Chapter 1

1.1

The total surface area for a cylinder is

$$A = \frac{\pi d^2}{4}(2) + \pi d \ell$$

while the volume is

$$V = \frac{\pi d^2}{4}\ell$$

The ratio of surface area to volume is then

$$\frac{A}{V} = \frac{4}{\pi d^2 \ell}\left(\frac{\pi d^2}{2} + \pi d \ell\right) = \frac{2}{\ell} + \frac{4}{d}$$

Alternatively, A/V can be expressed in terms of the volume, V, and the cylinder aspect ratio, $a = \ell/d$, as

$$\frac{A}{V} = \frac{2}{d}\left(\frac{1}{a} + 2\right) = \frac{2a^{-1/3}}{d}\left(a^{-2/3} + 2a^{1/3}\right)$$

$$= \frac{2}{d^{2/3}\ell^{1/3}}\left(a^{-2/3} + 2a^{1/3}\right) = \left(\frac{8}{d^2 \ell}\right)^{1/3}\left(a^{-2/3} + 2a^{1/3}\right)$$

$$= \left(\frac{2\pi}{V}\right)^{1/3}\left(a^{-2/3} + 2a^{1/3}\right)$$

Thus, when A/V is plotted as a function of a in units of $(2\pi/V)^{1/3}$, we get the curve shown in Figure 1.3. (This problem taken from Ref. (7).

1.2

Assuming that the particles are perfectly spherical, for a single large particle of diameter d_L, the total surface area is

$$A_L = \pi d_L^2$$

while the total volume is

$$V_L = \frac{\pi d_L^3}{6}$$

For a single small diameter sphere of diameter d_s, the corresponding surface area and volume are

$$A_s = \pi d_s^2 \quad \text{and} \quad V_s = \frac{\pi d_s^3}{6}$$

respectively. For N small diameter spheres having the same volume as a single large diameter sphere,

$$V_s N = V_L$$

so that

$$N = \frac{V_L}{V_s} = \left(\frac{d_L}{d_s}\right)^3$$

The ratio of the total surface area of N small diameter spheres to the surface area of a single large diameter sphere is

$$R = \frac{N A_s}{A_L} = N\left(\frac{d_s}{d_L}\right)^2 = \left(\frac{d_L}{d_s}\right)^3\left(\frac{d_s}{d_L}\right)^2 = \frac{d_L}{d_s}$$

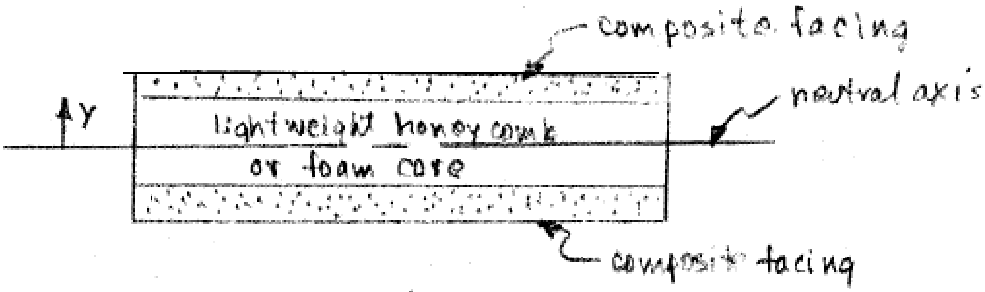
So, for example, if a large sphere is replaced by a group of N spheres having diameters 1000 times smaller, $d_L = 1000\, d_s$ and the total surface area for a constant volume will increase by a factor of 1000.

1.3

The flexural stiffness is defined by the product EI, where E is the effective flexural modulus of the sandwich structure and I is the area moment of inertia of the section about its neutral axis;

$$I = \int_A y^2 \, dA$$

where y is measured from the neutral axis.



Since most of the load-bearing area, A, is located at a large distance, y, and E for the composite facing is high, the flexural stiffness is high. The flexural stiffness-to-weight ratio, EI/w, is also high due to the fact that most of the cross-section is composed of lightweight honeycomb or foam core material. EI/w for such a sandwich structure is much greater than EI/w for a solid section having the same overall dimensions.

1.4

For dimensional stability, a very high stiffness is desired. The P-120S carbon fiber, with a tensile modulus of 120×10^6 psi (827 GPa), is the stiffest fiber listed in Table 1.1. Note that other fibers such as IM-7 carbon or even E-glass have higher tensile strengths than P-120S, but strength is not a critical property for this low stress application.

