Solutions to Problems in Chapter One

1.1 $W = mg = 3(32.2) = 96.6$ lb.

1.2 $m = W/g = 100/9.81 = 10.19$ kg. $W = 100(0.2248) = 22.48$ lb. $m = 10.19(0.06852) = 0.698$ slug.

1.3 $d = (50 + 5/12)(0.3048) = 15.37$ m.

1.4 $d = 3(100)(0.3048) = 91.44$ m

1.5 $d = 100(3.281) = 328.1$ ft

1.6 $d = 50(3600)/5280 = 34.0909$ mph

1.7 $v = 100(0.6214) = 62.14$ mph

1.8 $n = 1/[60(1.341 \times 10^{-3})] = 12.43$, or approximately 12 bulbs.

1.9 $5(70 - 32)/9 = 21.1^\circ$ C

1.10 $9(30)/5 + 32 = 86^\circ$ F

1.11 $\omega = 3000(2\pi)/60 = 314.16$ rad/sec. Period $P = 2\pi/\omega = 60/3000 = 1/50$ sec.

1.12 $\omega = 5$ rad/sec. Period $P = 2\pi/\omega = 2\pi/5 = 1.257$ sec. Frequency $f = 1/P = 5/2\pi = 0.796$ Hz.

1.13 Speed = 40(5280)/3600 = 58.6667 ft/sec. Frequency = 58.6667/30 = 1.9556 times per second.

1.14 $x = 0.005 \sin 6t$, $\dot{x} = 0.005(6) \cos 6t = 0.03 \cos 6t$. Velocity amplitude is 0.03 m/s. $\ddot{x} = -6(0.03) \sin 6t = -0.18 \sin 6t$. Acceleration amplitude is 0.18 m/s^2. Displacement, velocity and acceleration all have the same frequency.

1.15 Physical considerations require the model to pass through the origin, so we seek a model of the form $f = kx$. A plot of the data shows that a good line drawn by eye is given by $f = 0.2x$. So we estimate k to be 0.2 lb/in.

1.16 The script file is

```
x = [0:0.01:1];
subplot(2,2,1)
plot(x,sin(x),x,x),xlabel('x (radians)'),ylabel('x and sin(x)'),...
gtext('x'),gtext('sin(x)')
subplot(2,2,2)
plot(x,sin(x)-x),xlabel('x (radians)'),ylabel('Error: sin(x) - x')
subplot(2,2,3)
plot(x,100*(sin(x)-x)./sin(x)),xlabel('x (radians)'),...
ylabel('Percent Error'),grid
```

The plots are shown in the figure.

