$
$$\Rightarrow [x\in A \text{ and } x\notin A \text{ and } y\notin C \text{ and } y\in B]$$
$$\text{or } [x\in A \text{ and } y\in B-C]$$
$$\Rightarrow \text{False or } (x,y)\in A\times(B-C)$$
$$\Rightarrow (x,y)\in A\times(B-C)$$

Comment

Step 40 of 40

Therefore,

$(x,y)\in(A\times B)-(A\times C)\Rightarrow(x,y)\in A\times(B-C)$

Therefore,

$(A\times B)-(A\times C)\not\Rightarrow\subset A\times(B-C) \quad \rightarrow \ (II)$

Therefore,

By (I) and (II),

$A\times(B-C)=(A\times B)-(A\times C)$

Therefore,

The equality,

$A\times(B-C)=(A\times B)-(A\times C)$ is a true statement.

Comment

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