1.5

(a) Steel:

$$F.S. = 2.0 = \frac{\sigma_f A}{P}$$

$$A = \frac{(F.S.)(P)}{\sigma_f} = \frac{2.0(5000\,N)}{1030\times10^6 \frac{N}{m^2}} = 9.71\times10^{-6}\,m^2$$

$$W_{st} = AL\rho = (9.71\times10^{-6}\,m^2)(5\,m)\left(7.83\,\frac{g}{cm^3}\right)\left(\frac{cm}{10^{-2}\,m}\right)^3$$

$$W_{st} = 380\,g$$

AS-4 carbon:

$$A = \frac{(F.S.)(P)}{\sigma_f} = \frac{(2.0)(5000\,N)}{4475\times10^6 \frac{N}{m^2}} = 2.2346\times10^{-6}\,m^2$$

$$W_{AS\text{-}4} = AL\rho = (2.2346\times10^{-6})(5\,m)\left(1.79\,\frac{g}{cm^3}\right)\left(\frac{cm}{10^{-2}\,m}\right)^3$$

$$W_{AS\text{-}4} = 20\,g$$

$$\therefore \frac{W_{AS\text{-}4}}{W_{st}} = \frac{20}{380} = 0.0526$$

(b) $W = AL\rho = 380g = A(5m)(1.79)(10^6)$

or $A = 4.246\times10^{-5}\,m^2$

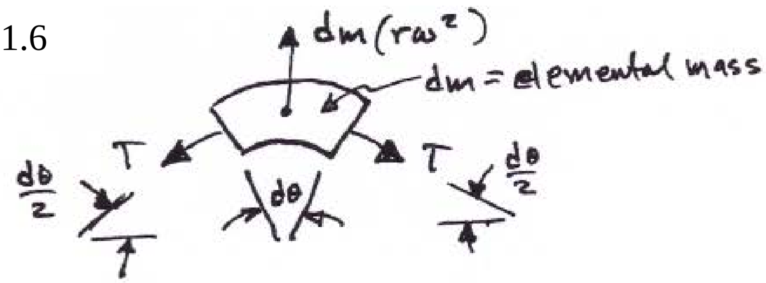
$$P = \frac{\sigma_f A}{F.S.} = \frac{(4475\times10^6 \frac{N}{m^2})(4.246\times10^{-5}\,m^2)}{2.0}$$

$P = 95{,}004\ N = 95.004\ kN$ for AS-4

compared with 5 kN for steel

$$\frac{P_{AS\text{-}4}}{P_{st}} = \frac{95.004}{5} = 19$$

1.6



$$\Sigma F_r = dm(r\omega^2) - 2T\sin\left(\frac{d\theta}{2}\right) = 0$$

for small $d\theta$, $\sin\frac{d\theta}{2} \simeq \frac{\theta}{2}$

and force $T = \sigma_f tL$ = tangential force

also, $dm = \rho r\, d\theta\, tL$

$\therefore \rho r\, d\theta\, tL\, r\omega^2 = \sigma_f\, tL\, d\theta$

and $\sigma_f = \rho(r\omega)^2 = \rho V^2$ = tangential stress

where $V = r\omega$ = tangential velocity

4340 Steel:

$$V_{st} = \sqrt{\frac{\sigma_f}{\rho}} = \sqrt{\frac{1030\times10^6\ N/m^2\,(kg\text{-}m/s^2)}{7.83\times10^3\ kg/m^3\ (N)}}$$

$$V_{st} = 363\ m/s$$

IM-7 carbon:

$$V_{IM\text{-}7} = \sqrt{\frac{\sigma_f}{\rho}} = \sqrt{\frac{5670\times10^6}{1.78\times10^3}}$$

$V_{IM\text{-}7} = 1784.8\ m/s$ so $\frac{V_{IM\text{-}7}}{V_{st}} = \frac{1784.8}{363} = 4.91$

Kinetic energy $K = \frac{1}{2}mV^2$

kinetic energy / unit mass $= \frac{K}{m} = \frac{1}{2}V^2$

$$\therefore \frac{(K/m)_{IM\text{-}7}}{(K/m)_{st}} = \left(\frac{V_{IM\text{-}7}}{V_{st}}\right)^2 = \left(\frac{1784.8}{363}\right)^2 = 24.17$$

1.7 the fiber volume fraction of the RRIM composite is limited by the viscosity of the chopped fiber/resin system and the resulting injection pressure required to produce flow in the mold. Nonuniform distribution of the chopped fibers in the resin results in nonuniform distribution of properties such as strength and modulus.

