1.19 Consider the function $f(x) = \frac{\sqrt{4+x}-2}{x}$.

(*a*) Use the decimal format with six significant digits (apply rounding at each step) to calculate (using a calculator) $f(x)$ for $x = 0.001$.

(*b*) Use MATLAB (`format long`) to calculate the value of $f(x)$. Consider this to be the true value, and calculate the true relative error due to rounding in the value of $f(x)$ that was obtained in part (*a*).

(*c*) Multiply $f(x)$ by $\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$ to obtain a form of $f(x)$ that is less prone to rounding errors. With the new form, use the decimal format with six significant digits (apply rounding at each step) to calculate (using a calculator) $f(x)$ for $x = 0.001$. Compare the value with the values in parts (*a*) and (*b*).

Solution

(*a*) To six significant figures, $f(x) = \frac{2.00025-2}{0.001} = \frac{0.00025}{0.001} = 0.25$

(*b*) Using the format long in MATLAB:

```
>> format long
>> x=0.001;
>> f=(sqrt(4+x)-2)/x
f =
   0.249984376953005
```

The true relative error, according to Eq. (1.17) is:

$$TrueRelativeError = \left|\frac{0.249984376953005-0.25}{0.249984376953005}\right| = 0.000062496$$

(*c*) Multiplying $f(x)$ by $\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$ yields:

$$f(x) = \frac{\sqrt{4+x}-2}{x}\left(\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}\right) = \frac{1}{\sqrt{4+x}+2}$$

Using the values to 6 significant figures from part (*a*),

$$f(0.001) = \frac{1}{\sqrt{4.001}+2} = \frac{1}{2.00025+2} = 0.249984$$

The true relative error is now:

$$TrueRelativeError = \left|\frac{0.249984376953005 - 0.249984}{0.249984376953005}\right| = 1.504 \times 10^{-6}$$

Clearly, the above form for $f(x)$ is more accurate than the form used in part (*a*) and the form given in the problem statement.