Contents

CHAPTER 1

OVERVIEW OF OR, ANALYTICS, AI, AND ML IN DECISION MAKING

Chapter 1

1

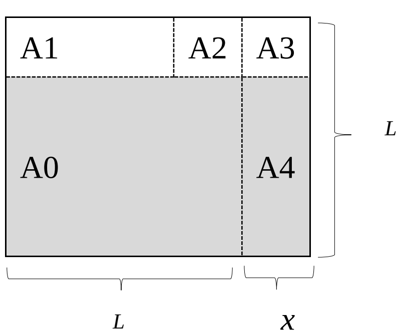
Buy five 1-way FYV-DEN and five 1-way DEN-FYV (not exactly the smartest option one would consider, but it is feasible).
Cost = 10x(.75x400) = $3000

2

Width, x	Height, h	Area
10	40	400
20	30	600
30	20	600
40	10	400

No, the calculations only indicate that the maximum *appears* to lie between x=20 and x=30. In general, functions may have local maxima, which may be detected conveniently in this one-dimensional case by graphing the function, an unnecessary time-consuming task. What is needed is a method that can locate the maximum, in the present example with calculus.

3

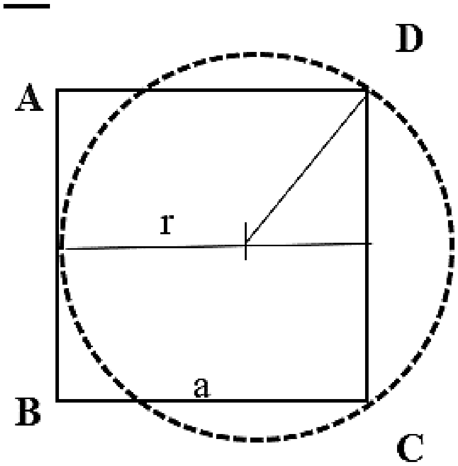


continued…

3 cont'd

Height of the (shaded) rectangle is x units shorter than L.
Base of rectangle is x units longer than L. Thus, A3 is square with side *x*. Construct A2, a mirror image of A3. Hence, area A1 = area A4.
Area of square of side L = **A0 + A1** + A2
Area of rectangle = A0 + A4 = **A0 + A1**.
Conclusion: Area of square is larger than area of rectangle by A2 (= x^2).
Algebraic solution: For $0 < x < L$, the square area L^2 is larger than the rectangle area of $(L + x)(L - x) = L^2 - x^2$, which is the same result shown by the graph.

4



Per triangle: $r^2 = (\frac{a}{2})^2 + (a - r)^2$

Hence $r = \frac{5a}{8}$

$\frac{\text{Circle circumference}}{\text{Square perimiter}} = \frac{2\pi r}{4a} = \frac{5\pi}{16} < 1$

$\Rightarrow$ square perimiter is larger

5

x = cumulative number of drops of balls #1 and #2 at any floor (problem unknown)

y_i = floor from which ith drop of ball #1 occurs.

Step 0: Set $y_0 = 0$, $y_1 = x$, and $i = 1$.

General step i: Drop ball#1 from floor yi. If it is dented, use ball#2 to check floors $y_{i-1} + 1$ to $y_i - 1$, in that order. Else, if #1 is not dented, set $i = i + 1$, and repeat step i.

Formula for determining y_i:

y_i must include the (cumulative) i #1-drops from floor y_1 to floor y_i. To maintain the same number of drops at any floor y_i, #2-drops cannot exceed $x - i$. Thus,

$$y_i = y_{i-1} + (x - i + 1)$$
$$= x + (x - 1) + (x - 2) + \ldots + (x - i + 1)$$
$$= ix - (1 + 2 + \ldots + i - 1) = ix - (i - 1)\,i/2$$

Maximum number of #1-drops is x (else $y_i \le y_{i-1}$ for $i > x$). Hence the highest floor from which #1 can be dropped is

$$y_x = x^2 - (x-1)x/2 = x^2 - x^2/2 + x/2 = (x^2 + x)/2$$

For a 100-storey building, $y_x \ge 100$, or $x^2 + x - 200 \ge 0$. The associated quadratic equation yields $x = 13.64$ and -14.64. The rounded positive value $x = 14$ is the smallest integer that satisfies the inequality.

