3-1 What is the significance of the "critical stress"?
(a) with respect to the structure of the concrete?
(b) with respect to spiral reinforcement?
(c) with respect to strength under sustained loads?

(a) A continuous pattern of mortar cracks begins to form. As a result, there are few undamaged portions to carry load and the stress-strain curve is highly nonlinear.

(b) At the critical stress the lateral strain begins to increase rapidly. This causes the concrete core within the spiral to expand, stretching the spiral. The tension in the spiral is equilibrated by a radial compression in the core. This in turn, biaxially compresses the core, and thus strengthens it.

(c) When concrete is subjected to sustained loads greater than the critical stress, it will eventually fail.

3-2 A group of 45 tests on a given type of concrete had a mean strength of 4820 psi and a standard deviation of 550 psi. Does this concrete satisfy the strength requirement for 4000-psi concrete?

$$f_c' = f_{cr} - 1.34s$$
$$\text{using } f_{cr} = 4280 \text{ psi}$$
$$\text{(for design) } f_c' = 4820 \text{ psi} - 1.34 \times 550 \text{ psi} = 4080 \text{ psi}$$

$$f_c' = f_{cr} - 2.33s + 500 \text{ psi}$$
$$\text{using } f_{cr} = 4280 \text{ psi}$$
$$\text{(for design) } f_c' = 4820 \text{ psi} - 2.33 \times 550 \text{ psi} + 500 \text{ psi} = 4040 \text{ psi}$$

Because both of these exceed 4000 psi, the concrete satisfies the strength requirement for 4000 psi concrete.

3-3 The concrete containing Type I cement in a structure is cured for 3 days at 70° F followed by 6 days at 40° F. Use the maturity concept to estimate its strength as a fraction of the 28-day strength under standard curing.

Note: $°C = \frac{5}{9}\left(°F - 32\right)$, so 70° F = 21.1° C and 40° F = 4.4° C

$$M = \sum_{i=1}^{n} (T_i + 10° \text{ C})(t_i)$$
$$= (21.1° \text{ C} + 10° \text{ C})(3 \text{ days}) + (4.4° \text{ C} + 10° \text{ C})(6 \text{ days}) = 180° \text{ C days}$$

From Fig. 3-8 the compressive strength will be between 0.60 and 0.70 times the 28-day strength under standard curing conditions.

3-4 Use Fig. 3-12a to estimate the compressive strength σ_2 for biaxially loaded concrete subject to: (a) $\sigma_1 = 0$. (b) $\sigma_1 = 0.50$ times the tensile strength, in tension. (c) $\sigma_1 = 0.75$ times the compressive strength, in compression.

(a) $\sigma_2 = f_c'$ (b) $\sigma_2 = 0.75\, f_t'$ (c) $\sigma_2 = 1.4\, f_c'$

3-5 The concrete in the core of a spiral is subjected to a uniform confining stress σ_3 of 680 psi. What will the compressive strength, σ_1 be? The unconfined uniaxial compressive strength is 5000 psi?

$$\sigma_1 = f_c' + 4.1\sigma_3 = 5000 \text{ psi} + 4.1 \times 680 \text{ psi} = 7790 \text{ psi}$$

3-6 What factors affect the shrinkage of concrete?

(a) Relative humidity. Shrinkage increases as the relative humidity decreases, reaching a maximum at RH $\leq$ 40%.

(b) The fraction of the total volume made up of paste. As this fraction increases, shrinkage increases.

(c) The modulus of elasticity of the aggregate. As *E* increases, shrinkage decreases.

(d) The water/cement ratio. As the water content increases, the aggregate fraction decreases, causing an increase in shrinkage.

(e) The fineness of the cement. Shrinkage increases for finely ground cement that has more surface area to attract and absorb water.

(f) The effective thickness or volume to surface ratio. As this ratio increases, the shrinkage occurs more slowly, and the total shrinkage is likely reduced.

(g) Exposure to carbon dioxide tends to increase shrinkage.

3-7 What factors affect the creep of concrete?

(a) The ratio of sustained stress to the strength of the concrete. The creep coefficient, □, is roughly constant up to a stress of $0.5\, f_c'$, but increases above that value.

(b) The humidity of the environment. The amount of creep decreases as the RH increases above 40%.

(c) As the effective thickness or volume to surface ratio increases, the rate at which creep develops decreases.

(d) Concretes with a high paste content creep more that concretes with a large aggregate fraction because only the paste creeps.

3-8 A structure is made from concrete containing Type I cement. The average ambient relative humidity is 70 percent. The concrete was moist-cured for 7 days. f_c' = 4000 psi.

(a) Compute the unrestrained shrinkage strain of a rectangular beam with cross-sectional dimensions 8 in. × 20 in. at 2 years after the concrete was placed.

(b) Compute the axial shortening of a 20 in. × 20 in. × 12 ft plain concrete column at age 3 years. A compression load of 400 kips was applied to the column at age 28 days.

