Chapter 1

1.1 Calculate the electron and hole concentrations in p-type silicon with $N_A = 5\times10^{17}$ cm^{-3} at $T = 280$ K, 300 K, and 330 K.

From the footnote on page 4 $n_i = 7\times10^{15}T^{3/2}\exp(-6600/T)$, and for p-type material we have from (1.2.6) and (1.2.7) that to a good approximation $p_o = N_A$ and $n_o = n_i^2/N_A$. Applying these gives

T (K)	n_i (cm^{-3})	n_o (cm^{-3})	p_o (cm^{-3})
280	1.901×10^{9}	7.225	5.000×10^{17}
300	1.015×10^{10}	2.059×10^{2}	5.000×10^{17}
330	8.649×10^{10}	1.496×10^{4}	5.000×10^{17}

Note that the hole concentration p_o is independent of temperature, but the intrinsic concentration n_i, and therefore the electron concentration n_o, vary strongly with temperature. Note also that the above are quoted to 4 digits of precision, however this is somewhat misleading as the constants for the intrinsic concentration are only provided to 1 and 2 digits of precision.

1.2 This problem is intended to give an idea of the order of magnitude involved for the various quantities discussed in Sec. 1.3. Consider the bar of Fig. 1.12 with $a = 100$ μm, $b = 10$ μm, $c = 2$ μm, and n-type silicon doping concentration of 10^{15} cm^{-3}; $V = 1$ V. Find the value of the conductivity, the mobility, the conductance, the sheet resistance, the total mobile charge, the mobile charge per unit area, the field intensity, the drift velocity, the transit time, and the current.

The magnitude of the electric field ("field intensity") is $E = V/a = 1/100 = 0.01$ V/μm, which is sufficiently small to justify the use of the low field approximation. From Fig. 1.13 the bulk mobility of 10^{15} cm^{-3} n-type silicon is $\mu_B \approx 1350$ cm^2/(V·s), the mobile charge concentration per unit volume n is from (1.2.4) just the donor concentration $N_D = 10^{15}$ cm^{-3}. The other required quantities are (using $q = 1.602\times10^{-19}$ C):

quantity	equation	formula	value	unit
conductivity	(1.3.14b)	$\sigma = \mu_B nq$	0.216	1/(Ω·cm)
conductance	(1.3.14a)	$G = \sigma(bc/a)$	4.33×10^{-6}	1/Ω
sheet resistance		$Rs = R(b/a) = 1/(\mu_B nqc)$	2.31×10^{4}	Ω/□
total mobile charge	(1.3.4)	$Q = -nq(abc)$	-3.20×10^{-13}	C
mobile charge density	(1.3.6)	$Q' = Q/(ab) = -nqc$	-3.20×10^{-16}	C/μm^{-2}
drift velocity	(1.3.8)	$v_d = \mu_B E$	1.35	μm/ns
transit time	(1.3.3)	$\tau = a/v_d$	74.1	ns
current	(1.3.1), (1.3.13)	$I = \lvert Q\rvert/\tau = GV$	4.33	μA

1.3 Prove that (1.3.23) is valid for the case of Fig. 1.15*c*.

Letting x and y be the directions associated with the dimensions a and b, respectively, with origin at the top left. The electron concentration varies with position x but is constant with respect to y and through the depth of the bar. The total charge is then the integral of the charge per unit area (the charge density) over the top area of the bar,

$$Q = \int_0^b \int_0^a Q' \, dx\, dy = b \int_0^a Q' \, dx \, .$$

Because the charge density is assumed to vary linearly with x the integral is just the area of the trapezoid in Fig. 1.15c, so

$$Q = ab \frac{Q'(0) + Q'(a)}{2} \, .$$

1.4 (a) Prove mathematically that (1.3.10) and (1.3.15) are valid even if the electron concentration varies with depth as long as it is uniform horizontally.
(b) Prove mathematically that (1.3.20) and (1.3.22) are valid for the conditions stated in the paragraph following (1.3.20).

(a) The solution is actually outlined in the footnote to the paragraph that follows (1.3.10). If y is position in the vertical direction, measured from the top surface, consider a thin horizontal slice of thickness Δy centered about y, within which $n(y)$ is considered to be constant. From (1.3.5b), (1.3.8) and (1.3.9) we have

$$\Delta I = \mu_B \frac{b}{a} V \cdot \left(qn(y) \Delta y \right)$$

for the current flowing through this slice, assuming that the flow is laminar and all electrons move horizontally. Allowing the differences to become differentials, i.e. taking the limit as $\Delta y \to 0$, and integrating from y=0 to y=c gives

$$I = \mu_B \frac{b}{a} V \cdot \int_0^c qn(y) dy$$

but the magnitude of the charge per unit area $|Q'|$ is just $\int_0^c qn(y) dy$ and substituting this into the above equation gives (1.3.10) directly. From (1.3.13) $G = I/V$ so (1.3.15) also holds in the case of nonuniform vertical doping.

(b) The solution for this is also outlined in footnote related to the paragraph following (1.3.20). Taking the same slice as in part (a), (1.3.20) still holds for that slice so we have

$$\Delta I = \mu_B \phi_t b \cdot \frac{d\Delta Q'}{dx}$$

where $\Delta Q' = qn(y)\Delta y$. In the limit as $\Delta y \to 0$, integrating from y=0 to y=c this gives

$$I = \mu_B \phi_t b \cdot \frac{d}{dx} \int_0^c qn(y) dy = \mu_B \phi_t b \cdot \frac{dQ'}{dx} \, .$$

For each of the slices (1.3.21) still holds, so using this in (1.3.20) and integrating over the thickness of the slab still therefore leads to (1.3.22).

