Preliminary Solutions to Problems and Questions
Chapter 1

Note: Printing errors and corrections are indicated in dark red. See Question 1.47. These are correct in the e-version of the textbook

1.1 Maxwell's wave equation and plane waves

(a) Consider a traveling sinusoidal wave of the form $E_x = E_o \cos(\omega t - kz + \phi_o)$. The latter can also be written as $E_x = E_o \cos[k(v t - z) + \phi_o]$, where $v = \omega/k$ is the velocity. Show that this wave satisfies Maxwell's wave equation, and show that $v = (\mu_o\varepsilon_o\varepsilon_r)^{-1/2}$.

(b) Consider a traveling function of any shape, even a very short delta pulse, of the form $E_x = f[k(v t - z)]$, where f is any function, which can be written is $E_x = f(\phi)$, $\phi = k(v t - z)$. Show that this traveling function satisfies Maxwell's wave equation. What is its velocity? What determines the form of the function f?

Solution
(a)

$$E_x = E_o \cos(\omega t - kz + \phi_o)$$

$\therefore$ $$\frac{\partial^2 E_x}{\partial x^2} = 0$$

and $$\frac{\partial^2 E_x}{\partial y^2} = 0$$

and $$\frac{\partial^2 E_x}{\partial z^2} = -k^2 E_0 \cos(\omega t - kz + \phi_0)$$

$\therefore$ $$\frac{\partial^2 E_x}{\partial t^2} = -\omega^2 E_0 \cos(\omega t - kz + \phi_0)$$

Substitute these into the wave equation $\dfrac{\partial^2 E}{\partial x^2} + \dfrac{\partial^2 E}{\partial y^2} + \dfrac{\partial^2 E}{\partial z^2} - \varepsilon_o\varepsilon_r\mu_o \dfrac{\partial^2 E}{\partial t^2} = 0$ to find

$$-k^2 E_0 \cos(\omega t - kz + \phi_0) + \varepsilon_o\varepsilon_r\mu_o + \omega^2 E_0 \cos(\omega t - kz + \phi_0) = 0$$

$\therefore$ $$\frac{\omega^2}{k^2} = \frac{1}{\varepsilon_o\varepsilon_r\mu_o}$$

$\therefore$ $$\frac{\omega}{k} = (\varepsilon_o\varepsilon_r\mu_o)^{-1/2}$$

$\therefore$ $$v = (\varepsilon_o\varepsilon_r\mu_o)^{-1/2}$$

(b) Let

$$E_x = f[k(v t - z)] = f(\phi)$$

Take first and second derivatives with respect to x, y, z and t.

$$\frac{\partial^2 E_x}{\partial x^2} = 0$$

$$\frac{\partial^2 E_x}{\partial y^2} = 0$$

$$\frac{\partial E_x}{\partial z} = -k\frac{df}{d\phi}$$

$$\frac{\partial^2 E_x}{\partial z^2} = k^2\frac{d^2 f}{d\phi^2}$$

$$\frac{\partial E_x}{\partial t} = k\mathrm{v}\frac{df}{d\phi}$$

$$\frac{\partial^2 E_x}{\partial t^2} = k^2\mathrm{v}^2\frac{d^2 f}{d\phi^2}$$

Substitute these into the wave equation $\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \varepsilon_o\varepsilon_r\mu_o\frac{\partial^2 E}{\partial t^2} = 0$ to find

$$k^2\frac{d^2 f}{d\phi^2} - \varepsilon_o\varepsilon_r\mu_o k^2\mathrm{v}^2\frac{d^2 f}{d\phi^2} = 0$$

∴ $$\mathrm{v}^2 = \frac{1}{\varepsilon_o\varepsilon_r\mu_o}$$

∴ $$\mathrm{v} = (\varepsilon_o\varepsilon_r\mu_o)^{-1/2}$$

1.2 Propagation in a medium of finite small conductivity An electromagnetic wave in an isotropic medium with a dielectric constant ε_r and a finite conductivity σ and traveling along z obeys the following equation for the variation of the electric field E perpendicular to z,

$$\frac{d^2 E}{dz^2} - \varepsilon_o\varepsilon_r\mu_o\frac{\partial^2 E}{\partial t^2} = \mu_o\sigma\frac{\partial E}{\partial t} \qquad (1)$$

