

Chapter 1

BASIC CONCEPTS

1.1

(i) Rectangular section

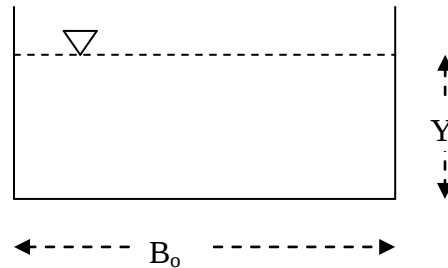
$$A = B_0 Y$$

$$P = 2Y + B_0$$

$$B = B_0$$

$$R = A/P = B_0 Y / (2Y + B_0)$$

$$D = A/B = B_0 Y / B_0 = Y$$



(ii) Trapezoidal section

$$A = B_0 Y + 2Y(SY/2)$$

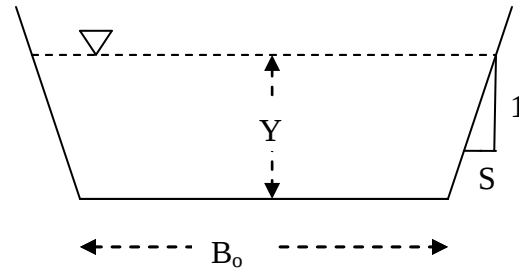
$$= Y(B_0 + SY)$$

$$P = B_0 + 2Y S(S^2 + 1)$$

$$R = A/P = Y(B_0 + SY) / [B_0 + 2Y S(S^2 + 1)]$$

$$B = B_0 + 2SY$$

$$D = A/B = Y(B_0 + SY) / (B_0 + 2SY)$$



(iii) Triangular section

We may use the same equation as that in the case of trapezoidal section with $B_0 = 0$.

Thus, $D = Y/2$

(iv) Partially full circular section

$$A = r^2 \theta / 2 + 2 (r \cos \alpha) / 2 (Y - D_0 / 2)$$

$$= D_0^2 \theta / 8 + (Y - D_0 / 2) (r \cos \alpha)$$

$$Y = D_0 / 2 + (D_0 / 2) \sin \alpha$$

$$A = D_0^2 \theta / 8 + D_0^2 / 4 \sin \alpha \cos \alpha$$

$$\begin{aligned} A &= \frac{D_o^2 \theta}{8} + \frac{D_o^2}{4} \sin \alpha \cos \alpha \\ &= \frac{D_o^2}{8} (\theta + \sin 2\alpha) \end{aligned}$$

But $\theta = 2\alpha + \pi$

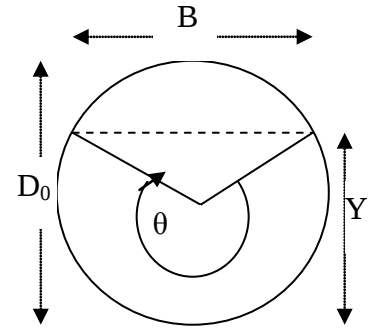
and $\sin \theta = \sin (2\alpha + \pi) = -\sin 2\alpha$

$$A = \frac{D_o^2}{8} (\theta - \sin \theta) \quad 0 \leq \theta \leq 2\pi$$

$$P = r\theta = \frac{D_o \theta}{4} \left(1 - \frac{\sin \theta}{\theta} \right)$$

$$B = 2r \cos \alpha = 2r \cos \left(\frac{\theta}{2} - \frac{\pi}{2} \right) = D_o \sin \frac{\theta}{2}$$

$$D = \frac{A}{B} = \frac{D_o}{8} \left(1 - \frac{\theta - \sin \theta}{\sin \frac{\theta}{2}} \right)$$



(v) Standard horseshoe section:

Length KB

$$\overline{OM} = \overline{MC} - \overline{OM}$$

$$\overline{OM} = d_0 / 2\sqrt{2}$$

$$\overline{MC} = \sqrt{\left(\overline{CG} \right)^2 - \left(\overline{GM} \right)^2} = \sqrt{d_0^2 - \left(\frac{d_0}{2\sqrt{2}} \right)^2} = d_0 \left(\sqrt{1 - \frac{1}{8}} \right)$$

$$\overline{MC} = \sqrt{\frac{7}{8}} d_0$$

$$\overline{OC} = d_0 \left(\sqrt{7/8} - \frac{1}{2\sqrt{2}} \right) = 0.58186 d_0 \text{ ----- (1)}$$