1.5 Verify (1.2.19).

From (1.2.15a) $p_1 = n_i e^{(E_{i1}-E_F)/(kT)}$ and $p_2 = n_i e^{(E_{i2}-E_F)/(kT)}$ where the Fermi level is, by definition, constant in equilibrium. Therefore

$$\frac{p_1}{p_2} = e^{(E_{i1}-E_{i2})/(kT)}$$

The band bending between points 1 and 2 is the same for the conduction, intrinsic, and valence band levels. The relation (1.2.17) is still valid, as although specified for electrons it relates potential difference to band energy difference, therefore $\psi_{21} = \psi_2 - \psi_1 = (E_{i1} - E_{i2})/q$. Using this relation and (1.2.12) we therefore have

$$\frac{p_1}{p_2} = e^{\psi_{21}/\phi_t}$$

which is just (1.2.19).

1.6 Calculate the contact potential of copper to *n*-type silicon with $N_D = 10^{17}$ cm^{-3} at 300 K, assuming that the work function potential for copper is 4.5 V.

The solution follows the procedure of Example 1.1. For *n*-type silicon from (1.2.14b) and the formula for n_i in the footnote on page 4, $\phi_F = -\phi_t \ln(N_D/n_i) = -0.0259 \ln(10^{17}/1.015\times10^{10}) = -0.416$ V, therefore from (1.4.1)

$$\phi_{Cu,S} = \phi_{W,S} - \phi_{W,Cu} = \chi + \frac{E_g}{2q} + \phi_F - \phi_{W,Cu} = 4.05 + 0.56 - 0.416 - 4.5 = -0.306 \text{ V}$$

1.7 A voltmeter with both of its leads made out of metal *X* measures a voltage *V* across a battery. Show that if one of the voltmeter's leads is replaced with a different material *Y*, the electrostatic potential across the voltmeter will not be affected.

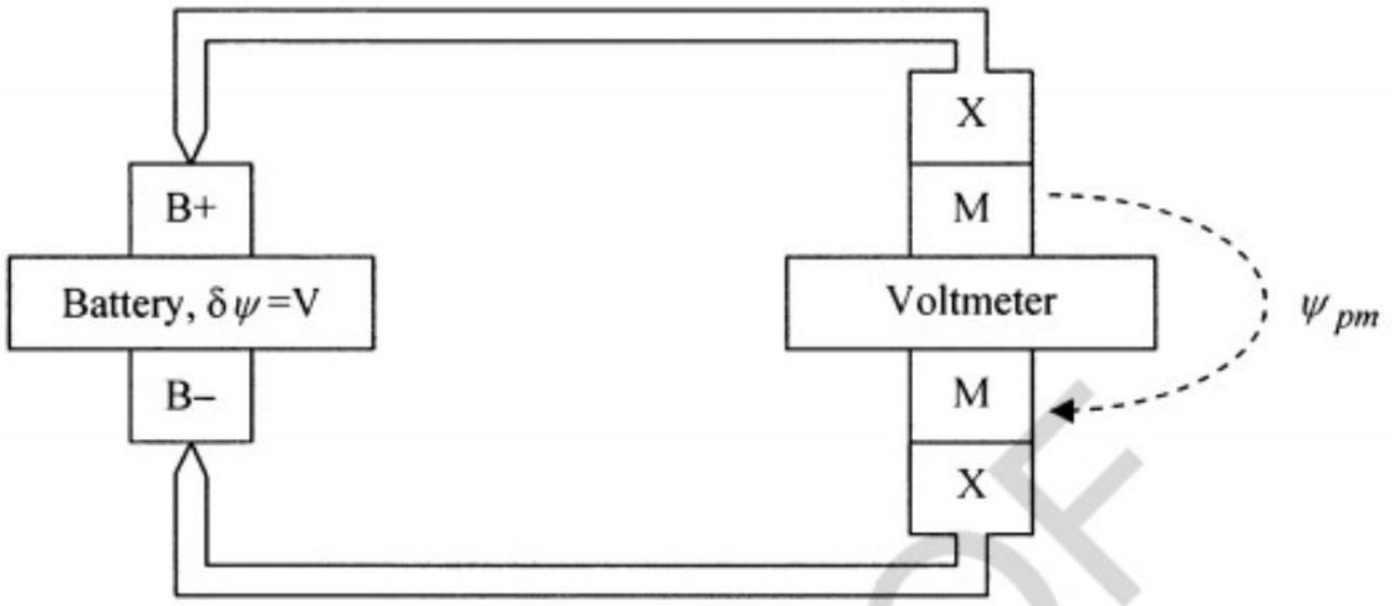
Assume the battery voltage (potential difference) is *V* and the battery is made of material type *B*, that the voltmeter leads are directly connected to the battery, and that the voltmeter is made out of material type *M*. For common metal *X* leads for the voltmeter, the potential difference ψ_{pm} sensed between the plus and minus terminals of the voltmeter is