Show that one possible solution is a plane wave whose amplitude decays exponentially with propagation along z, that is $E = E_o\exp(-\alpha z)\exp[j(\omega t - kz)]$. Here $\exp(-\alpha z)$ causes the envelope of the amplitude to decay with z (attenuation) and $\exp[j(\omega t - kz)]$ is the traveling wave portion. Show that in a medium in which α is small, the wave velocity and the attenuation coefficient are given by

$$\mathrm{v} = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o\varepsilon_o\varepsilon_r}} \qquad \text{and} \qquad \alpha = \frac{\sigma}{2\varepsilon_o cn}$$

where n is the refractive index ($n = \varepsilon_r^{1/2}$). (Metals with high conductivities are excluded.)

Solution

We can write $E = E_o\exp(-\alpha z)\exp[j(\omega t - kz)]$ as $E = E_o\exp[j\omega t - j(k - j\alpha)z]$. Substitute this into the wave resonance condition

$$[-j(k-j\alpha)]^2 E_o\exp[j\omega t - j(k-j\alpha)z] - (j\omega)^2\varepsilon_o\varepsilon_r\mu_o E_o\exp[j\omega t - j(k-j\alpha)z] = j\omega\mu_o\sigma E_o\exp[j\omega t - j(k-j\alpha)z]$$

$\therefore$ $$-(k-j\alpha)^2 + \omega^2\varepsilon_o\varepsilon_r\mu_o = j\omega\mu_o\sigma$$

$\therefore$ $$-k^2 + 2jk\alpha - \alpha^2 + \omega^2\varepsilon_o\varepsilon_r\mu_o = j\omega\mu_o\sigma$$

Rearrange into real and imaginary parts and then equating the real parts and imaginary parts

$\therefore$ $$-k^2 - \alpha^2 + \omega^2\varepsilon_o\varepsilon_r\mu_o + 2jk\alpha = j\omega\mu_o\sigma$$

Real parts

$$-k^2 - \alpha^2 + \omega^2\varepsilon_o\varepsilon_r\mu_o = 0$$

Imaginary parts

$$2k\alpha = \omega\mu_o\sigma$$

Thus, $$\alpha = \frac{\omega\mu_o\sigma}{2k} = \frac{\omega}{k}\cdot\frac{\mu_o\sigma}{2} = \frac{\mu_o c\sigma}{2n} = \frac{\sigma}{2\varepsilon_o n}$$

where we have assumed ω/k = velocity = c/n (see below).

From the imaginary part

$$k^2 = \omega^2\mu_o\varepsilon_o\varepsilon_r - \alpha^2$$

Consider the small α case (otherwise the wave is totally attenuated with very little propagation). Then

$$k^2 = \omega^2\mu_o\varepsilon_o\varepsilon_r$$

and the velocity is

$$v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o\varepsilon_o\varepsilon_r}}$$

1.3 Point light source What is the irradiance measured at a distance of 1 m and 2 m from a 1 W light point source?

Solution

Then the irradiance I at a distance r from O is

$$I = \frac{P_o}{4\pi r^2} = \frac{1\,\text{W}}{4\pi(1\,\text{m})^2} = 8.0\ \mu\text{W cm}^{-2}$$

which drops by a factor of 4 at r = 2 m to become 2.0 μW cm^{-2}

1.4 Gaussian Beam Estimate the divergence and Rayleigh range of a Gaussian beam from a He-Ne Laser with λ = 633 nm and a beam width of 1.00 mm at z = 0. After traversing 10m through vacuum, what will the beam width be?