(a)
Compute the humidity modification factor.

$$\gamma_{rh} = 1.40 - 0.01 \times RH = 1.40 - 0.01 \times 70\% = 0.70$$

Compute the volume/surface area ratio modification factor.

$$\text{Volume per foot of beam} = 12\text{ in.} \times 8\text{ in.} \times 20\text{ in.} = 1920\text{ in.}^3$$

$$\text{Surface area per foot of beam} = 2 \times [(12\text{ in.} \times 8\text{ in.}) + (12\text{ in.} \times 20\text{ in.})] = 672\text{ in.}^2$$

$$\gamma_{vs} = 1.2^{-0.12V/S} = 1.2^{-0.12 \times 1920\text{ in.}^3/672\text{ in.}^2} \cong 0.94$$

Compute the ultimate shrinkage strain:

$$(\varepsilon_{sh})_u = \gamma_{rh} \times \gamma_{vs} \times 780 \times 10^{-6} = 0.70 \times 0.94 \times 780 \times 10^{-6} = 513 \times 10^{-6}$$

Compute the shrinkage strain after 2 years:

$$t = 2\text{ yr} \times \frac{365\text{ days}}{\text{yr}} - 7\text{ days} = 723\text{ days}$$

$$(\varepsilon_{sh})_t = \frac{t}{35\text{ days} + t}(\varepsilon_{sh})_u = \frac{723\text{ days}}{35\text{ days} + 723\text{ days}} 513 \times 10^{-6} \cong 490 \times 10^{-6}$$

(b)
Compute the ultimate shrinkage strain coefficient, C_u.

$$\lambda_{rh} = 1.27 - 0.0067 \times RH = 1.27 - 0.0067 \times 70\% = 0.80$$

$$\lambda_{to} = 1.25 \times t_o^{-0.118} = 1.25 \times (28\text{ days})^{-0.118} = 0.84$$

$$\lambda_{vs} = 0.67(1 + 1.13^{-0.54V/S})$$

$$\text{Where } V = 20\text{ in.} \times 20\text{ in.} \times 12\text{ ft} \times 12\text{ in./ft} = 57{,}600\text{ in.}^3$$

$$S = 4\text{ sides} \times 20\text{ in.} \times 12\text{ ft} \times 12\text{ in./ft} = 11{,}500\text{ in.}^2$$

$$\lambda_{vs} = 0.67(1 + 1.13^{-0.54 \times 57{,}600\text{ in.}^3/11{,}500\text{ in.}^2}) = 1.15$$

$$C_u = 2.35 \times \lambda_{rh} \times \lambda_{to} \times \lambda_{vs} = 2.35 \times 0.80 \times 0.84 \times 1.15 = 1.82$$

Compute the creep coefficient for the time since loading, C_t.

$$t = 3\text{ yr} \times \frac{365\text{ days}}{\text{yr}} - 28\text{ days} = 1067\text{ days}$$

$$C_t = \frac{t^{0.6}}{10\text{ days} + t^{0.6}} C_u = \frac{(1067\text{ days})^{0.6}}{10\text{ days} + (1067\text{ days})^{0.6}} \times 1.82 \cong 1.58$$

Compute the total stress-dependent strain, ε_c (total).

First, calculate the creep strain since the load was applied:

$$f_{cm} = 1.2 f_c' = 1.2 \times 4000\text{ psi} = 4800\text{ psi}$$

$$E_c(28\text{ days}) = 57{,}000\sqrt{4800\text{ psi}} = 3.95 \times 10^6\text{ psi}$$

$$\varepsilon_{cc}(t,t_o) = \frac{\sigma_c(t_o)}{E_c(28\text{ days})} \times C_t = \frac{400{,}000\text{ lb}/(20\text{ in.} \times 20\text{ in.})}{3.95 \times 10^6\text{ psi}} \times 1.58 = 0.4 \times 10^{-3}$$

Then, calculate the initial strain when the load is applied:

$$f_c'(t_o) = f_c'(28\text{ days}) = 4000\text{ psi}$$

$$f_{cm}(t_o) = 1.2 f_c'(t_o) = 1.2 \times 4000\text{ psi} = 4800\text{ psi}$$

$$E_c(t_o) = 57{,}000\sqrt{f_{cm}(t_o)} = 57{,}000\sqrt{4800\text{ psi}} = 3.95 \times 10^6\text{ psi}$$

$$\varepsilon_c(t_o) = \frac{\sigma_c(t_o)}{E_c(t_o)} = \frac{400{,}000\text{ lb}/(20\text{ in.} \times 20\text{ in.})}{3.95 \times 10^6\text{ psi}} = 0.253 \times 10^{-3}$$

$$\varepsilon_c(\text{total}) = \varepsilon_c(t_o) + \varepsilon_{cc}(t,t_o) = 0.253 \times 10^{-3} + 0.4 \times 10^{-3} = 0.653 \times 10^{-3}$$

Compute the axial shortening

The column is 12 ft long, so the total expected shortening due to stress-dependent strain is,

$$\Delta\ell = \ell \times \varepsilon_c(\text{total}) = 144\text{ in.} \times 0.653 \times 10^{-3} = 0.094\text{ in.}$$

4-1 Figure P4-1 shows a simply supported beam and the cross-section at midspan. The beam supports a uniform dead load consisting of its own weight plus 1.5 kips/ft and a uniform live load of 1.2 kip/ft. The concrete strength is 4500 psi, and the yield strength of the reinforcement is 60,000 psi. The concrete is normal-weight concrete. For the midspan section shown in Fig. P4-1b, compute ϕM_n and show that it exceeds M_u.