$$\overline{KC} = \overline{FC}^2 - \overline{KF}^2 = d_0^2 - \left(d_0 - \overline{KB} \right)^2 \text{ ----- (2)}$$

$$\overline{KC}^2 = \overline{FC}^2 - \overline{OK}^2 = \overline{OC}^2 - \left(\overline{OB} - \overline{KB} \right)^2$$

$$\overline{KC}^2 = (0.58186 d_0)^2 - \left(\frac{d_0}{2} - \overline{KB} \right)^2 \text{ ----- (3)}$$

$$(2) \text{ and } (3) \quad d_0^2 - \left(d_0 - \overline{KB}\right)^2 = 0.33856 d_0^2 - \left(\frac{d_0}{2} - \overline{KB}\right)^2$$

$$d_0^2 - d_0^2 + 2d_0 \overline{KB} - \overline{KB}^2 = 0.33856 d_0^2 - \frac{d_0^2}{4} + d_0 \overline{KB} - \left(\overline{KB}\right)^2$$

$$\overline{KB} = 0.088562 d_0 \quad \overline{OK} = \frac{d_0}{2} - 0.088562 d_0 = 0.411438 d_0$$

$$\overline{KC}^2 = d_0^2 - (d_0 - 0.088562 d_0)^2 = 0.169281 d_0^2$$

$$KC = 0.4114377 d_0 \quad CC = 0.822875 d_0$$

$$\sin \frac{\theta_L}{2} = 0.4114377 \quad \frac{\theta_L}{2} = 24.295^\circ \quad \theta_L = 48.59^\circ$$

The standard horse shoe section is divided into three sections, i.e., upper section, middle section and lower section.

(a) Upper section

$$\pi \leq \theta_u \leq 2\pi$$

$$\text{Flow area, } A = \frac{D_0^2}{8} (\theta_u - \sin \theta_u) - \frac{\pi D_0^2}{8}$$

$$\text{Wetted perimeter, } P = \frac{D_0 \theta}{2}$$

$$\text{Hydraulic radius, } R = A/P = \frac{D_0}{4} \left(1 - \frac{\sin \theta}{\theta}\right)$$

$$\text{Top water surface, } B = D_0 \sin (\theta/2)$$

$$\text{Hydraulic depth, } D = A/B = \frac{D_0}{8} \left(\frac{\theta - \sin \theta}{\sin (\theta/2)}\right)$$

(b) Lower section

$$0 \leq \theta_L \leq 48.59^\circ$$

$$\text{Flow area, } A = \frac{D_0^2}{8} (\theta_L - \sin \theta_L) = \frac{d_0^2}{2} (\theta - \sin \theta)$$

$$\text{Wetted perimeter, } P = \frac{D_0 \theta}{2} = \theta d_0$$

$$\text{Hydraulic radius, } R = A/P = \frac{d_0}{2} \left(1 - \frac{\sin \theta}{\theta} \right)$$

$$\text{Top water surface, } B = 2d_0 \sin (\theta/2)$$

$$\text{Hydraulic depth, } D = A/B = \frac{d_0}{4} \left(\frac{\theta - \sin \theta}{\sin(\theta/2)} \right)$$

(c) Middle section

Assume trapezoidal section, $S = 0.215$

$$\text{Area, } A = Y(B_0 + SY)$$

$$A = Y(0.8229 d_0 + 0.215Y)$$

$$P = B_0 + 2Y \sqrt{S^2 + 1}$$

$$P = 0.8229 d_0 + 2Y \sqrt{0.215^2 + 1} = 0.8229 d_0 + 2.05Y$$

$$R = A/P = \frac{Y(0.8229 d_0 + 0.215Y)}{0.8229 d_0 + 2.05Y}$$

$$B = B_0 - 254 = 0.8229 d_0 + 2 \times 0.215Y = 0.8229 d_0 + 0.43Y$$

$$D = A/B = \frac{Y(0.8229 d_0 + 0.215Y)}{0.8229 d_0 + 0.43Y}$$

1.2

$$Q = K A R^{2/3}$$

$$A = (\theta - \sin \theta) D^2/8$$

$$R = A/P$$

$$P = D\theta/2$$

$$Q = \frac{KD^{\frac{10}{3}}}{8^{\frac{5}{3}}} \frac{(\theta - \sin \theta)^{\frac{5}{3}}}{(D/2)^{\frac{2}{3}} \theta^{\frac{2}{3}}} = C \frac{(\theta - \sin \theta)^{\frac{5}{3}}}{\theta^{\frac{2}{3}}}$$