Solution:

Divergence, $$2\theta = \frac{4\lambda}{\pi(2w_0)} = \frac{4(633\times10^{-9}m)}{\pi(1\times10^{-3}m)} = 805.96\times10^{-6} rad = {=}0.046^0$$

Rayleigh range, $$Z_0 = \frac{\pi w_0^2}{\lambda} = \frac{\pi\left[\frac{1}{2}(1\times10^{-3}m)\right]^2}{(633\times10^{-9}m)} = 1.24\ m$$

The beam width at a distance of 10 m is,

$$2w = 2w_\circ[1+(z/z_\circ)^2]^{\frac{1}{2}} = (1\times10^{-3}m)\left\{1+\left[\frac{10\ m}{1.24\ m}\right]^2\right\}^{1/2} = 8.126\times10^{-3}m$$

1.5 Gaussian beam in a cavity with spherical mirrors Consider an optical cavity formed by two aligned spherical mirrors facing each other as shown in Figure 1.54. Such an optical cavity is called a *spherical mirror resonator*, and is most commonly used in gas lasers. Sometimes, one of the reflectors is a plane mirror. The two spherical mirrors and the space between them form an optical resonator because only certain light waves with certain frequencies can exist in this optical cavity. The radiation inside a spherical mirror cavity is a *Gaussian beam*. The actual or particular Gaussian beam that fits into the cavity is that beam whose wavefronts at the mirrors match the curvature of the mirrors. Consider the symmetric resonator shown in Figure 1.54 in which the mirrors have the same radius of curvature *R*. When a wave starts at *A*, its wavefront is the same as the curvature of *A*. In the middle of the cavity it has the minimum width and at *B* the wave again has the same curvature as *B*. Such a wave in the cavity can replicate itself (and hence exist in the cavity) as it travels between the mirrors provided that it has right beam characteristics, that is the right curvature at the mirrors. The radius of curvature *R* of a Gaussian beam wavefront at a distance *z* along its axis is given by

$$R(z) = z[1 + (z_o/z)^2]\ ;\ z_o = \pi w_o^2/\lambda$$

is the Rayleigh range

Consider a confocal symmetric optical cavity in which the mirrors are separated by $L = R$.

(a) Show that the cavity length L is $2z_o$, that is, it is the same as the Rayleigh range, which is the reason the latter is called the **confocal length**.

(b) Show that the waist of the beam $2w_o$ is fully determined only by the radius of curvature R of the mirrors, and given by

$$2w_o = (2\lambda R/\pi)^{1/2}$$

(c) If the cavity length $L = R = 50$ cm, and $\lambda = 633$ nm, what is the waist of the beam at the center and also at the mirrors?

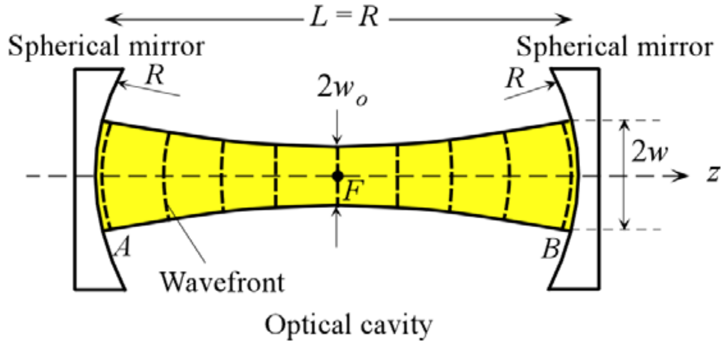


Figure 1.54 Two spherical mirrors reflect waves to and from each other. The optical cavity contains a Gaussian beam. This particular optical cavity is symmetric and confocal; the two focal points coincide at *F*.