6

(a) Let T = Total tie to move all four individuals to the other side of the river. the objective is to determine the transfer schedule that minimizes T.

(b) Let t = crossing time from one side to the other. Use codes 1, 2, 5, and 10 to represent Amy, Ravi, Ali, and Tao.

continued…

6 cont'd

(c)

East	Crossing	West
5,10	(1,2)→ **(t = 2)**	1,2
1,5,10	**(t = 1)**←(1)	2
1	(5,10)→ **(t = 10)**	2,5,10
1,2	**(t = 2)**←(2)	5,10
none	(1,2)→ **(t = 2)**	1,2,5,10
Total = 2 + 1 + 10 + 2 + 2 = **17 minutes**		

		Jorge	
		Curve	Fast
Jack(batter)	Curve	.500	.100
	Fast	.200	.300

7

(a) Alternatives:
Jorge(pitcher): Throw curve or fast ball.
Jack(batter): Prepare for curve or fast ball.

(b) Jack tries to improve his batting score and Jorge tries to counter Jack's action by selecting a less favorable strategy for Jack. This means that neither player will be satisfied with a single (pure) strategy.

The problem is not an optimization situation in the familiar sense in which the objective is maximized or minimized. Instead, the conflicting situation requires a compromise solution in which neither layer is tempted to change strategy. Game theory (Chapter 15) provides such a solution.

8

Let L=ops. 1 and 2=20 sec, C=ops. 3 and 4=25 sec, U=op. 5=20 sec

Gant chart: L1+load horse 1, L2=load horse 2, etc.

one joist: 0---L1---**20**---C1---45----U1+L1---**85**----U2+L2----125---U1+L1---**165**---U2+L2---*205*

20-L2-40 45---C2----70 **85**---C1---110 125---C2---140

165-C1-190

205---C2---230---U2---250

Total = 250

Loaders utilization=[250-(5+25)]/250=88%

Cutter utilization=[250-(20+15+15+15+15)]/250=68%

two joists: 0---2L1---**40**-----2C1-----**90**----2(U1+L1)---**170**----2C1----220---2U1---260

40---2L2---80 **90**---2C2----140 **170**---2U2---210

Total =260

Loaders utilization=[260-(10+10)]/260=92%

Cutter utilization=[260-(40+30+40)]/250=58%

three joists: 0---3L1---**60**------3C1------**135**------3C2---------210----3U2----270

60---3L2---120 **135**-----3U1-----195

Total =270

Loaders utilization=[270-(15+15)]/270=89%

Cutter utilization=[270-(60+60)]/270=56%

Recommendation: One joist at time gives the smallest time. The problem has other alternatives that combine 1, 2, and 3 joists. Cutter utilization indicates that cutter represents the bottleneck.

9

(a) Alternative 1: Move dots 5, 6, and 7 below bottom row, move dots 8 and 9 below new 5, 6, and 7. Move 10 to the bottom. Number of moves = 6. Alternative 2: See part (b)

(b) Three moves: Dot 1 up to the left of dot 8, dot 4 to the right of dot 8, and dot 10 below dots 2 and 3.

10

8 9

5 6 7

1 2 3 4

10

(a)Alternative 1: Break one end link of each chain and connect to another chain. Four breaks and re-solders, cost = 4 x (2 + 3) =20 cents. Alternative 2: See Part (b)
(b)Break three links in one chain and use them to connect the remaining three chains: Three breaks and re-solder, cost = 3 x (2 + 3) = 15 cents.

11

Represent the selected 2-digit number as 10x+y, x and y =1, 2, …, 9. The corresponding square number is 10x+y-(x+y)=9x . This means that the selected square will always be 9, 18, 27, …, or 81. By assigning zero dollars to these squares, the reward is always zero regardless of the rewards assigned to the remaining squares or the number of times the game is repeated.

12

Arrange cartons in a sequence that assign them the successive numbers $x \varepsilon X=\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. Take out x bottles from each carton $x \varepsilon X$, totaling $(1+2+\ldots+10) = (10\times11)/2=55$ bottles. In the absence of a defective carton these 55 bottles should weigh 550 oz. Else, they should weigh $W \leq 550$. Now weigh the 55 bottles. The defective carton number is $x = 550 - W$.

13

Mean waiting time < 5 min:

number of cashiers ≥ 4

Idleness % < 15%:

number of cashiers ≤ 3

The two conditions cannot be satisfied simultaneously. Relax one of the two conditions.

14

1. Credit history
2. Credit amount
3. Purpose of loan
4. Class of loan (good or bad)
5. Customer Age
6. Gender
7. Employment
8. Housing (rent or own)

15

(a) Tally data for 3-5 years in a spreadsheet on all employees
who left the organization, including (1) employee name,
date of birth, position, department, supervisor’s name,
(2) reason for termination, (3) Equal Employment Opportunity information, (4) date of termination, (5) tenure, (6) annual salary, and (7) total cost to replace
the employee.
(b) Examples of data analysis:
a. Departments with high turnover may require HR attention. Supervisors with high turnover may need management training. And positions with high turnover may need to be restructured to be more interesting.
b. If many employees are leaving for family reasons, the organization may need to adopt more family-friendly policies.
c. High turnover in minority groups may indicate policies that unintentionally have a disparate impact on those groups.
d. Check if date of termination occurs in the period immediately after annual bonuses are paid out.
e. Departures during an employee’s first year may indicate problems with the hiring process, orientation, or employee training.
f. A look at trends may help you anticipate times of high turnover, so you can plan both recruiting efforts and budgets. It will also show if your annual turnover rate is increasing or decreasing.

16

1. Identify the critical areas that need quick police deployment categorized by type of crime (e.g., auto theft, vehicle burglary, burglary, and gang activity).
2. Police reports, police body cameras, street-corner cameras, crime scene cameras. Track each crime and translate it into usable intelligence for the department
3. Provide information as to where and when a crime is likely to occur. Provide officers with biometrics and fingerprint alternatives (such as facial recognition) to assist officers issue citations on the scene for suspects with no IDs. Else officer will waste time transporting suspect to police station for processing.

17

Proposal (1):

a. The transportation paradigm "one car, one person, any time" is not sustainable in the long run. Public transportation and human-powered bikes are the answer.
b. Pricing is unfair to middle/low income who commute to/from work.
c. Traffic congestion caused by affluent car drivers creating time penalty on less affluent people riding the bus.
d. Catering to car travel in the city favors the upper middle classes and the rich over everyone else.
e. Paralyzed police cars, fire trucks, ambulances paralyzed in traffic with sirens blaring and unable to move.
f. Pollution cars and taxis cause.

Proposal (2)

a. This is a common-sense partial solution to congestion during regular business hours.
b. It is a first step toward disallowing traffic altogether in congested streets during business hours.

continued…

17 Cont'd

b. Success will depend in how many patrons are willing or able to participate in this type of setup.
c. The cost of staying open during after-hours just to receive shipments is a definite disadvantage.

18

	S1	S2	S3	S4	S5	
S1	0	1 **10/5**	3	2 **30/25**	4	**50**
S2	1	0	4	7	6	
S3	3	4	0	3 **10/0**	5	**30**
S4	2	7	3	0	2	
S5	4	6 **10/15**	5	2 **0/15**	0	**35**
		20		**40**		

Solution 1 (not underlined):
S1-S2 =10, S5-S2 =10; S1-S4=30. S3-S4=10
Cost = (10x1+10x6) + (30x2+10x3) = $160

Solution 2 (underlined):
S1-S2 =5, S5-S2=15; S1-S4=25, S5-S4=15
Cost=(5x1+15x6) + (25x2+15x2) = $175

Solution 1 is cheaper

19

Store	S1	S2	S3	S4	S5
Starting inventory	100	100	100	100	100
Projected demand during the season	110	130	160	50	65
Projected surplus				50	35
Projected shortage	10	30	60		

	S1	S2	S3	S4	S5	
S1	0	1	3	2	4	
S2	1	0	4	7	6	
S3	3	4	0	3	5	
S4	2 **10/5**	7 **20/15**	3 **20/30**	0	2	**50**
S5	4 **0/5**	6 **10/15**	5 **25/15**	2	0	**35**
	10	**30**	**60**			

Solution 1 (not underlined):
(S4-S1=10), (S4-S2=20, S5-S2=10),
(S4-S3=20, S5-S3=25) Shortage at S3=15
Cost=(10x2) + (20x7+10x6) + (20x3+25x5) = $405

Solution 1 (underlined):
(S4-S1=5, S5-S1=5)), (S4-S2=15, S5-S2=15),
(S4-S3=30, S5-S3=15) Shortage at S3=15
Cost=(5x2+5x4)+(15x7+15x6)+(30x3+15x5)=$390
Solution 2 is cheaper.

20

The need for an automated AI-based monitoring system appears critical in large cities.

The principal components of the proposed AI system are

1. Cameras installed at intersections provide live traffic data.
2. Vehicles breaking traffic laws are identified and tracked using computer vision machine learning (see Section 1.5.1) to gains understanding from digital images or videos to identify infractions.
3. Automatic number-plate recognition provides needed information to automate the ticket issuing process.
4. Central processor connects the cameras to a server that stores videos of infractions and issues fines to violators.

continued…

20 cont'd

The proposed system may pose privacy issues, as in having license plate numbers available in a public domain. Also, for this system to be effective, the cameras must be able to "see", a case that requires cooperation from nature. Snow and rain are obvious impediments in a system that is exclusively based on visual recognition. These issues must be addressed before the system is put into practical use. The hope is that future advances in software and hardware would remove such concerns.

21

$$\overbrace{P(A|B)}^{\text{Posterior}} = \frac{.5\times.7}{.55} = .636 \Rightarrow \text{revised } P(A) = .636$$

22

$$\overbrace{P(A|B)}^{\text{Posterior}} = \frac{.73\times.7}{.55} = .929 \Rightarrow \text{revised } P(A) = .929$$

Full parking + high rating = 92.9% chance food is good.

CHAPTER 2

Modeling with linear programming

Chapter 2

1

(a) $x_2 - x_1 \geq 1$ **or** $-x_1 + x_2 \geq 1$

(b) $x_1 + 2x_2 \geq 3$ **and** $x_1 + 2x_2 \leq 6$

$$\left.\begin{aligned} 6\times2+4\times2 &= 20 < 24 \\ 1\times1+2\times2 &= 6 = 6 \\ -1\times1+1\times2 &= 0 < 1 \\ 1\times2 &= 2 < 1 \end{aligned}\right\} \textit{feasible}$$

(c) $x_2 \geq x_1$ **or** $x_1 - x_2 \leq 0$

(d) $x_1 + x_2 \geq 3$

(e) $\dfrac{x_2}{x_1+x_2} \leq .5$ **or** $.5\,x_1 - x_2 \geq 0$

2

(a) $(x_1, x_2) = (1, 4)$

$(x_1, x_2) \geq 0$

$6\times1+4\times4 = 22 < 24$

$1\times1+2\times4 = 9 > 6$ $\Rightarrow$ infeasible

(b) $(x_1, x_2) = (2, 2)$

$(x_1, x_2) \geq 0$

$6\times2+4\times2 = 20 < 24$

$1\times2+2\times2 = 6 = 6$

$-1\times2+1\times2 = 0 < 1$

$1\times2 = 2 = 2$

Feasible, $z = 5\times2+4\times2 = \$18$

(c) $(x_1, x_2) = (3, 1.5)$

$(x_1, x_2) \geq 0$

$6\times3+4\times1.5 = 24 < 24$

$1\times3+2\times1.5 = 6 = 6$

$-1\times3+1\times1.5 = -1.5 < 1$

$1\times1.5 = 1.5 < 2$

Feasible, $z = 5\times3+4\times1.5 =$ **\$21**

(c) $(x_1, x_2) = (2, 1)$

$(x_1, x_2) \geq 0$

$6\times2+4\times1 = 16 < 24$

$1\times2+2\times1 = 4 < 6$

$-1\times2+1\times1 = -1 < 1$

$1\times1 = 1 < 2$

Feasible, $z = 5\times2+4\times1 = \$14$

(c) $(x_1, x_2) = (2, -1)$

$x_2 < 0$ $\Rightarrow$ infeasible

3

$(x_1, x_2) = (2, 2)$: Let s_1 and s_2 be the unused daily amounts of M1 and M2.

For M1: $s_1 = 24 - (6x_1 + 4x_2) = 24 - 2\times10 = 4$ tons/day

For M2: $s_2 = 6 - (x_1 + 2x_2) = 6 - 2\times3 = 0$ ton/day

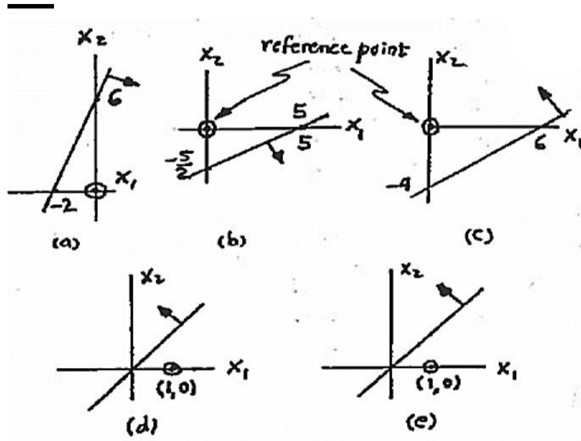
4

Quantity discount results in a nonlinear function:

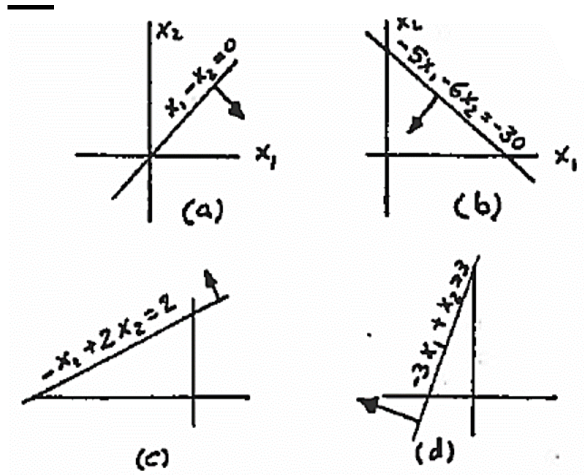
$$z = \begin{cases} 5x_1 + 4x_2, & 0 \leq x_1 \leq 2 \\ 4.5x_1 + 4x_2, & x_1 \geq 0 \end{cases}$$

The situation cannot be an LP. Mixed integer programming (Chapter 9) can handle this nonlinearity.

5



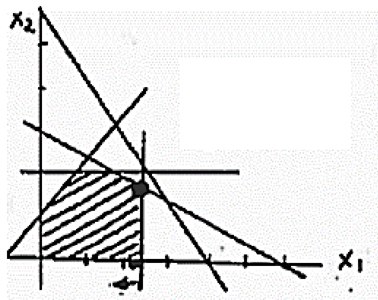
6



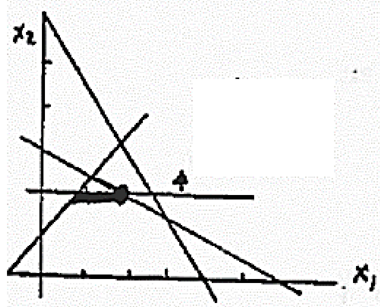
continue…

7

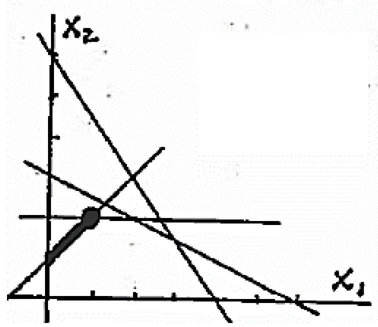
(a)
$x_1 \leq 2.5$
Optimum:
$x_1 = 2.5$
$x_2 = 1.75$
$z = \$13$



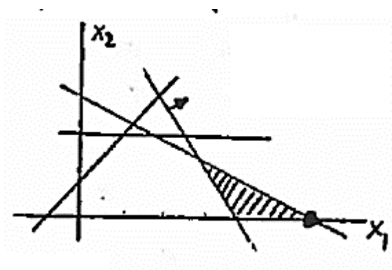
(b)
$x_2 \geq 2$
Optimum:
$x_1 = 2, x_2 = 2$
$z = \$18$



(c)
$-x_1 + x_2 = 1$
Optimum:
$x_1 = 1, x_2 = 2$
$z = \$13$



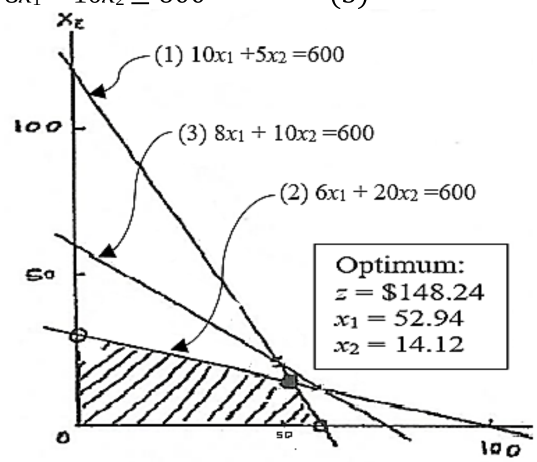
(d)
$6x_1 + 4x_2 \geq 24$
Optimum:
$x_1 = 6, x_2 = 0$
$z = \$30$



(e)
infeasible space

8

x_1=daily units of product 1
x_2=daily units of product 2
Max z = $2x_1 + 3x_2$ s.t.
$10x_1 + 5x_2 \leq 600$ (1)
$6x_1 + 20x_2 \leq 600$ (2)
$8x_1 + 10x_2 \leq 600$ (3)



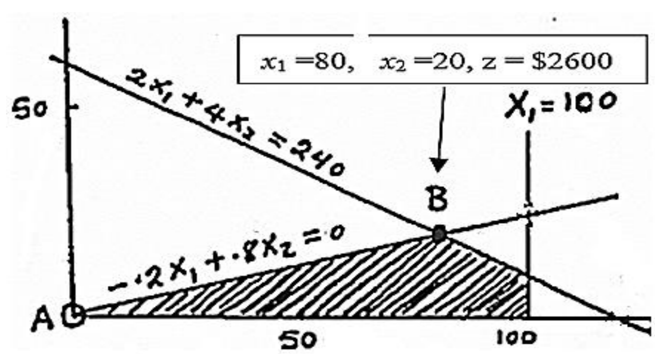
9

x_1= nbr. of units of A
x_2= nbr. of units of B
Max z = $20x_1 + 50x_2$ s.t. $x_1, x_2 \geq 0$

$$\frac{x_1}{x_1 + x_2} \geq .8 \quad \text{or} \quad -.2x_1 + .8x_2 \leq 0$$

$$x_1 \leq 100$$

$$2x_1 + 4x_2 \leq 240$$

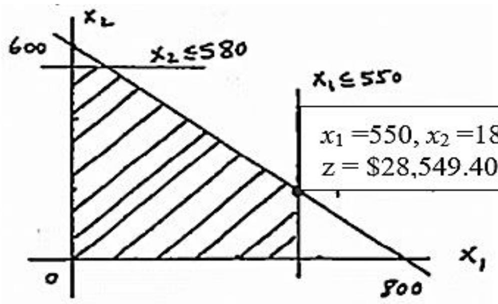


10

x_1=nbr. of sheets/day
x_2= nbr. of bars/day
Max z = $40x_1 + 35x_2$ s.t.