Calculate the dead load of the beam.
h = 24 in., b = 12 in., normal-weight concrete: 150 lb/ft^3

$$w_b = (12\text{ in.} \times 24\text{ in.}) \times (\text{ft}/12\text{ in.})^2 \times 150\text{ lb/ft}^3 \times \text{kip}/1000\text{ lb} = 0.3\text{ kip/ft}$$

Compute the factored moment, M_u:

$$w_u = 1.4D = 1.4(0.3\text{ kip/ft} + 1.5\text{ kip/ft}) = 2.52\text{ kip/ft}$$

$$w_u = 1.2D + 1.6L = 1.2(0.3\text{ kip/ft} + 1.5\text{ kip/ft}) + 1.6(1.2\text{ kip/ft}) = 4.08\text{ kip/ft (governs)}$$

$$M_u = \frac{w_u \ell^2}{8} = \frac{4.08\text{ kip/ft} \times (20\text{ ft})^2}{8} = 204\text{ kip-ft}$$

Compute the nominal moment capacity of the beam, M_n, and the strength reduction factor, ϕ. Tension steel area: three No. 8 bars

$$A_s = 3 \times 0.79\text{ in.}^2 = 2.37\text{ in.}^2$$

Compute the depth of the equivalent rectangular stress block, a, assuming that tension steel is yielding. From equilibrium:

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{2.37\text{ in.}^2 \times 60\text{ ksi}}{0.85 \times 4.5\text{ ksi} \times 12\text{ in.}} = 3.10\text{ in.}$$

$$\beta_1 = 0.85 - 0.05\frac{f'_c - 4000\text{ psi}}{1000\text{ psi}} = 0.85 - 0.05\frac{4500\text{ psi} - 4000\text{ psi}}{1000\text{ psi}} = 0.825$$

$$c = \frac{a}{\beta_1} = \frac{3.10\text{ in.}}{0.825} = 3.76\text{ in.}$$

Check whether tension steel is yielding:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60\text{ ksi}}{29{,}000\text{ ksi}} = 0.00207$$

$$\varepsilon_s = \varepsilon_t = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{21.5\text{ in.} - 3.76\text{ in.}}{3.76\text{ in.}}\right)0.003 = 0.014 \geq f_y = 0.00207\text{ (o.k.)}$$

Compute the moment strength:

$$M_n = A_s f_y (d - a/2) = 2.37\text{ in.}^2 \times 60\text{ ksi}(21.5\text{ in.} - 3.10\text{ in.}/2) \times (\text{ft}/12\text{ in.}) = 236\text{ kip-ft}$$

$$\varepsilon_t = 0.014 > 0.005 \therefore \phi = 0.9$$

$$\phi M_n = 0.9 \times 236\text{ kip-ft} = 213\text{ kip-ft} \geq M_u = 204\text{ kip-ft (o.k.)}$$

4-2 A cantilever beam shown in Fig. P4-2 supports a uniform dead load of 1 kip/ft plus its own weight and it supports a concentrated live load of 12 kips, as shown. The concrete is normal-weight concrete with f_c' = 4000 psi and the steel is Grade 60. For the section at the fixed support shown in Fig. P4-2b, compute ϕM_n and show that it exceeds M_u.

Calculate the dead load of the beam.
h = 18 in, b = 30 in, normal-weight concrete: 150 lb/ft^3

$$w_b = (18\text{ in.} \times 30\text{ in.}) \times (\text{ft/12 in.})^2 \times 150\text{ lb/ft}^3 \times \text{kip/1000 lb} = 0.563\text{ kip/ft}$$

Compute the factored moment, M_u:

Factored distributed dead load:

$$w_D = 1.2\left(0.563\text{ kip/ft} + 1\text{ kip/ft}\right) = 1.88\text{ kip/ft}$$

Factored concentrated live load:

$$P_L = 1.6 \times 12\text{ kip} = 19.2\text{ kip}$$

$$M_u = -\left(\frac{w_D \ell^2}{2}\right) - P_L(\ell - 1\text{ ft}) = -\left(\frac{1.88\text{ kip/ft} \times (10\text{ ft})^2}{2}\right) - 19.2\text{ kip}(10\text{ ft} - 1\text{ ft}) = -267\text{ kip-ft}$$

Compute the nominal moment capacity of the beam, M_n, and the strength reduction factor, ϕ. Tension steel area: six No. 8 bars

$$A_s = 6 \times 0.79\text{ in.}^2 = 4.74\text{ in.}^2$$

Compute the depth of the equivalent rectangular stress block, a, assuming that tension steel is yielding. From equilibrium:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{4.74\text{ in.}^2 \times 60\text{ ksi}}{0.85 \times 4\text{ ksi} \times 30\text{ in.}} = 2.79\text{ in.}$$

$$\beta_1 = 0.85$$

$$c = \frac{a}{\beta_1} = \frac{2.79\text{ in.}}{0.85} = 3.28\text{ in.}$$

Check whether tension steel is yielding:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60\text{ ksi}}{29{,}000\text{ ksi}} = 0.00207$$

$$\varepsilon_s = \varepsilon_t = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{15.5\text{ in.} - 3.28\text{ in.}}{3.28\text{ in.}}\right)0.003 = 0.011 \geq f_y = 0.00207\text{ (o.k.)}$$