Solution

(a) At $z = R/2$ we have $R(z) = R$. Substitute these into $R(z) = z[1 + (z_o/z)^2]$ to find

$$R = (R/2)[1 + (2z_o/R)^2]$$

∴ $$2 = 1 + \left(\frac{2z_o}{R}\right)^2$$

∴ $$\left(\frac{2z_o}{R}\right) = 1$$

∴ $$L = 2z_o$$

(b) $R = (R/2)[1 + (2z_o/R)^2]$

∴ $$2 = 1 + \left(\frac{2z_o}{R}\right)^2$$

∴ $$\left(\frac{2z_o}{R}\right) = 1$$

Now use $z_o = \pi w_o^2/\lambda$,

∴ $$\left(\frac{2\pi w_o^2}{R\lambda}\right) = 1$$

∴ $$2w_o = \sqrt{\frac{2R\lambda}{\pi}}$$

(c) Substitute $\lambda = 633$ nm, $L = R = 50$ cm into the above equation to find $2w_o = 449$ μm or **0.449 mm**. At the mirror, $z = R/2$, and also $z_o = R/2$ so that

$$2w = 2w_o\left[1 + \left(\frac{z}{z_o}\right)^2\right]^{1/2} = 2w_o\left[1 + \left(\frac{R/2}{R/2}\right)^2\right]^{1/2} = 2w_o(2^{1/2}) = \mathbf{0.635\ mm}$$

1.6 Cauchy dispersion equation Using the Cauchy coefficients and the general Cauchy equation, calculate refractive index of a silicon crystal at 200 μm and at 2 μm, over two orders of magnitude wavelength change. What is your conclusion?

Solution

At $\lambda = 200$μm, the photon energy is

$$h\upsilon = \frac{hc}{\lambda} = \frac{(6.62\times10^{-34}\ \text{J s})(3\times10^{8}\ \text{m s}^{-1})}{(200\times10^{-6}\ \text{m})} \times \frac{1}{1.6.\times10^{-19}\ \text{J eV}^{-1}} = 6.2062\times10^{-3}\,\text{eV}$$

Using the Cauchy dispersion relation for silicon with coefficients from Table 9.2,

$$n = n_{-2}(h\upsilon)^{-2} + n_0 + n_2(h\upsilon)^2 + n_4(h\upsilon)^4$$
$$= (-2.04\times10^{-8})(6.2062\times10^{-3})^{-2} + 3.4189 + (8.15\times10^{-2})(6.2062\times10^{-3})^2$$
$$+ (1.25\times10^{-2})(6.2062\times10^{-3})^4$$
$$= 3.4184$$

At $\lambda = 2$μm, the photon energy is

$$h\upsilon = \frac{hc}{\lambda} = \frac{(6.62\times10^{-34}\ \text{J s})(3\times10^{8}\ \text{m s}^{-1})}{(2\times10^{-6}\ \text{m})} \times \frac{1}{1.6\times10^{-19}\ \text{J eV}^{-1}} = 0.6206\text{eV}$$

Using the Cauchy dispersion relation for silicon with coefficients from Table 9.2,

$$\begin{aligned} n &= n_{-2}(h\upsilon)^{-2} + n_0 + n_2(h\upsilon)^2 + n_4(h\upsilon)^4 \\ &= (-2.04\times10^{-8})(0.6206)^{-2} + 3.4189 + (8.15\times10^{-2})(0.6206)^2 \\ &\quad + (1.25\times10^{-2})(0.6206)^4 \\ &= 3.4521 \end{aligned}$$

1.7 Sellmeier dispersion equation Using the Sellmeier equation and the coefficients, calculate the refractive index of fused silica (SiO_2) and germania GeO_2 at 1550 nm. Which is larger, and why?

Solution

The Sellmeier dispersion relation for fused silica is

$$n^2 = 1 + \frac{0.696749\lambda^2}{\lambda^2 - 0.0690660^2\,\mu\text{m}^2} + \frac{0.408218\lambda^2}{\lambda^2 - 0.115662^2\,\mu\text{m}^2} + \frac{0.890815\lambda^2}{\lambda^2 - 9.900559^2\,\mu\text{m}^2}$$