$$\text{Where } C = \frac{KD^{\frac{10}{3}}}{8^{\frac{5}{3}}(D/2)^{\frac{2}{3}}}$$

$$\frac{dQ}{d\theta} = 0 \text{ will give the angle } \theta \text{ corresponding to } Q_{\max}.$$

$$\frac{dQ}{d\theta} = C \left[-\frac{2}{3} \theta^{-\frac{5}{3}} (\theta - \sin \theta)^{\frac{5}{3}} + \frac{5}{3} \theta^{-\frac{2}{3}} (\theta - \sin \theta)^{\frac{2}{3}} (1 - \cos \theta) \right] = 0$$

$$\frac{dQ}{d\theta} = C \frac{\theta^{-2/3}}{3} (\theta - \sin\theta)^{2/3} [-2\theta^{-1}(\theta - \sin\theta) + 5(1 - \cos\theta)] = 0$$

$$\therefore 2(\theta - \sin\theta) = 5\theta(1 - \cos\theta)$$

$$\theta - \sin\theta = 5/2\theta - 5/2\theta \cos\theta$$

$$5/2 \cos\theta - \sin\theta / \theta - 1.5 = 0$$

Solving by trial and error or numerically $\theta = 302.41$

From the figure $Y = \frac{D}{2} + \frac{D}{2} \sin\alpha$ and $\alpha = \frac{\theta}{2} - \pi/2$

$$Y = \frac{D}{2} + \frac{D}{2} \sin\left(\frac{\theta}{2} - \pi/2\right)$$

$$Y = \frac{D}{2} \left[1 - \cos\frac{\theta}{2}\right]$$

Substituting θ value for the Q_{max}

$$Y = \frac{D}{2} \left[1 - \cos\frac{302.41}{2}\right]$$

$$Y = 0.938 D$$

1.3

$$(AR^{2/3})_F = \frac{\pi D^2}{4} \left(\frac{D}{4}\right)^{2/3} = \frac{\pi}{4^{5/3}} D^{8/3}$$

$$AR^{2/3} = \frac{D^2}{8} (\theta - \sin\theta) \left(\frac{D}{4}\right)^{2/3} \left(\frac{\theta - \sin\theta}{\theta^{2/3}}\right)^{2/3}$$

$$= \frac{2^{2/3}}{8^{5/3}} \frac{D^{8/3}}{\theta^{2/3}} (\theta - \sin\theta)^{5/3}$$

$$\frac{AR^{2/3}}{(AR^{2/3})_F} = \frac{1}{2\pi} \left(\frac{\theta - \sin\theta}{\theta^{2/3}}\right)^{5/3}$$

$$\frac{R}{R_F} = \frac{\theta - \sin\theta}{\theta}$$

$$Y = \frac{D}{2} + \frac{D}{2} \sin\alpha \quad \alpha = \frac{\theta}{2} - \pi/2$$

$$Y = \frac{D}{2} + \frac{D}{2} \sin\left(\frac{\theta}{2} - \frac{\pi}{2}\right)$$

$$Y = \frac{D}{2} \left[1 - \cos \frac{\theta}{2} \right]$$

$$\frac{Y}{D} = \frac{1}{2} \left[1 - \cos \frac{\theta}{2} \right]$$

Using trial and error

$$\frac{Y}{D} = 0.938 \quad \text{gives maximum value for } \frac{AR^{2/3}}{\left(AR^{2/3} \right)_F}$$

$$\frac{Y}{D} = 0.81 \quad \text{gives maximum value for } \frac{R^{2/3}}{\left(R^{2/3} \right)_F}$$

1.4

$$V = 5.75 V_0 \log \frac{30y}{K}$$

$$V_m = \frac{\int V dA}{dA} = \frac{\int V dy}{dy}$$

$$V_m = \frac{5.75 \int_0^{y_0} V_0 \log 30y/K dy}{\int_0^{y_0} dy} = \frac{5.75 V_0}{y_0} \int_0^{y_0} \log \frac{30y}{K} dy$$

$$\text{Let } x = 30y/K \quad dx = \frac{30}{K} dy \quad dy = \frac{K}{30} dx$$

$$\begin{aligned} V_m &= 5.75 V_0/Y_0 \frac{K}{30} \int \log x dx \\ &= 5.75 V_0/Y_0 \frac{K}{30} [x \log x - x]_0^{y_0} \\ &= 5.75 V_0/Y_0 \frac{K}{30} \left[\frac{30y}{K} \log \frac{30y}{K} - \frac{30y}{K} \right]_0^{y_0} \\ &= 5.75 V_0/Y_0 \left[y_0 \log \frac{30y_0}{K} - y_0 \right] \end{aligned}$$