Compute the moment strength:

$$M_n = A_s f_y (d - a/2) = 4.74\text{ in.}^2 \times 60\text{ ksi}(15.5\text{ in.} - 3.28\text{ in.}/2) \times (\text{ft/12 in.}) = 328\text{ kip-ft}$$

$$\varepsilon_t = 0.011 > 0.005 \therefore \phi = 0.9$$

$$\phi M_n = 0.9 \times 328\text{ kip-ft} = 295\text{ kip-ft} \geq M_u = 267\text{ kip-ft (o.k.)}$$

4-3 (a) Compare ϕM_n for singly reinforced rectangular beams having the following properties.

(b) Taking beam 1 as the reference point, discuss the effects of changing A_s, f_y, f_c', d, and b on ϕM_n. (Note that each beam has the same properties as beam 1 except for the italicized quantity.)

(c) What is the most effective way of increasing ϕM_n? What is the least effective way?

(a)

Beam No.	*b* (in.)	*d* (in.)	Bars	f_c' (psi)	f_y (psi)
1	12	22	3 No. 7	4,000	60,000
2	12	22	*2 No. 9 plus 1 No. 8*	4,000	60,000
3	12	22	3 No. 7	4,000	*80,000*
4	12	22	3 No. 7	*6,000*	60,000
5	12	*33*	3 No. 7	4,000	60,000
6	*18*	22	3 No. 7	4,000	60,000

Beam No. 1

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3 \times 0.60 \text{ in.}^2 \times 60 \text{ ksi}}{0.85 \times 4 \text{ ksi} \times 12 \text{ in.}} = 2.65 \text{ in.}$$

$$\beta_1 = 0.85$$

$$c = \frac{a}{\beta_1} = \frac{2.65 \text{ in.}}{0.85} = 3.12 \text{ in.}$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60 \text{ ksi}}{29{,}000 \text{ ksi}} = 0.00207$$

$$\varepsilon_s = \varepsilon_t = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{22 \text{ in.} - 3.12 \text{ in.}}{3.12 \text{ in.}}\right) 0.003 = 0.018 \geq \varepsilon_y = 0.00207 \text{ (o.k.)}$$

$$\varepsilon_t = 0.018 > 0.005 \therefore \phi = 0.9$$

$$\phi M_n = A_s f_y (d - a/2) = 0.9 \times 3 \times 0.60 \text{ in.}^2 \times 60 \text{ ksi}(22 \text{ in.} - 2.65 \text{ in.}/2) \times (\text{ft}/12 \text{ in.}) = 167 \text{ kip-ft}$$

Beam No. 2

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(2 \times 1\text{ in.}^2 + 0.79\text{ in.}^2) \times 60\text{ ksi}}{0.85 \times 4\text{ ksi} \times 12\text{ in.}} = 4.10\text{ in.}$$

$$\beta_1 = 0.85$$

$$c = \frac{a}{\beta_1} = \frac{4.10\text{ in.}}{0.85} = 4.82\text{ in.}$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60\text{ ksi}}{29{,}000\text{ ksi}} = 0.00207$$

$$\varepsilon_s = \varepsilon_t = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{22\text{ in.} - 4.82\text{ in.}}{4.82\text{ in.}}\right)0.003 = 0.0107 \geq f_y = 0.00207\text{ (o.k.)}$$

$$\varepsilon_t = 0.0107 > 0.005 \therefore \phi = 0.9$$

$$\begin{aligned}\phi M_n &= A_s f_y (d - a/2) \\ &= 0.9 \times (2 \times 1\text{ in.}^2 + 0.79\text{ in.}^2) \times 60\text{ ksi}(22\text{ in.} - 4.10\text{ in.}/2) \times (\text{ft/12 in.}) \\ &= 250\text{ kip-ft}\end{aligned}$$

Beam No. 3

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3 \times 0.60\text{ in.}^2 \times 80\text{ ksi}}{0.85 \times 4\text{ ksi} \times 12\text{ in.}} = 3.53\text{ in.}$$

$$\beta_1 = 0.85$$

$$c = \frac{a}{\beta_1} = \frac{3.53\text{ in.}}{0.85} = 4.15\text{ in.}$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{80\text{ ksi}}{29{,}000\text{ ksi}} = 0.00276$$

$$\varepsilon_s = \varepsilon_t = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{22\text{ in.} - 4.15\text{ in.}}{4.15\text{ in.}}\right)0.003 = 0.0129 \geq f_y = 0.00276\text{ (o.k.)}$$

$$\varepsilon_t = 0.0129 > 0.005 \therefore \phi = 0.9$$

$$\phi M_n = A_s f_y (d - a/2) = 0.9 \times 3 \times 0.60\text{ in.}^2 \times 80\text{ ksi}(22\text{ in.} - 3.53\text{ in.}/2) \times (\text{ft/12 in.}) = 219\text{ kip-ft}$$