$$n^2 = 1 + \frac{0.696749(1550\text{ nm})^2}{(1550\text{ nm})^2 - (69.0660\text{ nm})^2} + \frac{0.408218(1550\text{ nm})^2}{(1550\text{ nm})^2 - (115.662\text{ nm})^2} + \frac{0.890815(1550\text{ nm})^2}{(1550\text{ nm})^2 - (9900.559\text{ nm})^2}$$

so that

$n = 1.4443$

The Sellmeier dispersion relation for germania is

$$n^2 = 1 + \frac{0.8068664\lambda^2}{\lambda^2 - (0.0689726\,\mu\text{m})^2} + \frac{0.7181585\lambda^2}{\lambda^2 - (0.1539661\,\mu\text{m})^2} + \frac{0.8541683\lambda^2}{\lambda^2 - (11.841931\,\mu\text{m})^2}$$

$$n^2 = 1 + \frac{0.8068664(1550\text{ nm})^2}{(1550\text{ nm})^2 - (68.9726\text{ nm})^2} + \frac{0.7181585(1550\text{ nm})^2}{(1550\text{ nm})^2 - (153.9661\text{ nm})^2} + \frac{0.8541683(1550\text{ nm})^2}{(1550\text{ nm})^2 - (11841.931\text{ nm})^2}$$

so that n = **1.5871**

1.8 Sellmeier dispersion equation The Sellmeier dispersion coefficient for pure silica (SiO_2) and 86.5%SiO_2-13.5 mol.% GeO_2 re given in Table 1.2 Write a program on your computer or calculator, or use a math software package or even a spread sheet program (*e.g.* Excel) to obtain the refractive index n as a function of λ from 0.5 μm to 1.8 μm for both pure silica and 86.5%SiO_2-13.5%GeO_2. Obtain the group index, N_g, vs. wavelength for both materials and plot it on the same graph. Find the wavelength at which the material dispersion becomes zero in each material.

TABLE 1.2 Sellmeier and Cauchy coefficients

Sellmeier	A_1	A_2	A_3	λ_1 (μm)	λ_2 (μm)	λ_3 (μm)
SiO_2 (fused silica)	0.696749	0.408218	0.890815	0.0690660	0.115662	9.900559
86.5%SiO_2-13.5%GeO_2	0.711040	0.451885	0.704048	0.0642700	0.129408	9.425478
GeO_2	0.80686642	0.71815848	0.85416831	0.068972606	0.15396605	11.841931
Sapphire	1.023798	1.058264	5.280792	0.0614482	0.110700	17.92656
Diamond	0.3306	4.3356	–	0.1750	0.1060	–

Solution

Excel program to plot n and differentiate and find N_g

$$n^2 = 1 + \frac{A_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3\lambda^2}{\lambda^2 - \lambda_3^2}$$

Enter the Sellmeier coefficients for SiO_2 and 86.5%SiO_2-13.5%GeO

Worksheet 1: Data

	A	B	C	D	E	F	G
1		A1	A2	A3	λ1	λ2	λ3
2					μm	μm	μm
3	SiO2	0.696749	0.408218	0.890815	0.069066	0.115662	9.900559
4	86.5%SiO2-13.5%GeO2	0.71104	0.451885	0.704048	0.06427	0.129408	9.425478

Worksheet 2: SiO_2

Use Sellmeier equation and data from worksheet "Data"

$$n = \sqrt{1 + \frac{A_1\lambda^2}{\lambda^2 - \lambda_1^2} + \frac{A_2\lambda^2}{\lambda^2 - \lambda_2^2} + \frac{A_3\lambda^2}{\lambda^2 - \lambda_3^2}}$$

=SQRT(1+(Data!$B$3*SiO2!A2^2)/(SiO2!A2^2-Data!$E$3^2)+(Data!$C$3*SiO2!A2^2)/(SiO2!A2^2-Data!$F$3^2)+(Data!$D$3*SiO2!A2^2)/(SiO2!A2^2-Data!$G$3^2))!