$$V_m = 5.75 V_0 \left[\log \frac{30y_0}{K} - 1 \right]$$

Energy coefficient, α :

$$\alpha = \frac{\int V^3 dA}{V_m^3 A} = \frac{\int V^3 B dy}{V_m^3 B y_0} = \frac{\int \left[5.75 V_0 \log \frac{30y}{K} \right]^3 dy}{V_m^3 y_0}$$

$$\alpha = \frac{(5.75 V_0)^3 \int \left(\log \frac{30y}{K} \right)^3 dy}{V_m^3 y_0}$$

$$V_m^3 \alpha = \frac{b}{y_0} \int (\log x)^3 dx \quad \text{where, } b = (5.75 V_0)^3 K/30$$

$$\text{But, } \int (\log x)^n dx = x(\log x)^n - n \int (\log x)^{n-1} dx$$

$$\begin{aligned} V_m^3 \alpha &= \frac{b}{y_0} \left[\frac{30y}{K} \left(\log \frac{30y}{K} \right)^3 - 3 \left(\frac{30y}{K} \left(\log \frac{30y}{K} \right)^2 - 2 \frac{30y}{K} \log \frac{30y}{K} + 2 \frac{30y}{K} \right) \right]_0^{y_0} \\ &= \frac{(5.75 V_0)^3}{y_0} \frac{K}{30} \left[\frac{30y_0}{K} \left(\log \frac{30y_0}{K} \right)^3 - 3 \left\{ \frac{30y_0}{K} \left(\log \frac{30y_0}{K} \right)^2 - 2 \frac{30y_0}{K} \log \frac{30y_0}{K} + 2 \frac{30y_0}{K} \right\} \right] \\ \alpha &= \frac{(5.75 V_0)^3 \left[\left(\log \frac{30y_0}{K} \right)^3 - 3 \left(\log \frac{30y_0}{K} \right)^2 + 6 \log \left(\frac{30y_0}{K} \right) - 6 \right]}{(5.75 V_0)^3 \left[\log \frac{30y_0}{K} - 1 \right]^3} \end{aligned}$$

$$\text{Let, } (V_m)^3 = \left[\log \frac{30y_0}{K} - 1 \right]^3 = \left(\log \frac{30y_0}{K} \right)^3 - 3 \left(\log \frac{30y_0}{K} \right)^2 + \log(30y_0) - 1$$

$$\alpha = \frac{V_m^{*3} + 3 \left(\log \frac{30y_0}{K} - 1 \right) - 2}{V_m^{*3}} = 1 + \frac{3}{V_m^{*2}} - \frac{2}{V_m^{*3}}$$

$$\alpha = 1 + 3 \epsilon^2 - 2 \epsilon^3 \quad \text{where } \epsilon = \frac{1}{V_m^*}$$

Momentum coefficient, β :

$$\begin{aligned} \beta &= \frac{\int V^2 dA}{V_m^2 A} = \frac{\int V^2 B dy}{V_m^2 B y_0} = \frac{\int V^2 dy}{V_m^2 y_0} \\ &= \frac{\int \left[5.75 V_0 \log \frac{30y}{K} \right]^2 dy}{V_m^2 y_0} \end{aligned}$$

$$V_m^2 \beta = \frac{b}{y_0} \int (\log x)^2 dx \quad \text{where } b = (5.75 V_0)^2 K/30$$

$$\int (\log x)^n dx = x(\log x)^n - n \int (\log x)^{n-1} dx$$

$$V_m^2 \beta = \frac{b}{y_0} \left[\frac{30y}{K} \left(\log \frac{30y}{K} \right)^2 - 2 \frac{30y}{K} \log \frac{30y}{K} + 2 \frac{30y}{K} \right]_0^{y_0}$$

$$\beta = \frac{\frac{(5.75V_0)^2}{y_0} \frac{K}{30} \left[\frac{30y_0}{K} \left(\log \frac{30y_0}{K} \right)^2 - 2 \frac{30y_0}{K} \log \frac{30y_0}{K} + 2 \frac{30y_0}{K} \right]}{(5.75V_0)^2 \left[\log \frac{30y_0}{K} - 1 \right]^2}$$

