

**1.22** Taylor series expansion of the function  $f(x) = e^x$  is:

$$f(x) = e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots \quad (1.1)$$

Use Eq. (1.21) to calculate the value of  $e^{-2}$  for the following cases. Use decimal numbers with six significant numbers (apply rounding at each step). In each case calculate also the true relative error. Use MATLAB with `format long` to calculate the true value of  $e^{-2}$ .

(a) Use the first four terms. (b) Use the first six terms. (c) Use the first eight terms.

### Solution

(a) Using the first four terms and 6 significant figures,

$$e^{-2} = 1 - 2 + 2 - \frac{8}{6} = 1 - 1.33333 = -0.33333$$

Before calculating the true relative error, note that the sign of the answer is completely wrong since  $e^x > 0$  for all real values of  $x$ . Use of `format long` in MATLAB yields:

```
>> format compact
>> format long
>> exp(-2)
ans =
    0.13533528323661
```

Therefore, the true relative error is:

$$TrueRelativeError = \left| \frac{0.13533528323661 - (-0.33333)}{0.13533528323661} \right| = 3.46$$

or 346 %!

(b) With the first 6 terms and retaining 6 significant figures,

$$\begin{aligned} e^{-2} &= 1 - 2 + 2 - \frac{8}{6} + \frac{16}{24} - \frac{32}{120} = 1 - 1.33333 + 0.666667 - 0.266667 \\ &= 0.0666700 \end{aligned}$$

which yields a true relative error of  $\left| \frac{0.13533528323661 - 0.0666700}{0.13533528323661} \right| = 0.50737$  or less than 50.74%.

(c) With the first 8 terms and retaining 6 significant figures,

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$$\begin{aligned}e^{-2} &= 1 - 2 + 2 - \frac{8}{6} + \frac{16}{24} - \frac{32}{120} + \frac{64}{720} - \frac{128}{5040} \\&= 1 - 1.33333 + 0.666667 - 0.266667 + 0.0888889 - 0.0252968 \\&= 0.130262\end{aligned}$$

which yields a true relative error of  $\left| \frac{0.13533528323661 - 0.130262}{0.13533528323661} \right| = 0.03749$  or less than 3.75%.

It can be clearly seen from the answers of parts (a), (b), and (c), that as more terms are retained, the better the accuracy.