Enter the first wavelength (0.50 μm) and then increment by 0.001

= A2+0.001

	A	B	C
1	λ(μm)	n	Ng
2	0.5	1.462642	1.49024
3	0.501	1.462587	1.49008
4	0.502	1.462532	1.48992
5	0.503	1.462477	1.489762
6	0.504	1.462423	1.489605
7	0.505	1.462369	1.489448
8	0.506	1.462315	1.489293
9	0.507	1.462262	1.489139
10	0.508	1.462209	1.488986
11	0.509	1.462156	1.488834
12	0.51	1.462104	1.488682

=B2-(A2*(B3-B2)/0.001)

$$N_g = n - \lambda_o \frac{dn}{d\lambda_o}$$

$$N_{g1} = n_1 - \lambda_1 \frac{n_2 - n_1}{\lambda_2 - \lambda_1} = n_1 - \lambda_1 \frac{n_2 - n_1}{\Delta\lambda}$$

Differentiate by using finite difference

0.001

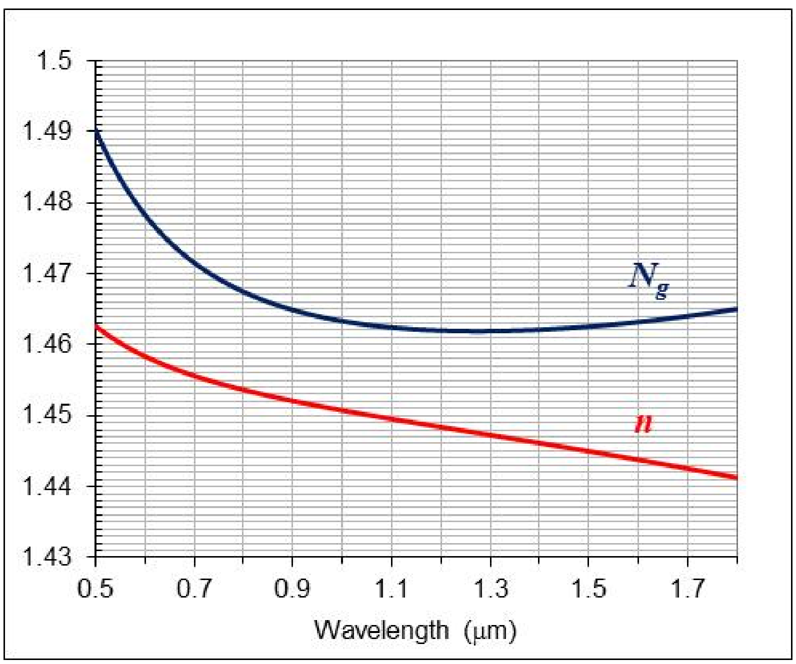


Figure 1Q8-1 Refractive index n and the group index N_g of pure SiO_2 (silica) glass as a function of wavelength (Excel). The minimum in N_g is around 1.3 μm. Note that the *smooth line* option used in Excel to pass a continuous smooth line through the data points. Data points are exactly on the line and are not shown for clarity.

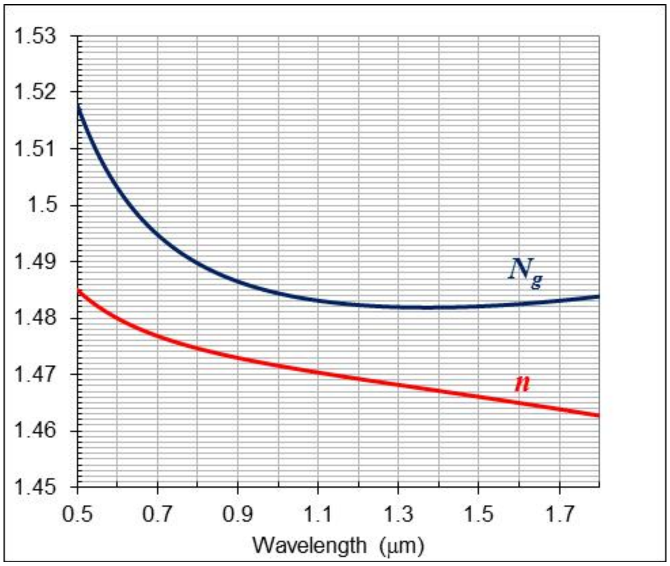


Figure 1Q8-2 Refractive index n and the group index N_g of 86.5%$SiO_2$13.5%GeO as a function of wavelength (Excel). The minimum in N_g is around 1.4 μm. Note that the *smooth line* option used in Excel to pass a continuous smooth line through the data points. Data points are exactly on the line and are not shown for clarity.