Beam No. 4

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3 \times 0.60 \text{ in.}^2 \times 60 \text{ ksi}}{0.85 \times 6 \text{ ksi} \times 12 \text{ in.}} = 1.76 \text{ in.}$$

$$\beta_1 = 0.85 - 0.05 \frac{f_c' - 4000 \text{ psi}}{1000 \text{ psi}} = 0.85 - 0.05 \frac{6000 \text{ psi} - 4000 \text{ psi}}{1000 \text{ psi}} = 0.75$$

$$c = \frac{a}{\beta_1} = \frac{1.76 \text{ in.}}{0.75} = 2.35 \text{ in.}$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60 \text{ ksi}}{29{,}000 \text{ ksi}} = 0.00207$$

$$\varepsilon_s = \varepsilon_t = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{22 \text{ in.} - 2.35 \text{ in.}}{2.35 \text{ in.}}\right)0.003 = 0.0251 \geq f_y = 0.00207 \text{ (o.k.)}$$

$$\varepsilon_t = 0.0251 > 0.005 \therefore \phi = 0.9$$

$$\phi M_n = A_s f_y (d - a/2) = 0.9 \times 3 \times 0.60 \text{ in.}^2 \times 60 \text{ ksi}(22 \text{ in.} - 1.76 \text{ in.}/2) \times (\text{ft}/12 \text{ in.}) = 171 \text{ kip-ft}$$

Beam No. 5

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3 \times 0.60 \text{ in.}^2 \times 60 \text{ ksi}}{0.85 \times 4 \text{ ksi} \times 12 \text{ in.}} = 2.65 \text{ in.}$$

$$\beta_1 = 0.85$$

$$c = \frac{a}{\beta_1} = \frac{2.65 \text{ in.}}{0.85} = 3.12 \text{ in.}$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60 \text{ ksi}}{29{,}000 \text{ ksi}} = 0.00207$$

$$\varepsilon_s = \varepsilon_t = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{33 \text{ in.} - 3.12 \text{ in.}}{3.12 \text{ in.}}\right)0.003 = 0.0287 \geq f_y = 0.00207 \text{ (o.k.)}$$

$$\varepsilon_t = 0.0287 > 0.005 \therefore \phi = 0.9$$

$$\phi M_n = A_s f_y (d - a/2) = 0.9 \times 3 \times 0.60 \text{ in.}^2 \times 60 \text{ ksi}(33 \text{ in.} - 2.65 \text{ in.}/2) \times (\text{ft}/12 \text{ in.}) = 257 \text{ kip-ft}$$

Beam No. 6

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{3 \times 0.60 \text{ in.}^2 \times 60 \text{ ksi}}{0.85 \times 4 \text{ ksi} \times 18 \text{ in.}} = 1.76 \text{ in.}$$

$$\beta_1 = 0.85$$

$$c = \frac{a}{\beta_1} = \frac{1.76 \text{ in.}}{0.85} = 2.07 \text{ in.}$$

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60 \text{ ksi}}{29{,}000 \text{ ksi}} = 0.00207$$

$$\varepsilon_s = \varepsilon_t = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{22 \text{ in.} - 2.07 \text{ in.}}{2.07 \text{ in.}}\right) 0.003 = 0.0289 \geq f_y = 0.00207 \text{ (o.k.)}$$

$$\varepsilon_t = 0.0289 > 0.005 \therefore \ \phi = 0.9$$

$$\phi M_n = A_s f_y (d - a/2) = 0.9 \times 3 \times 0.60 \text{ in.}^2 \times 60 \text{ ksi}(22 \text{ in.} - 1.76 \text{ in.}/2) \times (\text{ft}/12 \text{ in.}) = 171 \text{ kip-ft}$$

(b)

Beam No.	ϕM_n (kip-ft)
1	167
2	250
3	219
4	171
5	257
6	171

Effect of A_s (Beams 1 and 2)
An increase of 55% in A_s (from 1.80 in.^{2} to 2.79 in.^{2}) caused an increase of 50% in ϕM_n. Increasing the tension steel area causes a proportional increase in the strength of the section, with a loss of ductility. Note that in this case, the strength reduction factor was 0.9 for both sections.

Effect of f_y (Beams 1 and 3)
An increase of 33% in f_y caused an increase of 31% in ϕM_n. Increasing the steel yield strength has essentially the same effect as increasing the tension steel area.

Effect of f_c' (Beams 1 and 4)
An increase of 50% in f_c' caused an increase of 2% in ϕM_n. Changing the concrete strength has approximately no impact on moment strength, but does increase section ductility (increase in ε_t).

Effect of d (Beams 1 and 5)
An increase of 50% in d caused an increase of 54% in ϕM_n. Increasing the effective flexural depth of the section increases the section moment strength (without decreasing the section ductility).

Effect of b (Beams 1 and 6)

An increase of 50% in b caused an increase of 2% in ϕM_n. Changing the beam width has approximately no impact on moment strength, but does increase section ductility (increase in ε_t).

(c) Disregarding any other effects of increasing b, d, A_s, f_y, or f_c' such as changes in cost, etc., the most effective way to increase ϕM_n is to increase the effective flexural depth of the section, d, followed by increasing f_y and A_s. Note that increasing f_y and A_s too much will result in a decrease in ductility. The least effective ways of increasing ϕM_n is to increase f_c' and b. Note that increasing f_c' or b will cause a significant increase in ductility at failure.