1.8 Compression of sandwich structures during RTM mold closing may damage the foam or honeycomb core unless the mold closing force is limited. Sealing of the foam or honeycomb core is required to prevent resin infiltration during resin injection

1.9 There are obviously a number of possible fabrication processes and sequences that could be used. For example, the thick interior parts consisting of unidirectional glass/epoxy and hybrid glass/carbon/epoxy could be made by pultrusion, then bonded together with adhesive. the carbon/epoxy wraps could be filament wound over the other interior parts, then the ±45° woven glass/epoxy skins could be formed over the interior parts by autoclaving prepreg tape layups, or by compression molding. the titanium erosion shield and the electrical heater mat could be adhesively bonded to the remaining assembly.

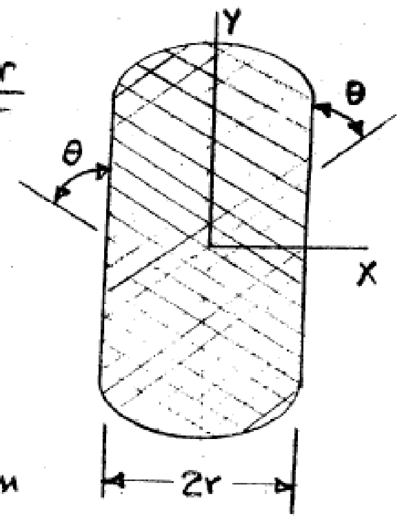
1.10

From elementary mechanics of materials, the static equilibrium of the pressure vessel leads to the following expressions for the stresses in the vessel wall:

σ_x = hoop (tangential) stress = $\frac{pr}{t}$

σ_y = axial stress = $\frac{pr}{2t}$

where p = internal pressure
r = mean radius of vessel wall
t = wall thickness (<< r)

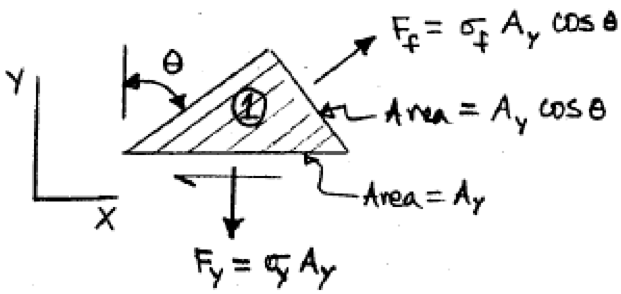


The above expressions are based on static equilibrium of the vessel wall for a thin walled vessel (t << r), and are valid for any material, including composites.

We now consider the static equilibrium of two elements in the wall of the filament wound composite vessel, assuming that all of the load is carried by the fibers.

continued

1.10 continued

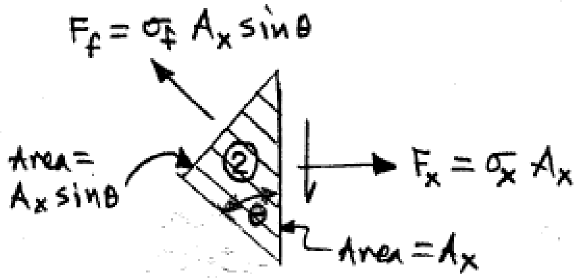


For static equilibrium of ①:

$$\Sigma F_y = \sigma_f A_y \cos^2\theta - \sigma_y A_y = 0$$

and $\sigma_y = \sigma_f \cos^2\theta$

where σ_f = stress in fibers



For static equilibrium of ②:

$$\Sigma F_x = \sigma_x A_x - \sigma_f A_x \sin^2\theta = 0$$

and $\sigma_x = \sigma_f \sin^2\theta$

For the hoop stresses and the axial stresses to be supported by the fibers alone, the ratio of the stresses must be

$$\frac{\sigma_x}{\sigma_y} = \frac{pr/t}{pr/2t} = 2 = \frac{\sigma_f \sin^2\theta}{\sigma_f \cos^2\theta} = \tan^2\theta$$