Material dispersion is proportional to derivative of group velocity over wavelength. The corresponding values are close to 1.3 and 1.4 μm.

1.9 The Cauchy dispersion relation for zinc selenide ZnSe is a II-VI semiconductor and a very useful optical material used in various applications such as optical windows (especially high power laser windows), lenses, prisms etc. It transmits over 0.50 to 19 μm. n in the 1 – 11 μm range described by a Cauchy expression of the form

$$n = 2.4365 + \frac{0.0485}{\lambda^2} + \frac{0.0061}{\lambda^4} - 0.0003\lambda^2 \qquad \textit{ZnSe dispersion relation}$$

in which λ in μm. What are the n_{-2}, n_0, n_2 and n_4 coefficients? What is ZnSe's refractive index n and group index N_g at 5 μm?

Solution

$$h\upsilon = \frac{hc}{\lambda}$$

$$hc = (6.62\times10^{-34}\ \mathrm{J\,s}\times\frac{1}{1.6\times10^{-19}\ \mathrm{J\,eV^{-1}}})(3\times10^{8}\ \mathrm{m\,s^{-1}}) = 1.24\times10^{-6}\,\mathrm{eVm}$$

so that

$$n = 2.4365 + \frac{0.0485}{(hc)^2}(h\nu)^2 + \frac{0.0061}{(hc)^4}(h\nu)^4 - 0.0003(hc)^2(h\nu)^{-2}$$

Comparing with Cauchy dispersion equation in photon energy: $n = n_{-2}(h\upsilon)^{-2} + n_0 + n_2(h\upsilon)^2 + n_4(h\upsilon)^4$, we have

$$n_0 = 2.4365$$

$$n_2 = \frac{0.0485}{(hc)^2} = \frac{0.0485}{(1.24\times10^{-6})^2} = 3.15\times10^{10}\,\text{eV}^{-2}$$

$$n_{-2} = 0.0003(hc)^2 = 0.0003\times(1.24\times10^{-6})^2 = 4.62\times10^{-16}\,\text{eV}^2$$

and $$n_4 = \frac{0.0061}{(hc)^4} = \frac{0.0061}{(1.24\times10^{-6})^4} = 2.58\times10^{21}\,\text{eV}^{-4}$$

At λ = 5 μm

$$n = 2.4365 + \frac{0.0485}{(5\mu m)^2} + \frac{0.0061}{(5\mu m)^4} - 0.0003(5\mu m)^2$$

$$= 2.4365 + \frac{0.0485}{25} + \frac{0.0061}{625} - 0.0003(25) = 2.43$$

Group index $$N_g = n - \lambda\frac{dn}{d\lambda}$$

and $$n = 2.4365 + \frac{0.0485}{\lambda^2} + \frac{0.0061}{\lambda^4} - 0.0003\lambda^2$$

∴ $$\frac{dn}{d\lambda} = \frac{-2\lambda\times0.0485}{\lambda^4} + \frac{-4\lambda^3\times0.0061}{\lambda^8} - 2\times0.0003\lambda$$

∴ $$\frac{dn}{d\lambda} = \frac{-0.097}{\lambda^3} + \frac{-0.0244}{\lambda^5} - 0.0006\lambda$$

At λ = 5 μm

$$\frac{dn}{d\lambda} = \frac{-0.097}{(5\mu\text{m})^3} + \frac{-0.0244}{(5\mu\text{m})^5} - 0.0006\times(5\mu\text{m})$$

∴ $$\frac{dn}{d\lambda} = -0.003783\mu\text{m}^{-1}$$

∴ $$N_g = n - \lambda\frac{dn}{d\lambda} = 2.43 - 5\mu\text{m}\times(-0.003783\mu\text{m}^{-1}) = 2.45$$

1.10 Refractive index, reflection and the Brewster angle

(a) Consider light of free-space wavelength 1300 nm traveling in pure silica medium. Calculate the phase velocity and group velocity of light in this medium. Is the group velocity ever greater than the phase velocity?

