

1.18 Consider the function $f(x) = \frac{1 - \cos(x)}{\sin(x)}$.

- (a) Use the decimal format with six significant digits (apply rounding) to calculate (using a calculator) $f(x)$ for $x = 0.007$.
- (b) Use MATLAB (use `format long`) to calculate the value of $f(x)$ and the true relative error, due to rounding, in the value of $f(x)$ that was obtained in part (a).
- (c) Multiply $f(x)$ by $\frac{1 + \cos(x)}{1 + \cos(x)}$ to obtain a form of $f(x)$ that is less prone to rounding errors. With the new form, use the decimal format with six significant digits (apply rounding) to calculate (using a calculator) $f(x)$ for $x = 0.007$. Compare the value with the values in parts (a) and (b).

Solution

(a) To 6 significant digits, $\cos(0.007) = 0.999976$ and $\sin(0.007) = 0.00699994$. Thus, retaining 6 significant figures,

$$f(0.007) = \frac{1 - 0.999976}{0.00699994} = 0.00342860$$

(b) Using MATLAB, with the format long, here is the value of $f(0.007)$:

```
>> format long
>> f=(1-cos(0.007))/sin(0.007)
f =
0.00350001429173
>>
```

The true relative error due to rounding is then determined from Eq. (1.17):

$$\begin{aligned} \text{TrueRelativeError} &= \left| \frac{\text{TrueSolution} - \text{NumericalSolution}}{\text{TrueSolution}} \right| \\ &= \left| \frac{0.00350001429173 - 0.00342860}{0.00350001429173} \right| = 0.0204 \end{aligned}$$

or 2.04%.

(c) Multiplying $f(x)$ by $\frac{1 + \cos(x)}{1 + \cos(x)}$ yields:

$$\begin{aligned} f(x) &= \frac{1 - \cos(x)}{\sin(x)} \left(\frac{1 + \cos(x)}{1 + \cos(x)} \right) = \frac{1 - \cos^2(x)}{\sin(x) + \sin(x)\cos(x)} \\ &= \frac{\sin^2(x)}{\sin(x) + \sin(x)\cos(x)} = \frac{\sin(x)}{1 + \cos(x)} \end{aligned}$$

Using the values of $\cos(0.007)$ and $\sin(0.007)$ to 6 significant figures from part (a),

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$$f(0.007) = \frac{\sin(0.007)}{1 + \cos(0.007)} = \frac{0.00699994}{1 + 0.999976} = 0.00350001$$

The true relative error is now:

$$TrueRelativeError = \left| \frac{0.00350001429173 - 0.00350001}{0.00350001429173} \right| = 1.22620 \times 10^{-6}$$

or less than $1.23 \times 10^{-4} \%$. Clearly, the above form for $f(x)$ is more accurate than the form used in part (a) and the form given in the problem statement.