4-4 A 12-ft-long cantilever supports its own weight plus an additional uniform dead load of 0.5 kip/ft. The beam is made from normal-weight 4000-psi concrete and has *b* = 16 in., *d* = 15.5 in., and *h* = 18 in. It is reinforced with four No. 7 Grade-60 bars. Compute the maximum concentrated live load that can be applied at 1 ft from the free end of the cantilever.

Compute the nominal moment capacity of the beam, M_n, and the strength reduction factor, ϕ. Tension steel area: four No. 7 bars

$$A_s = 4 \times 0.60\text{ in.}^2 = 2.4\text{ in.}^2$$

Compute the depth of the equivalent rectangular stress block, a, assuming that tension steel is yielding. From equilibrium:

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{2.40\text{ in.}^2 \times 60\text{ ksi}}{0.85 \times 4\text{ ksi} \times 16\text{ in.}} = 2.65\text{ in.}$$

$$\beta_1 = 0.85$$

$$c = \frac{a}{\beta_1} = \frac{2.65\text{ in.}}{0.85} = 3.12\text{ in.}$$

Check whether tension steel is yielding:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{60\text{ ksi}}{29{,}000\text{ ksi}} = 0.00207$$

$$\varepsilon_s = \varepsilon_t = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{15.5\text{ in.} - 3.12\text{ in.}}{3.12\text{ in.}}\right)0.003 = 0.0119 \geq \varepsilon_y = 0.00207 \text{ (o.k.)}$$

Compute the moment strength:

$$M_n = A_s f_y (d - a/2) = 2.40\text{ in.}^2 \times 60\text{ ksi}(15.5\text{ in.} - 2.65\text{ in.}/2) \times (\text{ft}/12\text{ in.}) = 170\text{ kip-ft}$$

$$\varepsilon_t = 0.0119 > 0.05 \therefore \phi = 0.9$$

$$\phi M_n = 0.9 \times 170\text{ kip-ft} = 153\text{ kip-ft}$$

Compute Live Load

$$M_u = \phi M_n = 153\text{ kip-ft}$$

Beam self-weight:

h = 18 in, b = 16 in, normal-weight concrete: 150 lb/ft^3

$$w_b = (18\text{ in.} \times 16\text{ in.}) \times (\text{ft/12 in.})^2 \times 150\text{ lb/ft}^3 \times \text{kip/1000 lb} = 0.3\text{ kip/ft}$$

$$w_D = 1.2(0.3\text{ kip/ft} + 0.5\text{ kip/ft}) = 0.96\text{ kip/ft}$$

$$M_D = \frac{w_D \ell^2}{2} = \frac{0.96\text{ kip/ft} \times (12\text{ ft})^2}{2} = 69.1\text{ kip-ft}$$

Therefore the maximum factored live load moment is:

$$\phi M_n - M_D = 153\text{ kip-ft} - 69.1\text{ kip-ft} = 83.9\text{ kip-ft}$$

Maximum factored concentrated live load at 1 ft from the tip:

$$M_L = \frac{83.9\text{ kip-ft}}{11\text{ ft}} = 7.63\text{ kip}$$

Maximum concentrated live load:

$$P_L = \frac{7.63\text{ kip}}{1.6} = 4.77\text{ kip}$$

4-5 Compute ϕM_n and check $A_{s,min}$ for the beam shown in Fig. P4-5. Use $f_c' = 5000$ psi and $f_y = 60{,}000$ psi. Assume effective depth, d, is 19 in for the beam in Fig P4-5.

Compute the nominal moment capacity of the beam, M_n, and the strength reduction factor, ϕ. Tension steel area: six No. 8 bars

$$A_s = 6 \times 0.79\text{ in.}^2 = 4.74\text{ in.}^2$$

The tension reinforcement for this section is provided in two layers, where the distance from the tension edge to the centroid of the total tension reinforcement is given as d = 19 in. Assuming that the depth of the Whitney stress block is less than or equal to the thickness of the compression flange, $a \le h_f$ and that the tension steel is yielding:

$$a = \frac{A_s f_y}{0.85 f_c' b_e} = \frac{4.74\text{ in.}^2 \times 60\text{ ksi}}{0.85 \times 5\text{ ksi} \times 48\text{ in.}} = 1.39\text{ in.} \le h_f = 6\text{ in. (o.k.)}$$

$$\beta_1 = 0.85 - 0.05\frac{f_c' - 4000\text{ psi}}{1000\text{ psi}} = 0.85 - 0.05\frac{5000\text{ psi} - 4000\text{ psi}}{1000\text{ psi}} = 0.80$$

$$c = \frac{a}{\beta_1} = \frac{1.39\text{ in.}}{0.80} = 1.74\text{ in.}$$

$$\varepsilon_s = \left(\frac{d - c}{c}\right)\varepsilon_{cu} = \left(\frac{19\text{ in.} - 1.74\text{ in.}}{1.74\text{ in.}}\right)0.003 = 0.0298 \ge 0.00207\text{ (o.k.)}$$