$$\text{Let } V_m^{*2} = \left(\log \frac{30y_0}{K} - 1 \right)^2 = \left(\log \frac{30y_0}{K} \right)^2 - 2 \log \frac{30y_0}{K} + 1$$

$$\beta = \frac{\left(\log \frac{30y_0}{K} \right)^2 - 2 \log \frac{30y_0}{K} + 1 + 1}{\left(\log \frac{30y_0}{K} \right)^2 - 2 \log \frac{30y_0}{K} + 1}$$

$$\beta = \frac{V_m^{*2} + 1}{V_m^{*2}} = 1 + \frac{1}{V_m^{*2}}$$

$$\beta = 1 + \epsilon^2 \quad \text{where } \epsilon = \frac{1}{V_m^*}$$

1.6

$$\alpha = \frac{\sum V_i^3 A_i \sum A_i^2}{(\sum V_i A_i)^3}$$

For i = 3

$$\alpha = \frac{(V_1^3 A_1 + V_2^3 A_2 + V_3^3 A_3)(A_1 + A_2 + A_3)^2}{(V_1 A_1 + V_2 A_2 + V_3 A_3)^3}$$

$$\beta = \frac{(V_1^2 A_1 + V_2^2 A_2 + V_3^2 A_3)(A_1 + A_2 + A_3)}{(V_1 A_1 + V_2 A_2 + V_3 A_3)^2}$$

Table 1

	A	V	VA	V ² A	V ³ A
1	40	3	120	360	1080
2	80	3	240	720	2160
3	80	3.1	248	768.8	2383.28
4	80	3.2	256	819.2	2621.44
5	80	3.3	264	871.2	2874.96
6	80	3.3	264	871.2	2874.96
7	80	3.2	256	819.2	2621.44
8	80	3.1	248	768.8	2383.28
9	40	3	120	360	1080
Σ	640		2016	6358.4	20079.36

The calculation is shown in Table -1.

$$\alpha = (20079.36)(640)^2/2016^3 = 1.0038$$

$$\beta = (6358.4)(640)/2016^2 = 1.00126$$

1.9

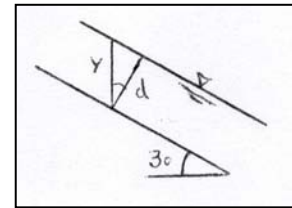
Using hydrostatic pressure distribution and depth = $y = 5\text{m}$

$$T = \gamma y^2/2 = 9810 (5^2/1000)/2 = 122.6 \text{ KN}$$

$$M = TY/3 = 122.6 \times 5/3 = 204.3 \text{ KN.m}$$

$$\text{But, } d = y \cos \theta = 5 \cos 30 = 4.33 \text{ m}$$

In this case the pressure distribution is not hydrostatic



Correction:

$$T_t = \rho d/2 = \gamma d^2 \cos \theta / 2 = 79.65 \text{ KN}$$

$$M_t = T_t d/3 = 79.65 (4.33)/3 = 114.96 \text{ KN m}$$

% error in the shearing force:

$$100(T_t - T)/T_t = (79.65 - 122.6)/79.65 = 53.9 \%$$

% error in the moment:

$$100(M_t - M)/M_t = (114.96 - 204.38) / 114.96 = 77 \%$$

1.10

$$\text{Centrifugal acceleration} = V^2/R$$

$$\text{Centrifugal force} = \rho y_s \Delta A V^2/R$$

$$\text{Pressure head due to centrifugal acceleration} = (1/g)y_s V^2/R$$

$$\text{Total pressure head, } y_t = y_s + (1/g)y_s V^2/R = y_s (1 + V^2/gR)$$

$$y_t = 5 (1 + 20^2/9.81/20) = 15.19 \text{ m}$$

$$\text{Pressure intensity at point } C = \gamma y_t = (9.81) (1000)(15.19/1000) = 149 \text{ kPa}$$

1.11

$$Q = K A R^{2/3}$$

$$A = [B - (h/\sqrt{3})]h$$

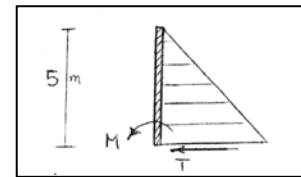
$$P = B + (4h/\sqrt{3})$$

$$Q = K [B - h/\sqrt{3}]^{5/3} h^{5/3} [B + 4h/\sqrt{3}]^{2/3}$$

$$= K [hB - h^2/\sqrt{3}]^{5/3} [B + 4h/\sqrt{3}]^{2/3}$$

$$Q \text{ is maximum or minimum if } dQ/dh = 0$$

$$dQ/dh = \frac{5}{3} K [hB - h^2/\sqrt{3}]^{2/3} [B - 2h/\sqrt{3}] [B + 4h/\sqrt{3}]^{-2/3}$$