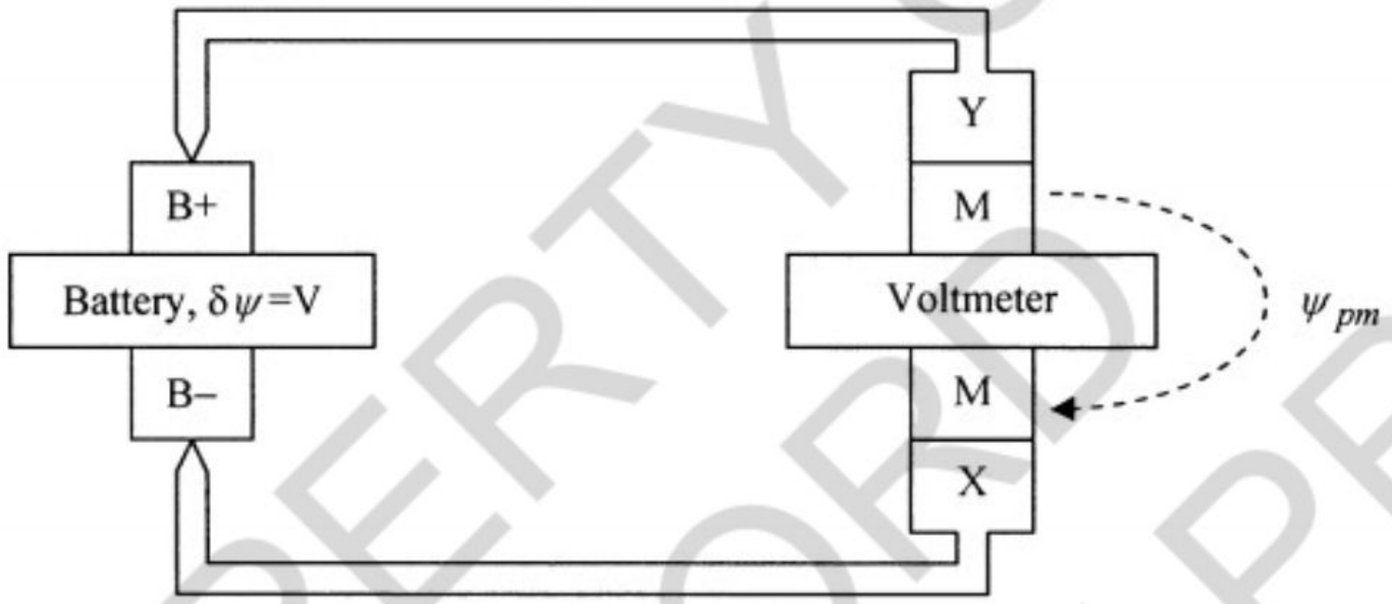
$$\psi_{pm} = \phi_{M,X} + \phi_{X,B} + V + \phi_{B,X} + \phi_{X,M} = V$$

Inserting the material *Y* makes the loop potential equation become

$$\begin{aligned}\psi_{pm} &= \phi_{M,X} + \phi_{X,B} + V + \phi_{B,Y} + \phi_{Y,M} \\ &= \phi_{M,X} + \phi_{X,B} + V + \phi_{B,Y} + \phi_{Y,X} + \phi_{X,Y} + \phi_{Y,M} \\ &= \phi_{M,X} + \phi_{X,B} + V + \phi_{B,X} + \phi_{X,M} = V\end{aligned}$$

hence the voltage sensed by the meter is independent of the material out of which its leads are made. Inserting some intermediate material in any loop makes no difference in the potential balance around the loop, the contact potentials between the material types at each end of the inserted material cancel, as in (1.4.4). The contact potential $\phi_{M,B}$ between the meter and the battery is independent of how many material system changes there are between the meter and the battery.





1.8 Assume that the two terminals of a voltage source are made out of different materials. Show that (1.4.9) and (1.4.10) are valid if V_{source} is defined as the voltage measured by an ideal voltmeter when it is attached to the terminals of the source.

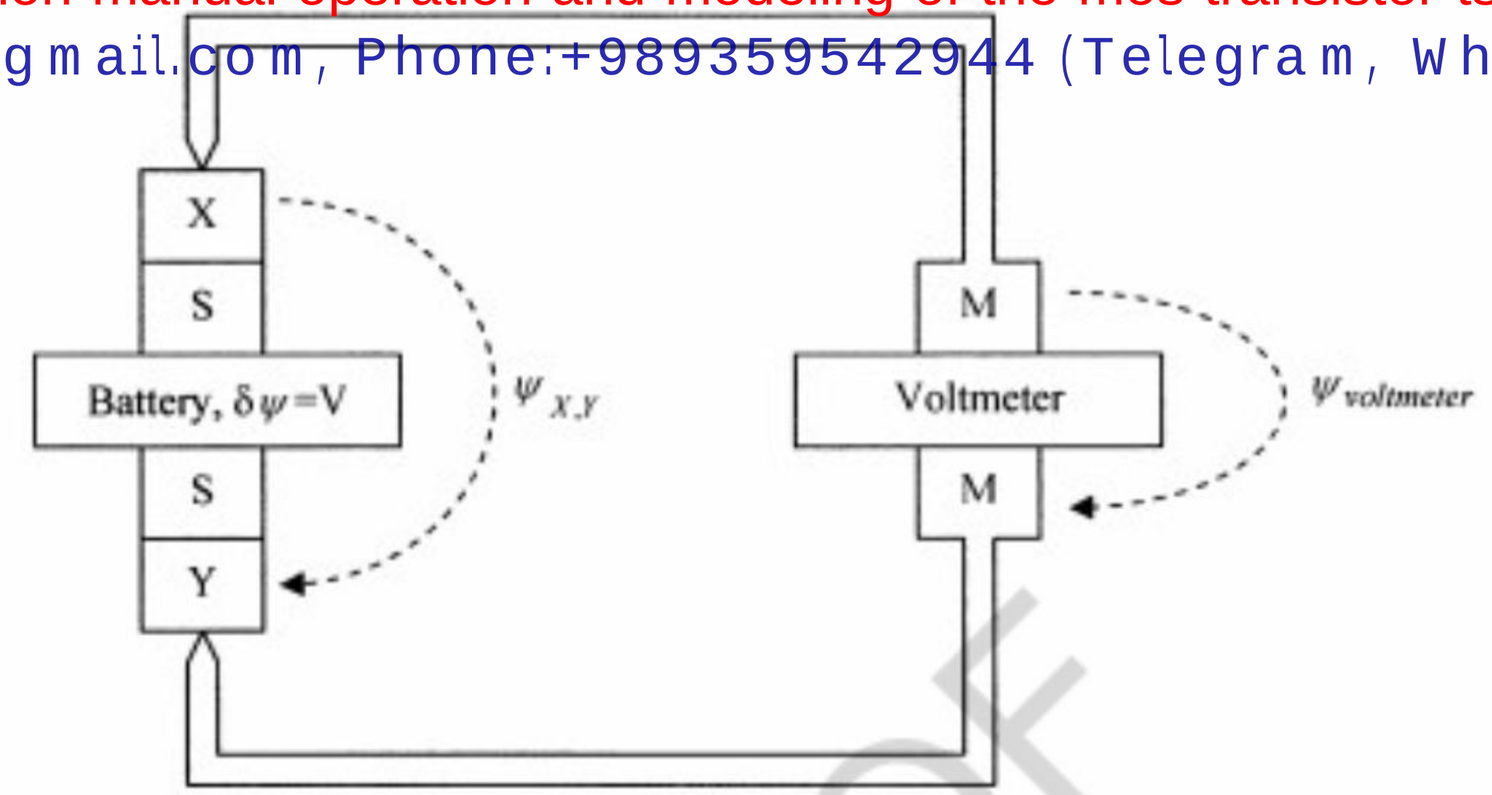
As in Prob. 1.7, insertion of a different material type does not affect the potential measured by a voltmeter as the work function of an inserted material gets added when calculating the contact potential at one end and subtracted when calculating the contact potential at the other end, and hence cancels when all of the contact potentials are summed. If the leads of the source are materials X and Y and S denotes the material of the source, then the potential difference between these is

$$\psi_{X,Y} = \phi_{X,S} + V_{source} + \phi_{S,Y} = V_{source} + (\phi_X - \phi_Y)$$

which is just (1.4.9). If the voltmeter leads are made of material M then the potential measured by the voltmeter is

$$\psi_{voltmeter} = \phi_{M,X} + \phi_{X,S} + V_{source} + \phi_{S,Y} + \phi_{Y,M} = V_{source}$$

which is (1.4.10).



1.9 For a two-sided step *pn* junction with neither of its sides degenerate, prove that the built-in potential ϕ_{bi} is given by $\phi_t \ln(N_A N_D / n_i^2)$, and that the total length of the depletion region and the charge per unit area are given by the formulas provided in the footnote to (1.5.11).

From (1.5.1), introducing (1.2.14a) and (1.2.14b), we have

$$\phi_{bi} = \phi_{Fp} - \phi_{Fn} = \phi_t \ln(N_A/n_i) + \phi_t \ln(N_D/n_i) = \phi_t \ln(N_A N_D/n_i^2)$$

Note this only holds for temperatures where the materials remain extrinsic, so $N_A >> n_i$ and $N_D >> n_i$.

Substituting (1.5.9) and (1.5.10) into (1.5.11) gives

$$\phi_1 + \phi_2 = \frac{q}{2\varepsilon_s}\left(N_D d_1^2 + N_A d_2^2\right) = \psi_c$$

The charge neutrality condition (1.5.6) gives the condition (1.5.7) which can be rearranged to form

$$d_1 = \frac{N_A}{N_D} d_2$$

and substituting this into the previous equation and rearranging gives

$$(N_A d_2)^2 \frac{N_A + N_D}{N_A N_D} = \frac{2\varepsilon_s}{q}\psi_c$$

from which

$$d_2 = \frac{1}{N_A}\sqrt{\frac{2\varepsilon_s}{q}\frac{N_A N_D}{N_A + N_D}\psi_c}$$

and therefore from (1.5.7)

$$d_1 = \frac{1}{N_D}\sqrt{\frac{2\varepsilon_s}{q}\frac{N_A N_D}{N_A + N_D}\psi_c}$$

Summing these individual depletion region distances gives the total length

$$d_1 + d_2 = \sqrt{\frac{2\varepsilon_s}{q}\frac{N_A N_D}{N_A + N_D}\psi_c}\left(\frac{1}{N_D} + \frac{1}{N_A}\right) = \sqrt{\frac{2\varepsilon_s}{q}\frac{N_A N_D}{N_A + N_D}\psi_c}\left(\frac{N_A + N_D}{N_A N_D}\right) = \sqrt{\frac{2\varepsilon_s}{q}\frac{N_A + N_D}{N_A N_D}\psi_c}$$

which is the result given in the footnote. The charge per unit area on the p side follows from (1.5.5) as

$$Q_2' = \frac{Q_2}{A} = -qN_A d_2 = -\sqrt{2q\varepsilon_s \frac{N_A N_D}{N_A + N_D}\psi_c}$$

where we have used the value for d_2 calculated above.

1.10 Plot the junction capacitance vs. reverse-bias voltage (from 0 to 2 V) for a silicon n^+p junction of area 200 μm^2 with $N_A = 5\times10^{17}$ cm^{-3}. Assume $\phi_{Fn} = -0.56$ V.

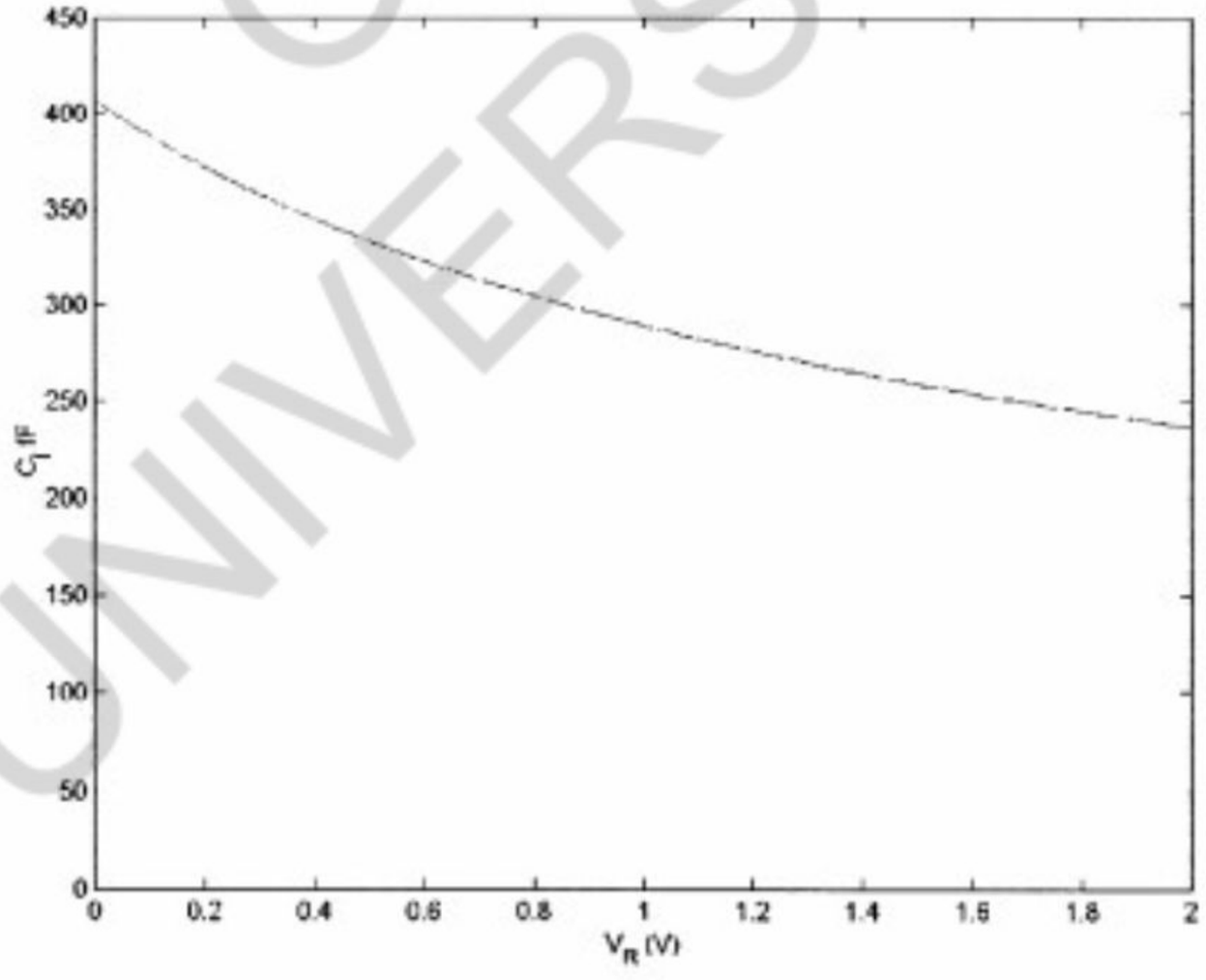
Because the n side of the junction is heavily doped we can use the one-sided step junction expressions for reverse bias from Sec. 1.5. The built-in potential is, from (1.5.1) and (1.2.14a), calculating n_i using the formula in the footnote on page 4 and assuming a temperature of 300 K,

$$\phi_{bi} = \phi_{Fp} - \phi_{Fn} = \phi_t \ln(N_A/n_i) + 0.56 = 0.02586 \ln(5\times10^{17}/1.015\times10^{10}) = 1.02 \text{ V}$$

The capacitance per unit area is given by (1.5.24), and multiplying this by the area gives

$$C = \frac{A\sqrt{2q\varepsilon_s N_A}}{2\sqrt{V_R + \phi_{bi}}} = \frac{4.11\times10^{-13}}{\sqrt{V_R + \phi_{bi}}} \text{ F}$$

This is plotted below.



1.11 Consider a semiconductor in equilibrium. Express the fact that the *total* current (drift plus diffusion components) must be zero, by using (1.3.10'), (1.3.6) and (1.3.17) Applying (1.2.18), show that the Einstein relation (1.3.18) results.

Using the differential form of (1.3.10), which is presented in a footnote to that equation, and substituting from (1.3.6), gives

$$I_{\text{drift}} = \mu_B q n(bc)\frac{d\psi}{dx}$$

The diffusion component of current is, from (1.3.17),

$$I_{\text{diffusion}} = -Dq(bc)\frac{dn}{dx}$$

therefore the total current being zero gives the relation

$$I_{\text{total}} = I_{\text{drift}} + I_{\text{diffusion}} = \mu_B q n(bc)\frac{d\psi}{dx} - Dq(bc)\frac{dn}{dx} = q(bc)\left(\mu_B n\frac{d\psi}{dx} - D\frac{dn}{dx}\right) = 0$$

Given that the Fermi energy is constant in equilibrium, differentiating (1.2.15b) w.r.t. position gives

$$\frac{dn}{dx} = -\frac{n}{kT}\frac{dE_i}{dx}$$

Because the band bending in Fig. 1.8 must be the same for the conduction, valence, and intrinsic energy levels, the change in E_i is related to the change in potential ψ through (1.2.17), therefore, recalling (1.2.12),

$$\frac{dn}{dx} = \frac{nq}{kT}\frac{d\psi}{dx} = \frac{n}{\phi_t}\frac{d\psi}{dx}.$$

Alternatively, from (1.2.18) it is apparent that n is proportional to $e^{\psi(x)/\phi_t}$ and differentiating w.r.t. position also gives the same result. Substituting this into the above expression for total current gives

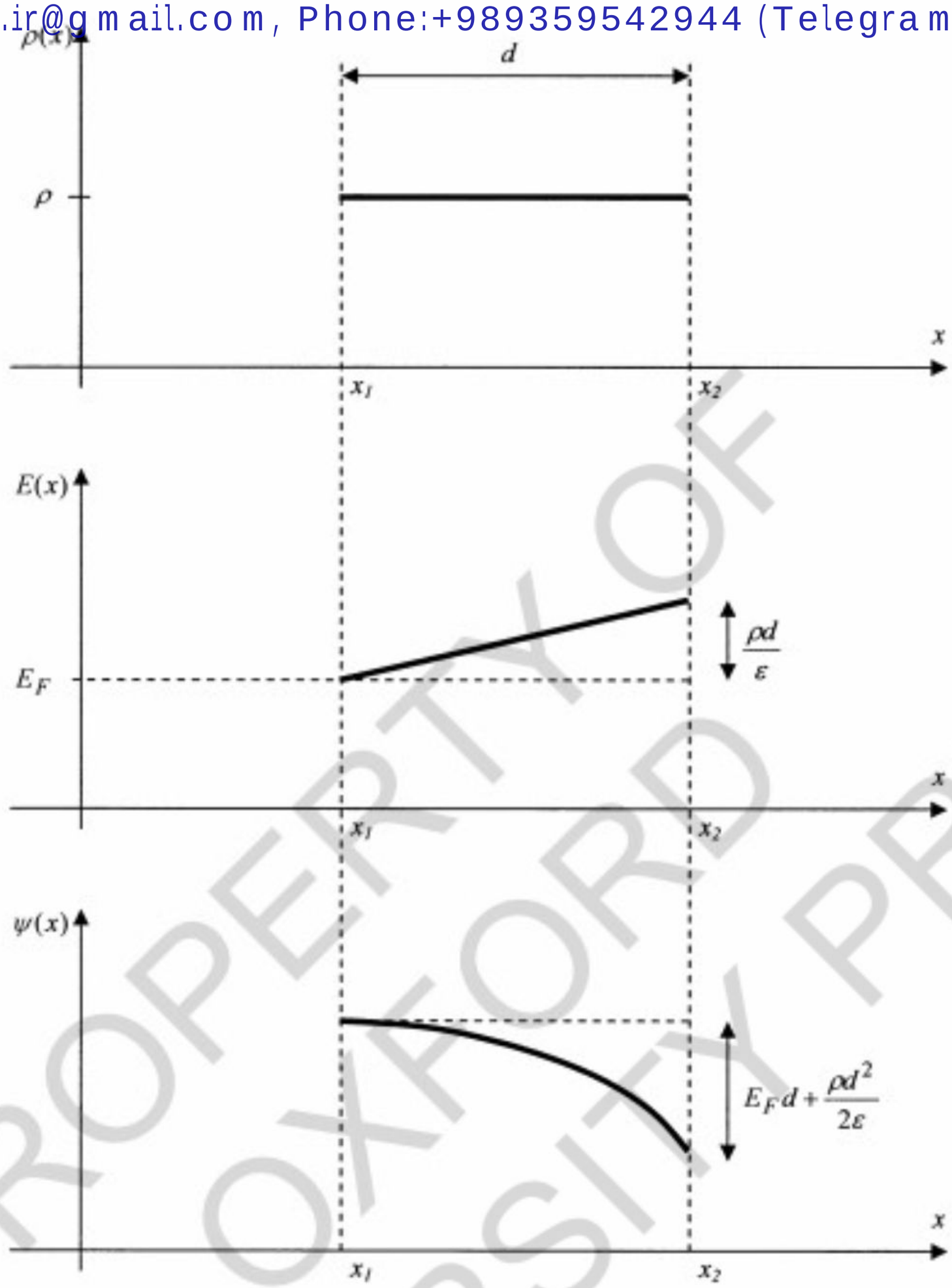
$$\left(\mu_B\frac{d\psi}{dx} - \frac{D}{\phi_t}\frac{d\psi}{dx}\right) = 0$$

and for this to be zero for all values of electric field $-d\psi/dx$ requires that $D = \phi_t\mu_B$, which is just the Einstein relationship (1.3.18).

1.12 Study the material on basic laws of electrostatics in Appendix A and provide detailed derivations for results (A.4), (A.6), and (A.7) given there.

Consider the region of uniform charge density as shown in Fig. A.3. From (A.1) and (A.2) it is apparent that for any region of uniform doping (and constant permittivity) the electric field E must vary linearly with position and the potential ψ must vary quadratically with position.

Denoting here E as the electric field, then applying the results in Appendix A to the situation shown in the figures on the next page, for which the charge density per unit volume is constant and of value ρ



$$E(x)=\frac{1}{\varepsilon}\int_{x_1}^{x}\rho\,dx+E(x_1)=\frac{\rho(x-x_1)}{\varepsilon}+E_F \text{ so therefore } E(x_2)=\frac{\rho d}{\varepsilon}+E_F$$

as shown in the middle figure. These results follow directly from integrating (A.1). Now we integrate again, based on (A.2), to calculate the potential as a function of position

$$\psi(x)=-\int_{x_1}^{x}E(x)\,dx+\psi(x_1)=-\int_{x_1}^{x}\left(\frac{\rho(x-x_1)}{\varepsilon}+E_F\right)dx+\psi(x_1)=-\frac{\rho(x-x_1)^2}{2\varepsilon}-E_F(x-x_1)+\psi(x_1) \text{ so therefore}$$

$$\psi(x_2)=-\frac{\rho d^2}{2\varepsilon}-E_F d+\psi(x_1) \text{ and } \psi_{FG}=\psi(x_1)-\psi(x_2)=\frac{\rho d^2}{2\varepsilon}+E_F d \text{ as shown.}$$

This is result (A.7).

Now integrating between two points x_1 and x_2, if the material is considered to be a parallelepiped of cross sectional area A as shown in Fig. A.2, then the total charge Q in the parallelepiped between the two points is

$$Q = \int_{x_1}^{x_2} \rho(x) A \, dx$$

so the charge per unit area, as seen from the side, is then

$$Q'_{12} = \frac{Q}{A} = \int_{x_1}^{x_2} \rho(x) \, dx .$$

Integrating (A.1) between x_1 and x_2 and introducing this relation gives

$$E(x_2) - E(x_1) = \frac{Q'_{12}}{\varepsilon}$$

which is (A.4). If the material at position x_1 has permittivity ε_1 different from the permittivity ε_2 at position x_2 then from (A.1) this needs to be modified to become

$$\varepsilon_2 E(x_2) - \varepsilon_1 E(x_1) = Q'_{12} .$$

which is (A.5). If there is no space charge density at some position x, so $Q'_{12} = 0$, but the permittivity changes from ε_1 just left of x to ε_2 just right of x, then if we denote the field just to the left of x as $E(x^-)$ and the field just to the right of x as $E(x^+)$, then the above relation implies there is a discontinuity in the electric field at x,

$$\frac{E(x^+)}{E(x^-)} = \frac{\varepsilon_1}{\varepsilon_2}$$

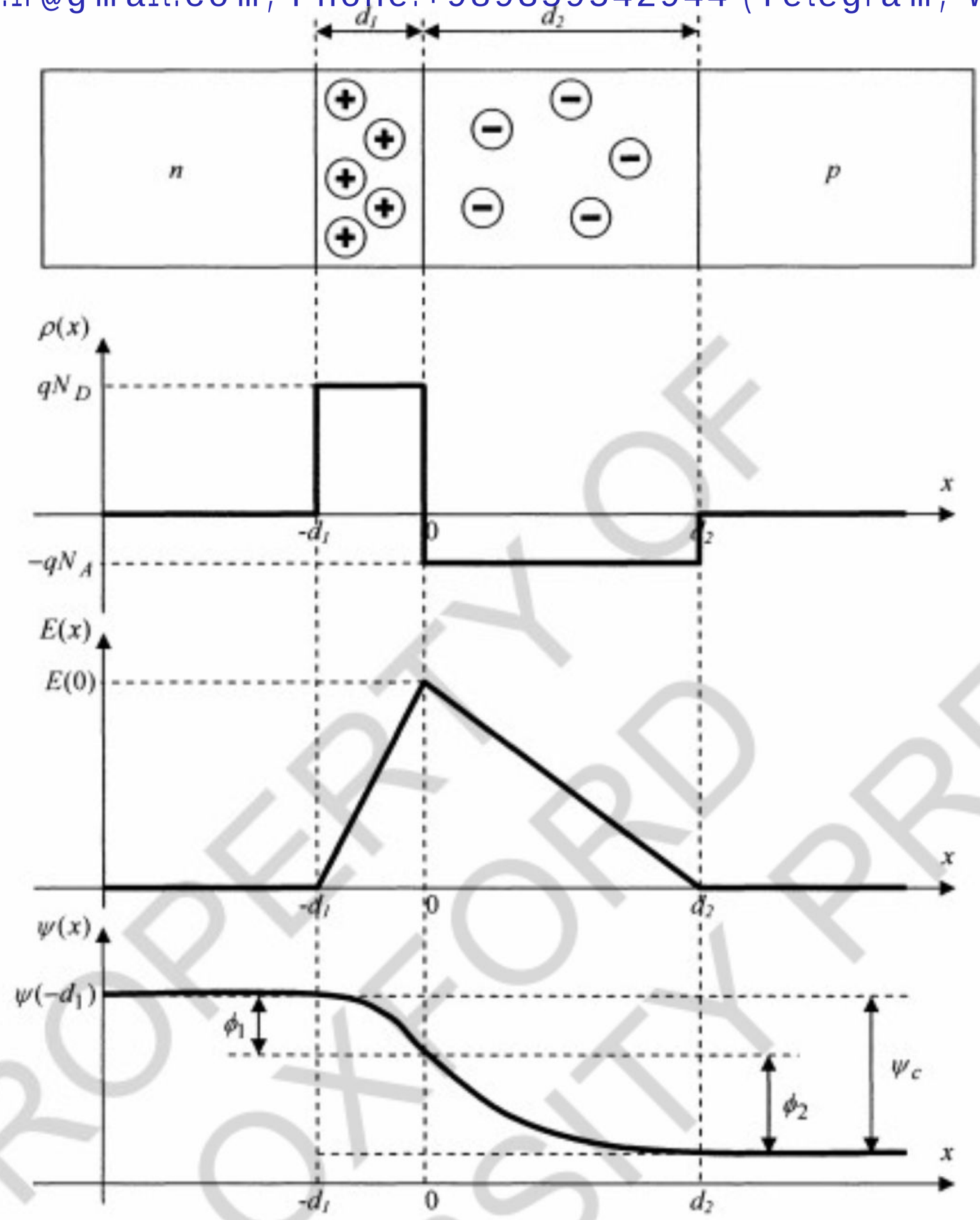
which is (A.6).

1.13 Provide detailed derivations for the results in Fig. 1.25.

These results are basic to anyone studying semiconductors and should be understood well. Again, from (A.1) and (A.2) it is apparent that for any region of uniform doping, such as in the space-charge region around a *pn* junction, the electric field E must vary linearly with position and the potential ψ must vary quadratically with position.

The junction, charge density, electric field, and potential plots are shown below. We consider that the metallurgical junction is as $x = 0$ and will assume constant doping and "hard" edges to the depletion region around the junction. The electric field is zero in the neutral semiconductor away from the space-charge (depletion) region around the junction.

Integrating (A.1) from $x = -d_1$ (the position of the left edge of the depletion region in the *n*-type material) we have, for $-d_1 \le x \le 0$



$$E(x) = \frac{1}{\varepsilon_s}\int_{-d_1}^{x} qN_D\,dx + E(-d_1) = \frac{qN_D(x+d_1)}{\varepsilon_s} + 0 = \frac{qN_D(x+d_1)}{\varepsilon_s}$$

and this achieves a peak value of $E(0) = qN_D d_1/\varepsilon_s$ at $x = 0$. Similarly, integrating (A.1) from $x = 0$ into the depletion region on the p-side of the junction, we have, for $0 \le x \le d_2$

$$E(x) = -\frac{1}{\varepsilon_s}\int_{0}^{x} qN_A\,dx + E(0) = -\frac{qN_A x}{\varepsilon_s} + \frac{qN_D d_1}{\varepsilon_s}.$$