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right)\varepsilon_{cu} = \left(\frac{20\text{ in.} - 1.74\text{ in.}}{1.74\text{ in.}}\right)0.003 = 0.0315 \ge 0.00207\text{ (o.k.)}$$

$$\varepsilon_t = 0.0315 > 0.005 \therefore \phi = 0.9$$

Calculate M_n:

$$M_n = A_s f_y (d - a/2) = 4.74\text{ in.}^2 \times 60\text{ ksi}(19\text{ in.} - 1.39\text{ in.}/2) \times (\text{ft/12 in.}) = 434\text{ kip-ft}$$

$$\phi M_n = 0.9 \times 434\text{ kip-ft} = 391\text{ kip-ft}$$

Check of $A_{s,min}$: The section is subjected to positive bending and tension is at the bottom of this section, so we should use b_w:

$$A_{s,\min} \geq \begin{cases} \dfrac{3\sqrt{f_c'}}{f_y} b_w d = \dfrac{3\sqrt{5000 \text{ psi}}}{60{,}000 \text{ psi}} \times 12 \text{ in.} \times 19 \text{ in.} = 0.81 \text{ in.}^2 \text{(governs)} \\ \dfrac{200 \text{ psi}}{f_y} b_w d = \dfrac{200 \text{ psi}}{60{,}000 \text{ psi}} \times 12 \text{ in.} \times 19 \text{ in.} = 0.76 \text{ in.}^2 \end{cases}$$

$$A_s = 4.74 \text{ in.}^2 \geq A_{s,\min} \text{ (o.k.)}$$

4-6 Compute ϕM_n and check $A_{s,min}$ for the beam shown in Fig. P4-6. Use f_c' = 5000 psi and f_y = 60,000 psi. Assume effective depth, *d*, is 18.5 in for the beam in Fig P4-6.

Compute the nominal moment capacity of the beam, M_n, and the strength reduction factor, ϕ. Tension steel area: six No. 8 bars

$$A_s = 6 \times 0.79 \text{ in.}^2 = 4.74 \text{ in.}^2$$

The tension reinforcement for this section is provided in two layers, where the distance from the tension edge to the centroid of the total tension reinforcement is given as d = 18.5 in. Assuming that the depth of the Whitney stress block is less than or equal to the thickness of the compression flange, $a \leq h_f$ and that the tension steel is yielding:

$$a = \frac{A_s f_y}{0.85 f_c' b_e} = \frac{4.74 \text{ in.}^2 \times 60 \text{ ksi}}{0.85 \times 5 \text{ ksi} \times 20 \text{ in.}} = 3.35 \text{ in.} \leq h_f = 5 \text{ in. (o.k.)}$$

$$\beta_1 = 0.85 - 0.05 \frac{f_c' - 4000 \text{ psi}}{1000 \text{ psi}} = 0.85 - 0.05 \frac{5000 \text{ psi} - 4000 \text{ psi}}{1000 \text{ psi}} = 0.80$$

$$c = \frac{a}{\beta_1} = \frac{3.35 \text{ in.}}{0.80} = 4.19 \text{ in.}$$

$$\varepsilon_s = \left(\frac{d - c}{c}\right) \varepsilon_{cu} = \left(\frac{18.5 \text{ in.} - 4.19 \text{ in.}}{4.19 \text{ in.}}\right) 0.003 = 0.0102 \geq 0.00207 \text{ (o.k.)}$$

$$\varepsilon_t = \left(\frac{d_t - c}{c}\right) \varepsilon_{cu} = \left(\frac{19.5 \text{ in.} - 4.19 \text{ in.}}{4.19 \text{ in.}}\right) 0.003 = 0.011 \geq 0.00207 \text{ (o.k.)}$$

$$\varepsilon_t = 0.011 > 0.005 \therefore \phi = 0.9$$

Calculate M_n:

$$M_n = A_s f_y (d - a/2) = 4.74 \text{ in.}^2 \times 60 \text{ ksi} (18.5 \text{ in.} - 3.35 \text{ in.}/2) \times (\text{ft}/12 \text{ in.}) = 399 \text{ kip-ft}$$

$$\phi M_n = 0.9 \times 399 \text{ kip-ft} = 359 \text{ kip-ft}$$

Check of $A_{s,min}$: The section is subjected to positive bending and tension is at the bottom of this section, so we should use b_w:

$$A_{s,\min} \geq \begin{cases} \dfrac{3\sqrt{f_c'}}{f_y} b_w d = \dfrac{3\sqrt{5000 \text{ psi}}}{60{,}000 \text{ psi}} \times 10 \text{ in.} \times 18.5 \text{ in.} = 0.65 \text{ in.}^2 (\text{governs}) \\ \dfrac{200 \text{ psi}}{f_y} b_w d = \dfrac{200 \text{ psi}}{60{,}000 \text{ psi}} \times 10 \text{ in.} \times 18.5 \text{ in.} = 0.62 \text{ in.}^2 \end{cases}$$

$$A_s = 4.74 \text{ in.}^2 \geq A_{s,\min} \text{ (o.k.)}$$

4-7 Compute the negative-moment capacity, ϕM_n, and check $A_{s,min}$ for the beam shown in Fig. P4-7. Use f_c' = 4500 psi and f_y = 60,000 psi.