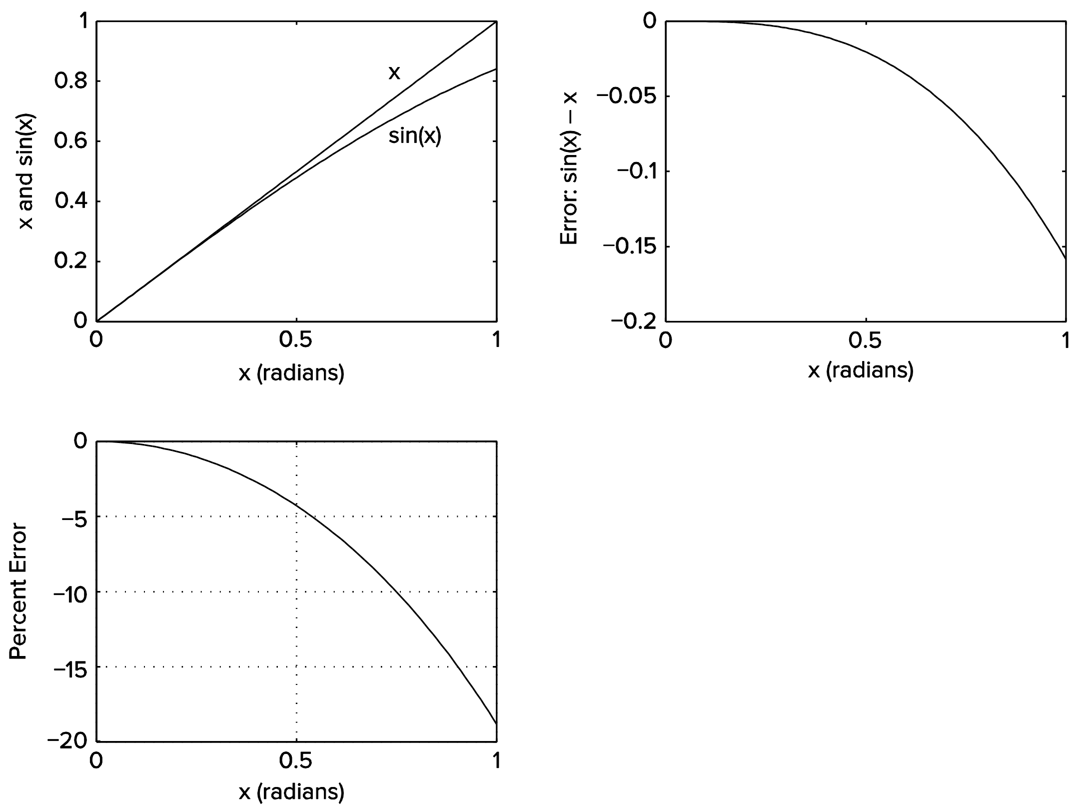

Figure : for Problem 1.16.

From the third plot we can see that the approximation $\sin x \approx x$ is accurate to within 5% if $|x| \leq 0.5$ radians.

1.17 For θ near $\pi/4$,

$$f(\theta) \approx \sin\frac{\pi}{4} + \left(\cos\frac{\pi}{4}\right)\left(\theta - \frac{\pi}{4}\right)$$

For θ near $3\pi/4$,

$$f(\theta) \approx \sin\frac{3\pi}{4} + \left(\cos\frac{3\pi}{4}\right)\left(\theta - \frac{3\pi}{4}\right)$$

1.18 For θ near $\pi/3$,

$$f(\theta) \approx \cos\frac{\pi}{3} - \left(\sin\frac{\pi}{3}\right)\left(\theta - \frac{\pi}{3}\right)$$

For θ near $2\pi/3$,

$$f(\theta) \approx \cos\frac{2\pi}{3} - \left(\sin\frac{2\pi}{3}\right)\left(\theta - \frac{2\pi}{3}\right)$$

1.19 For h near 25,

$$f(h) \approx \sqrt{25} + \frac{1}{2\sqrt{25}}(h - 25) = 5 + \frac{1}{10}(h - 25)$$

1.20 For r near 5,

$$f(r) \approx 5^2 + 2(5)(r - 5) = 25 + 10(r - 5)$$

For r near 10,

$$f(r) \approx 10^2 + 2(10)(r - 10) = 100 + 20(r - 10)$$

1.21 For h near 16,

$$f(h) \approx \sqrt{16} + \frac{1}{2\sqrt{16}}(h - 16) = 4 + \frac{1}{8}(h - 16)$$

and $f(h) \geq 0$ if $h > -16$.

1.22 Construct a straight line the passes through the two endpoints at $p = 0$ and $p = 900$. At $p = 0$, $f(0) = 0$. At $p = 900$, $f(900) = 0.002\sqrt{900} = 0.06$. This straight line is

$$f(p) = \frac{0.06}{900}p = \frac{1}{15{,}000}p$$

1.23 a)

$$4\int_2^x dx = 3\int_0^t t\,dt$$

$$x(t) = 2 + \frac{3}{8}t^2$$

b)

$$5\int_3^x dx = 2\int_0^t e^{-4t}\,dt$$

$$x(t) = 3.1 - 0.1e^{-4t}$$

c) Let $v = \dot{x}$.

$$3\int_7^v dv = 5\int_0^t t\,dt$$

$$v(t) = \frac{dx}{dt} = 7 + \frac{5}{6}t^2$$

$$\int_2^x dx = \int_0^t \left(7 + \frac{5}{6}t^2\right) dt$$

$$x(t) = 2 + 7t + \frac{5}{18}t^3$$

d) Let $v = \dot{x}$.

$$4\int_2^v dv = 7\int_0^t e^{-2t}\,dt$$

$$v(t) = \frac{23}{8} - \frac{7}{8}e^{-2t}$$

$$\int_4^x dx = \int_0^t \left(\frac{23}{8} - \frac{7}{8}e^{-2t}\right) dt$$

$$x(t) = \frac{57}{16} + \frac{23}{8}t + \frac{7}{16}e^{-2t}$$

e) $\dot{x} = C_1$, but $\ddot{x}(0) = 5$, so $C_1 = 5$. $x = 5t + C_2$, but $x(0) = 2$, so $C_2 = 2$. Thus $x = 5t + 2$.