1.12

(i) nonuniform

(ii) nonuniform

(iii) nonuniform

(iv) uniform

1.13

(i) unsteady

(ii) unsteady

(iii) steady

(iv) unsteady

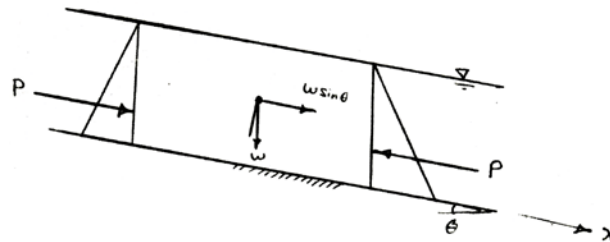
1.14

$$(i) R_e = Vy/\nu = (1)(1)/(0.11 * 10^{-5}) = 9.0 * 10^5 \text{ Turbulent}$$

$$(ii) R_e = Vy/v = \frac{0.1 \times 2 \times 10^{-3}}{0.11 \times 10^{-5}} = 181.8 \text{ Laminar}$$

1.15

It is not possible to have uniform flow in a frictionless sloping channel. The forces in the x direction will not be balanced.



1.16

It is not possible to have uniform flow in a horizontal channel. There is no acceleration force.

1.17

y = flow depth measured vertically; θ = angle between water surface and horizontal; ϕ = angle between channel bed and horizontal. Let d = perpendicular on channel bed.

Thus, pressure at channel bed is

$$p = \rho g d \cos \phi$$

From the geometry, $y = d \cos \phi + d \sin \phi \tan \theta$

$$\text{Or, } d \cos \phi = \frac{y}{1 + \tan \phi \tan \theta}$$

$$\text{Or, } p = \rho g \frac{y}{1 + \tan \phi \tan \theta}$$

1.18

$$V = 5.75 V_f \log (30y/K) \quad (1)$$

$$V_m = \frac{\int V dA}{\int dA} = \frac{\int V dy}{\int dy} = \frac{5.75 \int_0^{y_0} V_f \log(30y/K) dy}{\int_0^{y_0} dy} = \frac{5.75V_f}{y_0} \int_0^{y_0} \log(30y/K) dy$$

Let $x = 30y/K$, $dx = (30/K)dy$ or, $dy = (K/30)dx$

$$V_m = \frac{\int V dA}{\int dA} = \frac{5.75V_f}{y_0} \frac{K}{30} \int_0^{y_0} \log x dx = \frac{5.75V_f}{y_0} \frac{K}{30} [x \log x - x]_0^{y_0}$$

$$V_m = \frac{5.75V_f}{y_0} \frac{K}{30} \left[\frac{30y}{K} \log \frac{30y}{K} - \frac{30y}{K} \right]_0^{y_0}$$

$$V_m = 5.75V_f [\log(30y_0/K) - 1]$$

In Eq 1, at $y = y_0$, $V = V_{max}$

$$V_{max} = 5.75V_f \log(30y_0/K)$$

Let $\gamma = (V_{max}/V_m) - 1 = 5.75V_f \log(30y_0/K) / 5.75V_f [\log(30y_0/K) - 1] - 1$

$$\gamma = 1 / [\log(30y_0/K) - 1]$$

Similar to the solution of problem 1.4 this will lead to

$$\alpha = 1 + 3\gamma^2 - 2\gamma^3$$

$$\beta = 1 + \gamma^2$$

1.19

$$d = y \cos \theta$$

$$p = (\gamma d^2 / 2) (\cos \theta)$$

$$p = (\gamma y^2 / 2) (\cos^2 \theta \cos \theta)$$

$$p = (\gamma y^2 / 2) (\cos^3 \theta)$$

$$M = pd^3 / 3 = (py \cos \theta) / 3$$

$$= (\gamma / 2) (y^2 \cos^3 \theta) (\gamma \cos \theta / 3)$$

$$M = (\gamma y^3 / 6) (\cos^4 \theta)$$

$$\text{Shearing force} = p = (\gamma y^2 / 2) (\cos^3 \theta)$$