Compute the nominal moment capacity of the beam, M_n, and the strength reduction factor, ϕ. This section is subjected to negative bending, so tension will develop in the top flange and the compression zone is at the bottom of the section. ACI Code requires that a portion of the tension reinforcement be distributed in the flange, so assuming that the No. 7 bars in the flange are part of the tension reinforcement: six No. 7 bars

$$A_s = 6 \times 0.60 \text{ in.}^2 = 3.60 \text{ in.}^2$$

$$a = \frac{A_s f_y}{0.85 f_c' b_e} = \frac{3.60 \text{ in.}^2 \times 60 \text{ ksi}}{0.85 \times 4.5 \text{ ksi} \times 12 \text{ in.}} = 4.71 \text{ in.}$$

$$\beta_1 = 0.85 - 0.05 \frac{f_c' - 4000 \text{ psi}}{1000 \text{ psi}} = 0.85 - 0.05 \frac{4500 \text{ psi} - 4000 \text{ psi}}{1000 \text{ psi}} = 0.825$$

$$c = \frac{a}{\beta_1} = \frac{4.71 \text{ in.}}{0.825} = 5.71 \text{ in.}$$

$$\varepsilon_s = \varepsilon_t = \left(\frac{d - c}{c}\right) \varepsilon_{cu} = \left(\frac{19.5 \text{ in.} - 5.71 \text{ in.}}{5.71 \text{ in.}}\right) 0.003 = 0.0072 \geq 0.00207 \text{ (o.k.)}$$

$$\varepsilon_t = 0.0072 > 0.005 \therefore \phi = 0.9$$

Calculate M_n:

$$M_n = A_s f_y (d - a/2) = 3.60 \text{ in.}^2 \times 60 \text{ ksi}(19.5 \text{ in.} - 4.71 \text{ in.}/2) \times (\text{ft}/12 \text{ in.}) = 309 \text{ kip-ft}$$

$$\phi M_n = 0.9 \times 309 \text{ kip-ft} = 278 \text{ kip-ft}$$

Check of $A_{s,min}$:

The flanged portion of the beam section is in tension because the beam is subjected to negative bending. Therefore, the value of $A_{s,min}$ will depend on whether the beam is statically determinate. Assuming that the beam is part of a continuous, statically indeterminate floor system, the minimum tension reinforcement should be calculated using b_w:

$$A_{s,\min} \ge \begin{cases} \dfrac{3\sqrt{f_c'}}{f_y} b_w d = \dfrac{3\sqrt{4500 \text{ psi}}}{60{,}000 \text{ psi}} \times 12 \text{ in.} \times 19.5 \text{ in.} = 0.78 \text{ in.}^2 \text{(governs)} \\ \dfrac{200 \text{ psi}}{f_y} b_w d = \dfrac{200 \text{ psi}}{60{,}000 \text{ psi}} \times 12 \text{ in.} \times 19.5 \text{ in.} = 0.78 \text{ in.}^2 \end{cases}$$

$$A_s = 3.60 \text{ in.}^2 \ge A_{s,\min} \text{ (o.k.)}$$

However, for a statically determinate beam:

$$b \le \begin{cases} 2b_w = 2 \times 12 \text{ in.} = 24 \text{ in. (governs)} \\ b_e = 48 \text{ in.} \end{cases}$$

$$A_{s,\min} \ge \begin{cases} \dfrac{3\sqrt{f_c'}}{f_y} bd = \dfrac{3\sqrt{4500 \text{ psi}}}{60{,}000 \text{ psi}} \times 24 \text{ in.} \times 19.5 \text{ in.} = 1.57 \text{ in.}^2 \text{(governs)} \\ \dfrac{200 \text{ psi}}{f_y} bd = \dfrac{200 \text{ psi}}{60{,}000 \text{ psi}} \times 24 \text{ in.} \times 19.5 \text{ in.} = 1.56 \text{ in.}^2 \end{cases}$$

$$A_s = 3.60 \text{ in.}^2 \ge A_{s,\min} \text{ (o.k.)}$$

4-8 For the beam shown in Fig. P4-8, f_c' = 4000 psi and f_y = 60,000 psi.

(a) Compute the effective flange width at midspan.

(b) Compute ϕM_n for the positive- and negative-moment regions and check $A_{s,min}$ for both sections. At the supports, the bottom bars are in one layer; at midspan, the No. 8 bars are in the bottom, the No. 7 bars in a second layer.

(a) Determine the effective compression flange, b_e, for a flanged section that is part of a continuous floor system are:

$$b_e \le \begin{cases} b_w + 2\left(\dfrac{\ell_n}{8}\right) \\ b_w + 2(8h_f) \\ b_w + 2\left(\dfrac{s_w}{2}\right) \end{cases}$$

Because the columns are 18 in. wide, the longitudinal clear span is:

$$\ell_n = 22 \text{ ft} - 2\left(\frac{18 \text{ in.} \times \text{ft}/12 \text{ in.}}{2}\right) = 20.5 \text{ ft} \times 12 \text{ in./ft} = 246 \text{ in.}$$

The clear transverse distance for the 9 ft 6 in. span is:

$$s_w = 9.5 \text{ ft} \times 12 \text{ in./ft} - 2\left(\frac{12 \text{ in.}}{2}\right) = 102 \text{ in.}$$

and for the 11 ft span is: