Contents

Chapter 1

What is Operations Research?

1

Weeks 1-4: 2 weekend trips FYV-DEN-FYV and 2 weekend trips DEN-FYV-DEN. Week 5: 1 regular trip.
Cost: 4x320+1x400= $2200

2

(a) Given a fence of length L:
(1) $h = .3L$, $w = .2L$, Area $= .06L^2$
(2) $h = .1L$, $w = .4L$, Area $= .04L^2$
Solution (2) is better because the area is larger

(b) $L = 2(w + h)$, $w = L/2 - h$
$z = wh = h(L/2 - h) = Lh/2 - h^2$
$\delta z/\delta h = L/2 - 2h = 0$
Thus, $h = L/4$ and $w = L/4$.
Solution is optimal because z is a concave function

3

x = cumulative number of drops of balls #1 and #2 at any floor (problem unknown)
y_i = floor from which ith drop of ball #1 occurs.
Step 0: Set $y_0 = 0$, $y_1 = x$, and $i = 1$.
General step i: Drop ball#1 from floor yi. If it is dented, use ball#2 to check floors $y_{i-1} + 1$ to $y_i - 1$, in that order. Else, if #1 is not dented, set $i = i + 1$, and repeat step i.
Formula for determining y_i:
y_i must include the (cumulative) i #1-drops from floor y_1 to floor y_i. To maintain the same number of drops at any floor y_i, #2-drops cannot exceed $x - i$. Thus,

$$y_i = y_{i-1} + (x - i + 1)$$
$$= x + (x - 1) + (x - 2) + \ldots + (x - i + 1)$$
$$= ix - (1 + 2 + \ldots + i - 1) = ix - (i - 1)\, i/2$$

Maximum number of #1-drops is x (else $y_i \le y_{i-1}$ for $i > x$). Hence the highest floor from which #1 can be dropped is

$$y_x = x^2 - (x-1)x/2 = x^2 - x^2/2 + x/2 = (x^2 + x)/2$$

For a 100-storey building, $y_x \ge 100$, or $x^2 + x - 200 \ge 0$. The associated quadratic equation yields $x = 13.64$ and -14.64. The rounded positive value $x = 14$ is the smallest integer that satisfies the inequality.

4

(a) Let T = total time to move all four individuals to the other side of the river. The objective is to determine the transfer schedule that minimizes T.
(b) Let t = crossing time from one side to the other. Use codes 1, 2, 5, and 10 to represent Amy, Jim, John, and Kelly.

East	Crossing	West
5,10	(1,2)→ (t = 2)	1,2
1,5,10	(t = 1)←(1)	2
1	(5,10)→ (t = 10)	2,5,10
1,2	(t = 2)←(2)	5,10
none	(1,2)→ (t = 2)	1,2,5,10
Total = 2 + 1 + 10 + 2 + 2 = 17 minutes		

5

		Jim	
		Curve	Fast
Joe	Curve	.500	.200
	Fast	.100	.300

(a) Alternatives:
Jim: Throw curve of fast ball.
Joe: Prepare for curve or fast ball.
(b) Joe tries to improve his batting score and Jim tries to counter Joe's action by selecting a less favorable strategy. This means that neither player will be satisfied with a single (pure) strategy.

The problem is not an optimization situation in the familiar sense in which the objective is maximized or minimized. Instead, the conflicting situation requires a compromise solution in which neither player is tempted to change strategy. Game theory (Chapter 14) provides such a solution.

6

\L L Let L=ops. 1 and 2=20 sec, C=ops. 3 and 4=25 sec, U=op. 5=20 sec

Gant chart: L1=load horse 1, L2=load horse 2, etc.

one joist: 0---L1---**20**---C1---45----U1+L1---**85**----U2+L2----125---U1+L1---**165**---U2+L2---*205*

20-L2-40 45---C2----70 **85**---C1---110 125---C2---140

165-C1-190

205---C2---230---U2---250

Total = 250

Loaders utilization=[250-(5+25)]/250=88%

Cutter utilization=[250-(20+15+15+15+15)]/250=68%

two joists: 0---2L1---**40**-----2C1-----**90**----2(U1+L1)---**170**----2C1----220---2U1---260

40---2L2---80 **90**---2C2----140 **170**---2U2---210

Total =260

Loaders utilization=[260-(10+10)]/260=92%

Cutter utilization=[260-(40+30+40)]/250=58%

three joists: 0---3L1---**60**------3C1------**135**------3C2---------210----3U2----270

60---3L2---120 **135**-----3U1-----195

Total =270

Loaders utilization=[270-(15+15)]/270=89%

Cutter utilization=[270-(60+60)]/270=56%

Recommendation: One joist at time gives the smallest time. The problem has other alternatives that combine 1, 2, and 3 joists. Cutter utilization indicates that cutter represents the bottleneck.

7

Note that all 'dots' are indistinguishable even if they are designated as 1, 2, 3, …, 10.

(a) Alternative 1: Move dots 5, 6, and 7 below bottom row, move dots 8 and 9 below new 5, 6, and 7. Move 10 to the bottom. Number of moves = 6. Alternative 2: See part (b).

(b) Three moves: Move dot 1 up to the left of dot 8, dot 4 to the right of dot 9, and dot 10 below dots 2 and 3.

```
      10
    8    9
  5    6    7
1    2    3    4
```

8

(a) Alternative 1: Break one end link of each chain and connect to another chain. Four breaks and re-solders, cost = 4 x (2 + 3) =20 cents. Alternative 2: See Part (b)

(b) Break three links in one chain and use them to connect the remaining three chains: Three breaks and re-solder, cost = 3 x (2 + 3) = 15 cents.

9

Represent the selected 2-digit number as $10x+y$. The corresponding square number is $10x+y-(x+y)=9x$. This means that the selected square will always be 9, 18, 27, ..., or 81. By assigning zero dollars to these squares, the reward is always zero regardless of the rewards assigned to the remaining squares or the number of times the game is repeated.

10

Assign a sequential number x to each cartons, $x \varepsilon X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

(a) Let Y be the set of cartons so far weighed (initially $Y = \emptyset$).

General Step: Randomly select a carton $y \varepsilon X-Y$. If y weighs 90 oz, stop. Else, augment y to Y and repeat the General Step. $1 \leq$ number of times scale is used ≤ 10.

(b) Exactly once! Take x bottles from carton $x \varepsilon X$ to end up with $(1+2+\ldots+10) = (10+11)/2=55$ bottles. Weigh the 55 bottles. If the weight $= 550 - x$, carton x is the defective one.

CHAPTER 2

Modeling with Linear Programming

1

(a) $x_2 - x_1 \geq 1$ or $-x_1 + x_2 \geq 1$

(b) $x_1 + 2x_2 \geq 3$ and $x_1 + 2x_2 \leq 6$

(c) $x_2 \geq x_1$ or $x_1 - x_2 \leq 0$

(d) $x_1 + x_2 \geq 3$

(e) $\frac{x_2}{x_1 + x_2} \leq .5$ or $.5x_1 - .5x_2 \geq 0$

2

(a) $(x_1, x_2) = (1, 4)$

$(x_1, x_2) \geq 0$

$6 \times 1 + 4 \times 4 = 22 < 24$

$1 \times 1 + 2 \times 4 = 9 \not\leq 6$ infeasible

(b) $(x_1, x_2) = (2, 2)$

$(x_1, x_2) \geq 0$

$6 \times 2 + 4 \times 2 = 20 < 24$

$1 \times 2 + 2 \times 2 = 6 = 6$

$-1 \times 2 + 1 \times 2 = 0 < 1$

$1 \times 2 = 2 = 2$

feasible

$Z = 5 \times 2 + 4 \times 2 = \$18$

(c) $(x_1, x_2) = (3, 1.5)$

$x_1, x_2 \geq 0$

$6 \times 3 + 4 \times 1.5 = 24 = 24$

$1 \times 3 + 2 \times 1.5 = 6 = 6$

$-1 \times 3 + 1 \times 1.5 = -1.5 < 1$

$1 \times 1.5 = 1.5 < 2$

feasible

$Z = 5 \times 3 + 4 \times 1.5 = \$21$

(d) $(x_1, x_2) = (2, 1)$

$x_1, x_2 \geq 0$

$6 \times 2 + 4 \times 1 = 16 < 24$

$1 \times 2 + 2 \times 1 = 4 < 6$

$-1 \times 2 + 1 \times 1 = -1 < 1$

$1 \times 1 = 1 < 2$

feasible

$Z = 5 \times 2 + 4 \times 1 = \$14$

(e) $(x_1, x_2) = (2, -1)$

$x_1 \geq 0$, $x_2 < 0$, infeasible

Conclusion: (c) gives the best feasible solution

3

$(x_1, x_2) = (2, 2)$

Let S_1 and S_2 be the unused daily amounts of M1 and M2.

For M1: $S_1 = 24 - (6x_1 + 4x_2) = 4$ tons/day

For M2: $S_2 = 6 - (x_1 + 2x_2)$

$= 6 - (2 + 2 \times 2) = 0$ tons/day

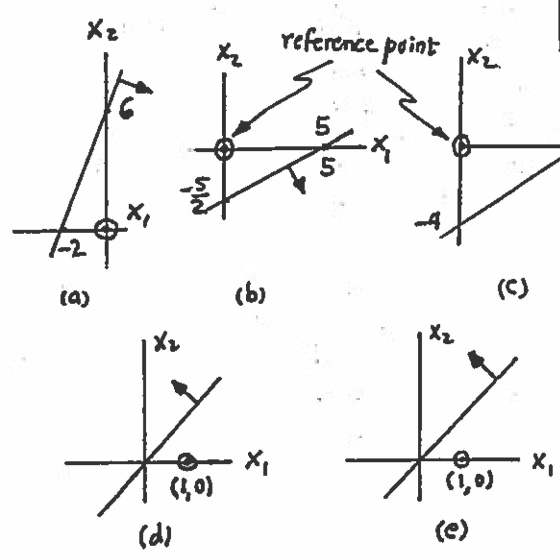
4

Quantity discount results in the following nonlinear objective function:

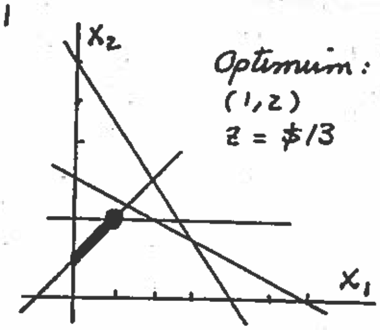
$$Z = \begin{cases} 5x_1 + 4x_2, & 0 \leq x_1 \leq 2 \\ 4.5x_1 + 4x_2, & x_1 > 2 \end{cases}$$

The situation cannot be treated as a linear program. Nonlinearity can be accounted for in this case using mixed integer programming (chapter 9).

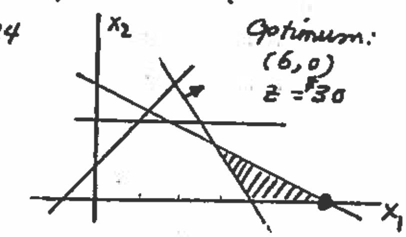
5



(c) $-x_1 + x_2 = 1$

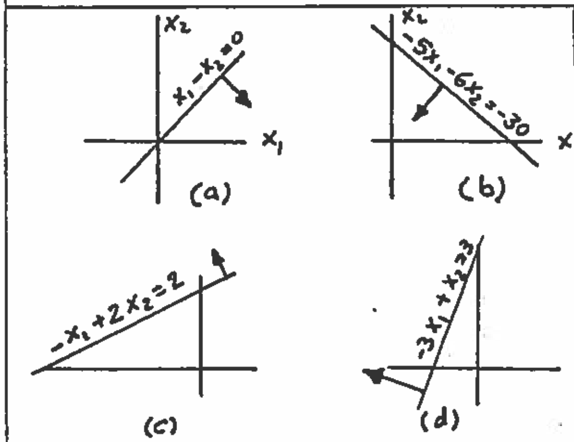


(d) $6x_1 + 4x_2 \geq 24$



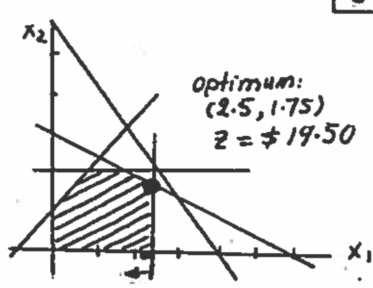
(e) No feasible space

6

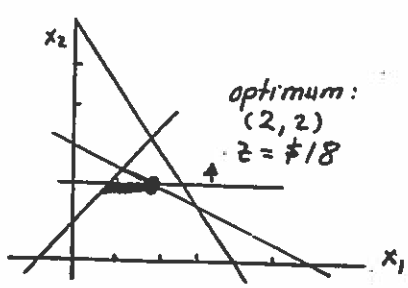


7

(a) $x_1 \leq 2.5$
$x_2 \leq 1.9$



(b) $x_2 \geq 2$



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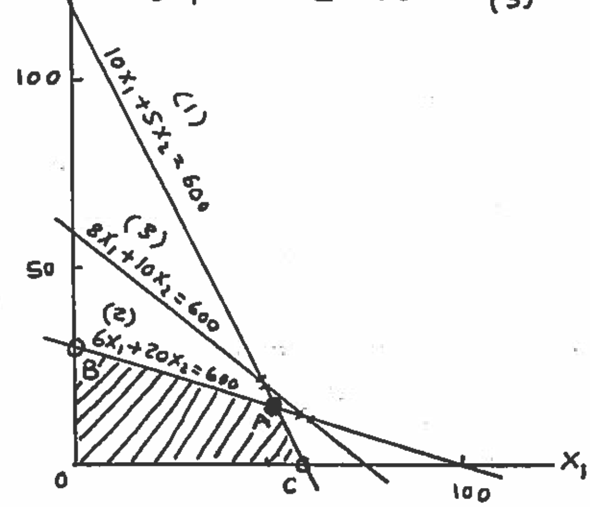
8

x_1 = daily units of product 1
x_2 = daily units of product 2

Maximize $z = 2x_1 + 3x_2$

s.t.

$$10x_1 + 5x_2 \leq 600 \quad (1)$$
$$6x_1 + 20x_2 \leq 600 \quad (2)$$
$$8x_1 + 10x_2 \leq 600 \quad (3)$$



Optimum occurs at A:
$x_1 = 52.94$
$x_2 = 14.12$
$z = \$148.24$

9

x_1 = number of units of A
x_2 = number of units of B

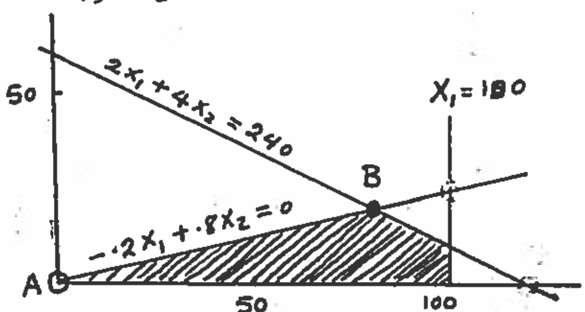
Maximize $z = 20x_1 + 50x_2$

$\frac{x_1}{x_1+x_2} \geq .8$ or $-.2x_1 + .8x_2 \leq 0$

$x_1 \leq 100$

$2x_1 + 4x_2 \leq 240$

$x_1, x_2 \geq 0$



Optimal occurs at B:
$x_1 = 80$ units
$x_2 = 20$ units
$z = \$2,600$

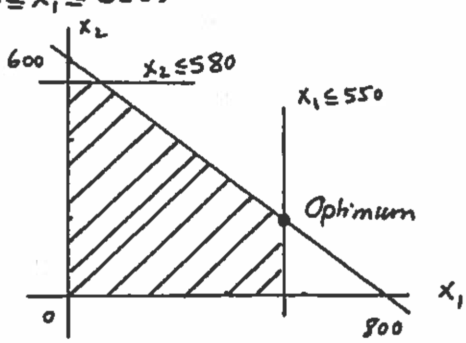
10

x_1 = number of sheets/day
x_2 = number of bars/day

Maximize $z = 40x_1 + 35x_2$

s.t. $\frac{x_1}{800} + \frac{x_2}{600} \leq 1$

$0 \leq x_1 \leq 550,\quad 0 \leq x_2 \leq 580$



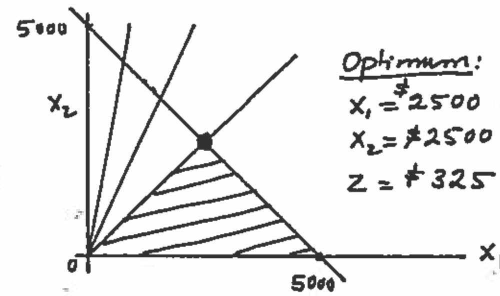
Optimum solution:
$x_1 = 550$ sheets
$x_2 = 187.13$ bars
$z = \$28,549.40$

11

x_1 = \$ invested in A
x_2 = \$ invested in B

Maximize $z = .05x_1 + .08x_2$

s.t.
$x_1 \geq .25(x_1 + x_2)$
$x_2 \leq .5(x_1 + x_2)$
$x_1 \geq .5x_2$
$x_1 + x_2 \leq 5000$
$x_1, x_2 \geq 0$



Optimum:
$x_1 = \$2500$
$x_2 = \$2500$
$z = \$325$

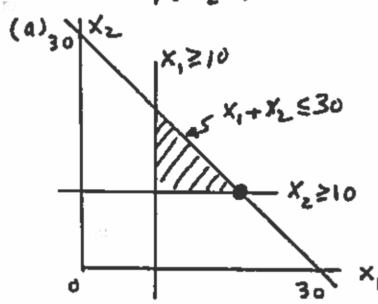
12

x_1 = number of practical courses
x_2 = number of humanistic courses

Maximize $z = 1500x_1 + 1000x_2$

s.t.
$x_1 + x_2 \leq 30$
$x_1 \geq 10$
$x_2 \geq 10$
$x_1, x_2 \geq 0$

(a)



Optimum:
$x_1 = 20$
$x_2 = 10$
$z = \$40,000$

(b) Change $x_1 + x_2 \leq 30$ to $x_1 + x_2 \leq 31$

Optimum $z = \$41,500$

$\Delta z = \$41,500 - 40,000 = \$1500$

Conclusion: Any additional course will be of the practical type.

13

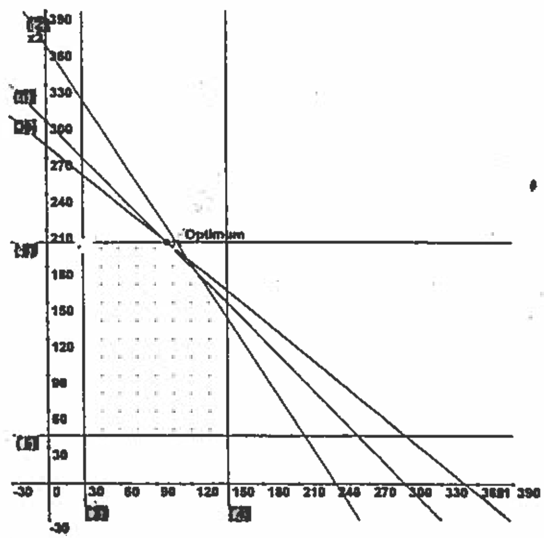
x_1 = units of solution A
x_2 = units of solution B

Maximize $z = 8x_1 + 10x_2$

Subject to

$$.5x_1 + .5x_2 \le 150$$
$$.6x_1 + .4x_2 \le 145$$
$$x_1 \ge 30$$
$$x_1 \le 150$$
$$x_2 \ge 40$$
$$x_2 \le 200$$
$$x_1, x_2 \ge 0$$

Summary of Optimal Solution:
Objective Value = 2800.00
x1 = 100.00
x2 = 200.00



14

x_1 = nbr. of grano boxes
x_2 = nbr. of wheatie boxes

Maximize $Z = x_1 + 1.35x_2$

s.t. $.2x_1 + .4x_2 \le 60$
$$x_1 \le 200$$
$$x_2 \le 120$$
$$x_1, x_2 \ge 0$$

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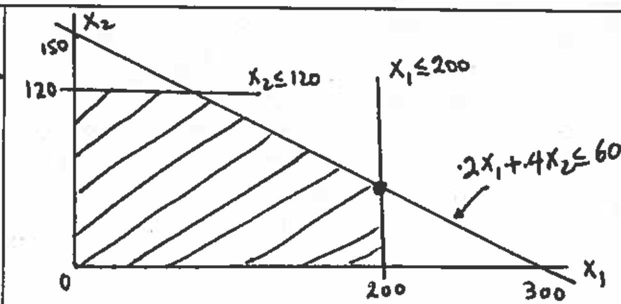


Optimum: $x_1 = 200$, $x_2 = 50$, $z = \$267.50$

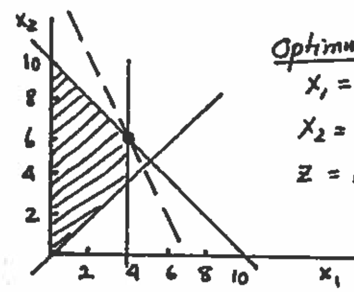
Area allocation: 67% grano, 33% wheatie

15

x_1 = play hours per day
x_2 = work hours per day

Maximize $z = 2x_1 + x_2$

s.t.
$$x_1 + x_2 \le 10$$
$$x_1 - x_2 \le 0$$
$$x_1 \le 4$$
$$x_1, x_2 \ge 0$$



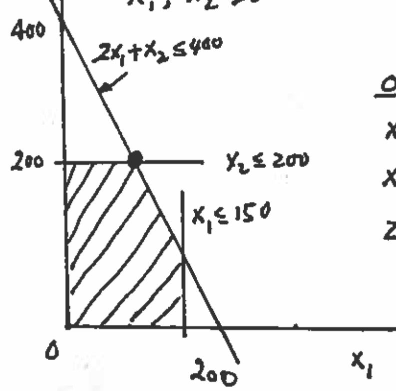
Optimum solution:
$x_1 = 4$ hours
$x_2 = 6$ hours
$z = 14$ "pleasurits"

16

x_1 = Daily nbr. of type 1 hat
x_2 = Daily nbr. of type 2 hat

Maximize $Z = 8x_1 + 5x_2$

s.t. $2x_1 + x_2 \le 400$
$$x_1 \le 150$$
$$x_2 \le 200$$
$$x_1, x_2 \ge 0$$



Optimum:
$x_1 = 100$ type 1
$x_2 = 200$ type 2
$Z = \$1800$

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