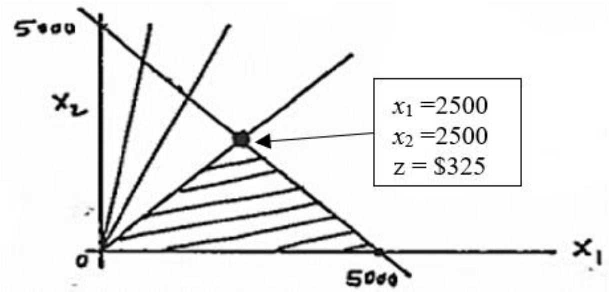
$\frac{x_1}{800} + \frac{x_2}{600} \leq 1$

$0 \leq x_1 \leq 550, 0 \leq x_2 \leq 580$



11

x_1=$ invested in A
x_2= $ invested in B
Max z = $.05x_1 + .08x_2$ s.t.
$x_1 \geq .25(x_1 + x_2)$
$x_2 \leq .5(x_1 + x_2)$
$x_1 \geq .5x_2$
$x_1 + x_2 \leq 5000$
$x_1, x_2 \geq 0$



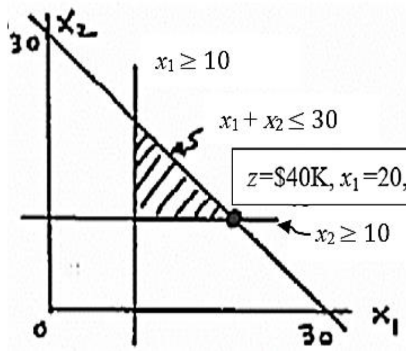
12

x_1= nbr of practical courses
x_2= nbr of humanistic courses
Max z = $1500x_1 + .1000x_2$ s.t.
$x_1 \geq .25(x_1 + x_2)$
$x_1 + x_2 \leq 30$
$x_1 \geq 10$,

continued…

12 cont'd

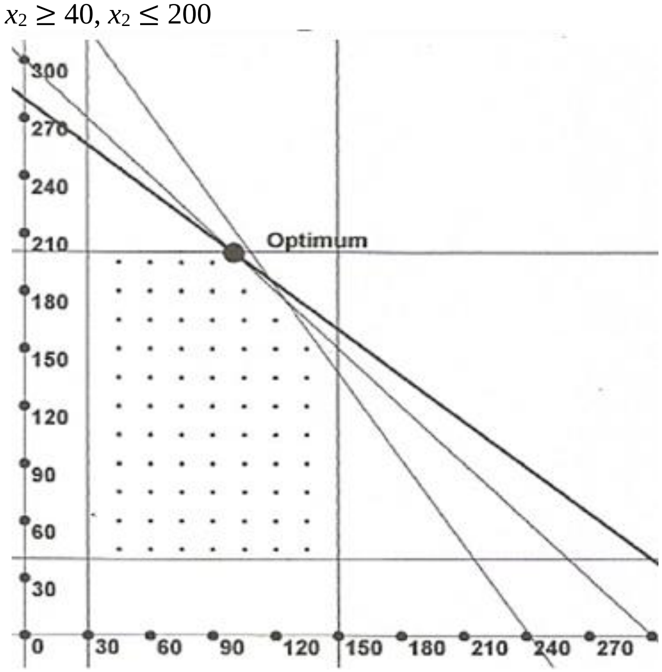
(a)



(b) Change $x_1 + x_2 \leq 30$ to $x_1 + x_2 \leq 31$
Optimum: z=$41,500, Δz=41500-40,000=$1500
Conclusion: any additional course will be 'practical'

13

x_1=units of solution A
x_2= units of solution B
Max z = $8x_1 + 10x_2$ s.t.
$.5x_1 + .5x_2 \leq 159$
$.6x_1 + .4x_2 \leq 145$
$x_1 \geq 30, x_1 \leq 150$
$x_2 \geq 40, x_2 \leq 200$



Optimum: z = 2800, x_1 = 100, x_2 = 200

continued…