thus, the fiber angle must be

$$\theta = \tan^{-1}\theta = \tan^{-1}(\sqrt{2}) = 54.74^\circ$$

1.11

For the E-glass/epoxy vessel, the hoop stress is

$$\sigma_x = \frac{pr}{t} = \sigma_f \sin^2\theta$$

If we assume that the fibers carry all of the load, the maximum fiber stress for E-glass is (Table 1.1)

$$\sigma_{f_{max}} = 500 \times 10^3 \text{ psi } (3448 \text{ MPa})$$

continued

1.11 continued

Thus, the internal pressure corresponding to fiber failure is

$$p = \frac{\sigma_{f\,max}\, t}{r} \sin^2\theta = \frac{(500\times10^3)(0.25)}{25} \sin^2(54.74°)$$

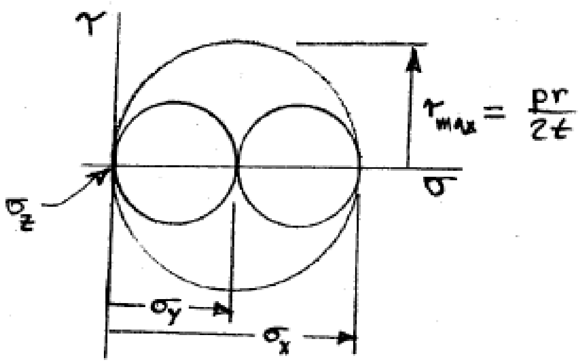
$$p = 3{,}333 \text{ psi}$$

For a safety factor of 2, $p = \frac{3{,}333}{2} = 1{,}666$ psi

Now for the aluminum vessel, the principal stresses are

$$\sigma_x = \frac{pr}{t}, \quad \sigma_y = \frac{pr}{2t}, \quad \sigma_z = 0 \text{ (plane stress)}$$

The Mohr's circles are shown below.



The maximum shear stress is

$$\tau_{max} = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{\sigma_x - \sigma_z}{2} = \frac{pr}{2t}$$

For the maximum shear stress yield criterion,

$$\tau_{max} = \frac{Y}{2} \quad \text{where } Y = \text{tensile yield stress}$$

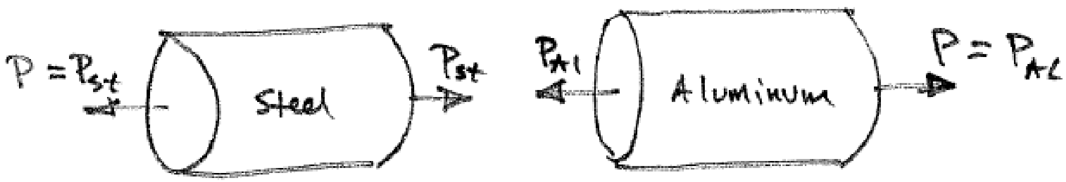
The allowable internal pressure is then

$$p = \frac{tY}{r} = \frac{(0.25)(40{,}000)}{25} = 400 \text{ psi}$$

For a safety factor of 2, $p = \frac{400}{2} = 200$ psi

Thus, the E-glass/epoxy vessel will withstand a much higher pressure than the aluminum vessel.

1.12 Free body diagrams:



Static equilibrium and Newton's 3rd Law:

$$P = P_{Al} = P_{st}$$

Stresses:

since $P_{st} = P_{Al}$ and $A_{st} = A_{Al}$

$$\sigma_{st} = \frac{P_{st}}{A_{st}} = \sigma_{Al} = \frac{P_{Al}}{A_{Al}}$$

Hooke's Law:

$$\sigma_{Al} = E_{Al}\,\epsilon_{Al} = (70\times10^{9})(1000\times10^{-6})$$

$$= 70\times10^{6}\,\frac{N}{m^2} = 70\ MPa = \sigma_{st}$$

Elongation:

$$\delta_{total} = \delta_{st} + \delta_{Al} = \left(\frac{PL}{AE}\right)_{st} + \left(\frac{PL}{AE}\right)_{Al}$$

$$= \sigma_{Al}\left(\frac{L_{Al}}{E_{Al}}\right) + \sigma_{st}\left(\frac{L_{st}}{E_{st}}\right)$$

$$= 70\times10^{6}\left(\frac{0.5}{70\times10^{9}} + \frac{1.5}{210\times10^{9}}\right)$$

$$= 0.001\ m = 1\ mm$$