1.24 From (1.4.4),

$$\alpha = \frac{(10)^2}{2(2)(1000)} = 0.025$$

$$f = (\alpha + 1)mg = 1.025(5000)(2) = 10{,}250 \text{ N}$$

From (1.4.2),

$$t_f = \frac{v_0}{\alpha g} = \frac{10}{0.025(2)} = 200 \text{ s}$$

1.25 Let $\dot{h}(0) = b = -v_0$.

$$\dot{h}(t_f) = 0 = at_f - v_0$$

so

$$a = \frac{v_0}{t_f}$$

Thus

$$h(t) = \frac{a}{2}t^2 - v_0 t + h_0 = \frac{v_0}{2t_f}t^2 - v_0 t + h_0$$

and

$$h(t_f) = \frac{v_0}{2t_f}t_f^2 - v_0 t_f + h_0 = 0$$

which gives

$$t_f = \frac{2h_0}{v_0}$$

and

$$a = -\frac{v_0}{t_f} = \frac{v_0^2}{2h_0}$$

Also, since $\ddot{h} = a$,

$$f = m(\ddot{h} + g) = m(a + g) = m\left(\frac{v_0^2}{2h_0} + g\right)$$

1.26 We require that $\dot{h}(t_f) = -v_f$. From (1.4.3),

$$h(t) = \alpha g \frac{t^2}{2} - v_0 t + h_0$$

$$\dot{h} = \alpha g t - v_0$$

$$\dot{h}(t_f) = \alpha g t_f - v_0 = -v_f$$

so

$$t_f = \frac{v_0 - v_f}{\alpha g} = (v_0 - v_f)\frac{2h_0}{v_0^2}$$

Equation (1.4.4) stays the same, so the thrust stays the same as (1.4.6):

$$f = (\alpha + 1)mg$$

1.27 a.

$$f = m(\ddot{h} + g) = m(2at + b + g)$$

Find a and b.

$$\dot{h}(t_f) = at_f^2 + bt_f - v_0 = 0 \quad (1)$$

$$h(t) = \frac{a}{3}t^3 + \frac{b}{2}t^2 - v_0 t + h_0$$

and

$$h(t_) = \frac{a}{3}t_f^3 + \frac{b}{2}t_f^2 - v_0 t_f + h_0 = 0 \quad (2)$$

Solve (1) and (2) for a and b:

$$a = \frac{6h_0 - 3v_0 t_f}{t_f^3}$$

$$b = \frac{4v_0}{t_f} - \frac{6h_0}{t_f^2}$$

Thus t_f is free to be chosen.

b. 1.

$$a = \frac{6(500) - 3(5)(200)}{(200)^3} = 0$$

$$b = \frac{4(5)}{200} - \frac{6(500)}{(200)^2} = 0.025$$

$$f = 5062.5$$

2.

$$a = \frac{6(500) - 3(5)(100)}{(100)^3} = 0.0015$$

$$b = \frac{4(5)}{100} - \frac{6(500)}{(100)^2} = -0.1$$

$$f = 2500(0.003t + 1.9) = 7.5t = 4750$$

1.28 The total inertia is $I = I_m + I_S + I_L = 0.005 + 0.002 + 0.008 = 0.015$. From Figure 1.5.4 we obtain the required angular acceleration from the slope of the velocity curve. This gives

$$\dot{\omega} = \begin{cases} \dfrac{200}{0.1} = 2000 & 0 \leq t \leq 0.1 \\ 0 & 0.1 < t < 4.5 \\ -\dfrac{200}{0.1} = -2000 & 4.5 \leq t \leq 4.6 \end{cases}$$

Thus the required torque is

$$T = I\dot{\omega} = \begin{cases} 30 & 0 \leq t \leq 0.1 \\ 0 & 0.1 < t < 4.5 \\ -30 & 4.5 \leq t \leq 4.6 \end{cases}$$

So the maximum required torque is 30 N·m.

The rms average torque is calculated as follows:

$$T_{rms} = \sqrt{\frac{1}{4.6}[(30)^2(0.1) + (0)^2(4.4) + (-30)^2(0.1)]} = 6.255 \ \ \mathrm{N \cdot m}$$

1.29 The equation of motion is now

$$I\dot{\omega} = T - T_F \qquad \omega > 0$$

Thus

$$T = I\dot{\omega} + T_F = \begin{cases} 30 + 3 = 33 & 0 \le t \le 0.1 \\ 3 & 0.1 < t < 4.5 \\ -30 + 3 = -27 & 4.5 \le t \le 4.6 \end{cases}$$

So the maximum required torque is 33 and the rms torque is

$$T_{rms} = \sqrt{\frac{1}{4.6}[(33)^2(0.1) + (3)^2(4.4) + (-27)^2(0.1)]} = 6.938 \ \ \mathrm{N \cdot m}$$

1.30 The total load displacement is given as $\theta_f = 10\pi$ rad. We also have $t_f = 0.5$ s and a slew time of $t_2 - t_1 = 0.4$ s. Using a symmetrical velocity profile requires that $t_1 = 0.05$ and $t_2 = 0.45$. The total inertia is $I = 6 \times 10^{-3} + 5 \times 10^{-4} + 10^{-3} = 7.5 \times 10^{-3}$. From Table 1.5.1 we have $\omega_{max} = 10\pi/0.4 = 69.813$ rad/s or $60\omega_{max}/(2\pi) = 666.667$ rpm. So the load will rotate at that speed for 0.4 s.

We are given that $T_d = 2$ N·m. So the maximum torque required is $T_{max} = 12.4720$ N·m and the rms average torque is $T_{rms} = 5.0924$ N·m.

1.31 The total load displacement is given as $\theta_f = 4\pi$ rad. We also have $t_f = 1$ s and a slew time of $t_2 - t_1 = 0.6$ s. Using a symmetrical velocity profile requires that $t_1 = 0.2$ and $t_2 = 0.8$. The total inertia is $I = 10^{-2} + 6 \times 10^{-4} + 2 \times 10^{-3} = 11.6 \times 10^{-3}$. From Table 1.5.1 we have $\omega_{max} = 4\pi/0.8 = 15.7083$ rad/s or $60\omega_{max}/(2\pi) = 150$ rpm. So the load will rotate at that speed for 0.6 s.

We are given that $T_d = 0.5$ N·m. So the maximum torque required is $T_{max} = 1.4111$ N·m and the rms average torque is $T_{rms} = 0.7629$ N·m.

1.32 The equivalent inertia of the load mass is, from (1.5.11),

$$I_L = \frac{50(0.008)^2}{4\pi^2} = 8.106 \times 10^{-7}$$

The mass of the screw is

$$m_r = \rho V = 7700\pi(0.01)^2(0.7) = 1.6933 \text{ kg}$$

and the inertia of the screw is

$$I_r = \frac{1}{2}1.6933(0.01)^2 = 8.467 \times 10^{-5}$$

So the total system inertia is

$$I = I_m + I_L + I_r = 2 \times 10^{-4} + 8.106 \times 10^{-7} + 8.467 \times 10^{-5} = 2.855 \times 10^{-4} \quad \text{kg} \cdot \text{m}^2$$

From the equation of motion, the required torque is $T = I\dot{\omega}$. The linear displacement x of the mass is related to the rotation angle θ of the screw by $x = L\theta/2\pi$. Thus the linear acceleration $\ddot{x}$ is related to the angular acceleration $\ddot{\theta} = \dot{\omega}$ by $\dot{\omega} = 2\pi\ddot{x}/L$.

The given linear acceleration is $\ddot{x} = (0.3 - 0)/0.2 = 2 = 1.5 \text{ m/s}^2$. Thus

$$\dot{\omega} = \frac{2\pi(1.5)}{0.008} = 1178.1 \ \ \text{rad/s}^2$$

Thus the required torque is $T = 2.855 \times 10^{-4}(1178.1) = 0.3363$ N·m.

1.33

```
>> x=2;y=5;(y*x^3)/(x-y)
ans =
-13.3333
>> 3*x/(2*y)
ans =
0.6000
>> (3.2)*x*y
ans =
32
>> x^5/(x^5-1)
ans =
1.0323
```

1.34

```
>> x=-7-5j;y=4+3j;
>> x+y
ans =
-3.0000 - 2.0000i
>> x*y
ans =
-13.0000 -41.0000i
>> x/y
ans =
-1.7200 + 0.0400i
```

1.35

```
>> (3+6j)/(-7-9j)
ans =
-0.5769 - 0.1154i
>> (5+4j)/(5-4j)
ans =
0.2195 + 0.9756i
>> (3/2)*j
ans =
0.0000 + 1.5000i
>> 3/(2j)
ans =
0.0000 - 1.5000i
```

1.36

```
>> x=5+8j;y=-6+7j;
v>> u=x+y
u =
v -1.0000 +15.0000i
>> v=x*y
v =
-86.0000 -13.0000i
>> w=x/y
w =
0.3059 - 0.9765i
>> z=exp(x)
z =
-2.1594e+01 + 1.4683e+02i
>> r=sqrt(y)
r =
1.2688 + 2.7586i
>> s=x*y^2
s =
6.0700e+02 - 5.2400e+02i
```

1.37

```
>> exp(2)
ans =
7.3891
>> log10(2)
ans =
0.3010
>> log(2)
ans =
0.6931
```

1.38

```
>> cos(pi/3)
ans =
0.5000
>> cosd(80)
ans =
0.1736
>> acos(0.7)
ans =
0.7954
>> acosd(0.6)
ans =
53.1301
```

1.39

```
>> atan(2)
ans =
1.1071
>> atan(100)
ans =
1.5608
```

The answers for (c), (d), and (e) are in degrees.

```
>> atan2d(3,2)
ans =
56.3099
>> atan2d(3,-2)
ans =
123.6901
>> atan2d(-3,2)
ans =
-56.3099
```

1.40

```
>> x=1:0.2:5;
>> y=7*sin(4*x);
>> length(y)
ans =
21
>> y(3)
ans =
-4.4189
```

1.41

```
roots([13,182,-184,2503])
ans =
-15.6850 + 0.0000i
0.8425 + 3.4008i
0.8425 - 3.4008i
```

1.42

```
roots([70,24,-10,20])
ans =
-0.8771 + 0.0000i
0.2671 + 0.5044i
0.2671 - 0.5044i
```

1.43

```
>>poly([-2+5j,-2-5j,-7])
ans =
1 11 57 203
```

This gives the polynomial

$$x^3 + 11x^2 + 57x + 203$$

To check,

```
>> roots(ans)
ans =
-7.0000 + 0.0000i
-2.0000 + 5.0000i
-2.0000 - 5.0000i
```

1.44

```
>> x=linspace(0,2,300);u=2*log10(60*x+1);v=3*cos(6*x);
>> plot(x,u,x,v,'–'),xlabel('Miles'),ylabel('Mph'),gtext('u'),gtext('v')
```

1.45

```
>> h=polyval([0.0125,-5,500],t);
>> plot(t,h),xlabel('Time(s)'),ylabel('Height (m)')
```

1.46 The given parameters are m, g, h_0, and v_0. Follow Example 1.4.1 and program equations (1.4.3) for α and (1.4.2) for t_f.

1.47

```
t1=0:0.001:0.5;
om1=300*t1;
t2=0.501:0.001:2.5;
om2=150*ones(size(t2));
t3=2.501:0.001:3;
om3=-300*(t3-2.5)+150;
t=[t1,t2,t3];
om=[om1,om2,om3];
plot(t,om)
```

1.48 Here is a fragment of the program. You must create code to input the following parameters: θ_f, T_d, t_1, t_2, t_f, and I.

```
om_max=theta_f/t2
alpha_max=theta_f/(t1*t2)
T_max=I*theta_f/(t1*t2)+Td
T_rms = sqrt(2*I^2*theta_f^2/(tf*t1*t2^2)+Td^2)
```

1.49

Follow Example 1.5.5. The equations are:

$$I_L = \frac{mL^2}{4\pi^2} \qquad m_s = 7700\pi r^2 L$$

$$I_s = \frac{1}{2}m_s r^2 \qquad I = I_m + I_L + I_s$$

$$\ddot{x} = \frac{v_s}{t_1} \qquad \dot{\omega} = \frac{2\pi\ddot{x}}{L}$$

$$T = I\dot{\omega}$$