From charge neutrality we also have

$$N_D d_1 = N_A d_2$$

hence we can write $E(0) = qN_A d_2/\varepsilon_s$ therefore for $0 \le x \le d_2$

$$E(x) = \frac{qN_A(d_2 - x)}{\varepsilon_s}.$$

We now integrate the negative of the electric field to get potential as a function of position, from (A.2) for $-d_1 \le x \le 0$

$$\psi(x) = -\int_{-d_1}^{x} E(x)\,dx + \psi(-d_1) = -\int_{-d_1}^{x} \frac{qN_D(x+d_1)}{\varepsilon_s}\,dx + \psi(-d_1) = -\frac{qN_D(x+d_1)^2}{2\varepsilon_s} + \psi(-d_1).$$

The potential drop on the *n* side of the junction is thus

$$\phi_1 = \psi(-d_1) - \psi(0) = \frac{qN_D d_1^2}{2\varepsilon_s}.$$

Similarly applying (A.2) for $0 \le x \le d_2$ gives

$$\psi(x) = -\int_{0}^{x} E(x)\,dx + \psi(0) = \int_{0}^{x} \frac{qN_A(x-d_2)}{\varepsilon_s}\,dx + \psi(-d_1) - \frac{qN_D d_1^2}{2\varepsilon_s} = \frac{qN_A(x-d_2)^2}{2\varepsilon_s} + \psi(-d_1) - \frac{qN_D d_1^2}{2\varepsilon_s} - \frac{qN_A d_2^2}{2\varepsilon_s}.$$

The potential drop on the *p* side of the junction is thus

$$\phi_2 = \psi(d_2) - \psi(0) = \frac{qN_A d_2^2}{2\varepsilon_s}$$

and the built-in potential of the junction is then

$$\phi_{bi} = \phi_1 + \phi_2 = \frac{qN_D d_1^2}{2\varepsilon_s} + \frac{qN_A d_2^2}{2\varepsilon_s}.$$

where the distances d_1 and d_2 are calculated with no external bias applied to the junction.

If the potential difference, including a possible externally applied bias, across the junction is ψ_c then we have

$$\psi_c = \frac{qN_D d_1^2}{2\varepsilon_s} + \frac{qN_A d_2^2}{2\varepsilon_s}$$

and introducing in this $N_D d_1 = N_A d_2$ we can solve for the individual distances,

$$d_1 = \sqrt{\frac{2\varepsilon_s \psi_c}{q}\frac{N_A}{N_D(N_A+N_D)}},\quad d_2 = \sqrt{\frac{2\varepsilon_s \psi_c}{q}\frac{N_D}{N_A(N_A+N_D)}}$$

and the total extent of the depletion regions is

$$d_1 + d_2 = \sqrt{\frac{2\varepsilon_s \psi_c}{q}\frac{N_A N_D}{N_A + N_D}\left(\frac{1}{N_D} + \frac{1}{N_A}\right)} = \sqrt{\frac{2\varepsilon_s \psi_c}{q}\frac{N_A N_D}{N_A + N_D}\left(\frac{N_A + N_D}{N_A N_D}\right)} = \sqrt{\frac{2\varepsilon_s \psi_c}{q}\frac{N_A + N_D}{N_A N_D}}\,.$$

The charge per unit area on either side has magnitude

$$\left|Q'\right| = qN_D d_1 = qN_A d_2 = \sqrt{\frac{2\varepsilon_s \psi_c}{q}\frac{N_A N_D}{N_A + N_D}}\,.$$

For a one-sided junction, where $N_D >> N_A$, the main extent of the depletion region is in the p side of the junction and it has a spatial extent of

$$d_2 \approx \sqrt{\frac{2\varepsilon_s \psi_c}{q}\frac{1}{N_A}}\,.$$

The capacitance per unit area is

$$C' = \frac{d\left|Q'\right|}{d\psi_c} = \frac{\sqrt{2\varepsilon_s N_A}}{2\sqrt{\psi_c}}$$

therefore for a one-sided abrupt junction the capacitance is determined by the doping on the lowly doped side of the junction.

1.14 (a) Show that, within the weak inversion saturation region marked in Fig. 1.36*a*, I_D is of the form $I_1 \exp(V_{GS}/V_1)$ and deduce the values for I_1 and V_1.
(b) Show that within the strong inversion saturation region marked in Fig. 1.36*b*, I_D is of the form $k_1(V_{GS} - V_T)^2$ and deduce the values for k_1 and V_T.

(a) From Fig. 1.37*a*, the saturation current in the weak inversion region is, by eye, spaced linearly for the fixed increments of V_{GS} shown. Because the y-axis is logarithmic, this means that $\log(I_D)$ depends linearly on V_{GS}, which is equivalent to I_D varying exponentially with V_{GS}. Introducing I_1 as the weak inversion current that would theoretically be measured at V_{GS}=0 (although this may not always be possible to measure directly) and the current can be written as $I_1 \exp(V_{GS}/V_1)$ where $1/V_1$ is the slope of $\log(I_D)$ plotted versus V_{GS}. From the figure, the saturation current for V_{GS}=0.2 V is about and 2 nA and for V_{GS}=0.3 V is about and 30 nA, using these values gives V_1=0.0369 V and I_1=8.89 pA.

(b) From Fig. 1.37*b*, the saturation current in strong inversion for different values of V_{GS} can be seen to have a spacing that increases as V_{GS} increases; because the y-axis for this plot is linear, this means that the drain current is increasing at a rate that is faster than linear with V_{GS}. Assuming the dependence of the saturated I_D on V_{GS} is polynomial in nature, it is reasonable to approximate the dependence is quadratic, so $I_D = k_1(V_{GS} - V_T)^2$. Later analysis will show that this form is reasonable for long channel devices. By eye, the saturated currents for V_{GS} values of 0.8 and 1.2 V are approximately 20 and 75 μA, respectively. Adding additional subscripts for these two bias points, from the quadratic model

$$\sqrt{\frac{I_{D1}}{I_{D2}}} = \frac{V_{GS1} - V_T}{V_{GS2} - V_T}$$

and from the numerical values this gives V_T=0.373 V. Calculation from either of the data points then gives k_1=110 μA.