1.18 Consider the function $f(x) = \frac{1-\cos(x)}{\sin(x)}$.

(*a*) Use the decimal format with six significant digits (apply rounding) to calculate (using a calculator) $f(x)$ for $x = 0.007$.

(*b*) Use MATLAB (use `format long`) to calculate the value of $f(x)$ and the true relative error, due to rounding, in the value of $f(x)$ that was obtained in part (*a*).

(*c*) Multiply $f(x)$ by $\frac{1+\cos(x)}{1+\cos(x)}$ to obtain a form of $f(x)$ that is less prone to rounding errors. With the new form, use the decimal format with six significant digits (apply rounding) to calculate (using a calculator) $f(x)$ for $x = 0.007$. Compare the value with the values in parts (*a*) and (*b*).

Solution

(*a*) To 6 significant digits, $\cos(0.007) = 0.999976$ and $\sin(0.007) = 0.00699994$. Thus, retaining 6 significant figures,

$$f(0.007) = \frac{1-0.999976}{0.00699994} = 0.00342860$$

(*b*) Using MATLAB, with the format long, here is the value of f(0.007):

```
>> format long
>> f=(1-cos(0.007))/sin(0.007)
f =
0.00350001429173
>>
```

The true relative error due to rounding is then determined from Eq. (1.17):

$$TrueRelativeError = \left|\frac{TrueSolution - NumericalSolution}{TrueSolution}\right|$$
$$= \left|\frac{0.00350001429173 - 0.00342860}{0.00350001429173}\right| = 0.0204$$

or 2.04%.

(*c*) Multiplying $f(x)$ by $\frac{1+\cos(x)}{1+\cos(x)}$ yields:

$$f(x) = \frac{1-\cos(x)}{\sin(x)}\left(\frac{1+\cos(x)}{1+\cos(x)}\right) = \frac{1-\cos^2(x)}{\sin(x)+\sin(x)\cos(x)}$$
$$= \frac{\sin^2(x)}{\sin(x)+\sin(x)\cos(x)} = \frac{\sin(x)}{1+\cos(x)}$$

Using the values of $\cos(0.007)$ and $\sin(0.007)$ to 6 significant figures from part (*a*),

$$f(0.007) = \frac{\sin(0.007)}{1 + \cos(0.007)} = \frac{0.00699994}{1 + 0.999976} = 0.00350001$$

The true relative error is now:

$$TrueRelativeError = \left|\frac{0.00350001429173 - 0.00350001}{0.00350001429173}\right| = 1.22620 \times 10^{-6}$$

or less than 1.23×10^{-4} %. Clearly, the above form for $f(x)$ is more accurate than the form used in part (*a*) and the form given in the problem statement.