(b) What is the Brewster angle (the polarization angle θ_p) and the critical angle (θ_c) for total internal reflection when the light wave traveling in this silica medium is incident on a silica/air interface. What happens at the polarization angle?

(c) What is the reflection coefficient and reflectance at normal incidence when the light beam traveling in the silica medium is incident on a silica/air interface?

(d) What is the reflection coefficient and reflectance at normal incidence when a light beam traveling in air is incident on an air/silica interface? How do these compare with part (c) and what is your conclusion?

Solution

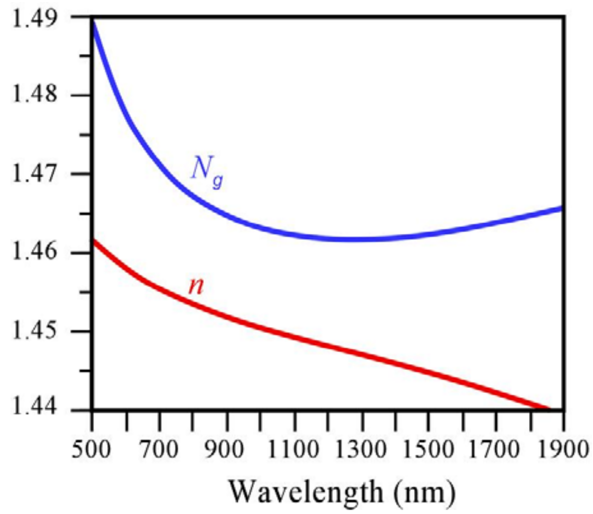


Figure 1.8 Refractive index n and the group index N_g of pure SiO_2 (silica) glass as a function of wavelength.

(a) From Figure 1.8, at λ = 1300 nm, n = 1.447, N_g = 1.462, so that
The phase velocity is given by

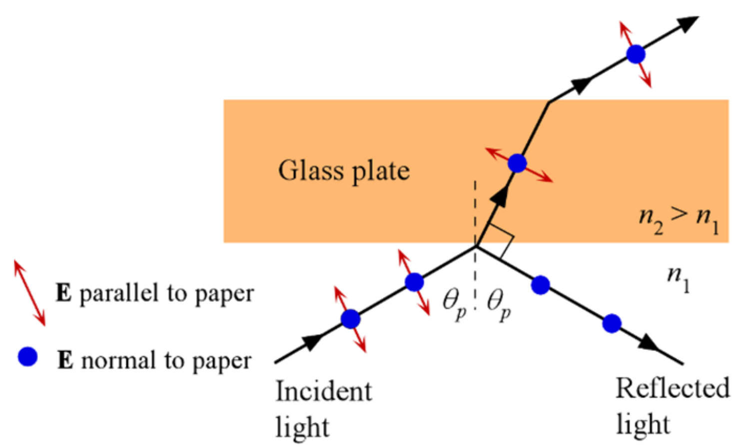
$$v = c/n = (3\times10^8 \text{ m s}^{-1})/(1.447) = 2.073\times10^8 \text{ m s}^{-1}.$$

The group velocity is given by

$$v_g = c/N_g = (3\times10^8 \text{ ms}^{-1})/(1.462) = 2.052\times10^8 \text{ m s}^{-1}.$$

The group velocity is about ~1% smaller than the phase velocity.

(b)



The Brewster angle θ_p is given by

$$\tan\theta_p = \frac{n_2}{n_1} = \frac{1}{1.447} = 0.691$$