

1.21 The Taylor series expansion of $\cos(x)$ is given by:

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + \dots \quad (1.1)$$

Use the first three terms in Eq. (1.20) to calculate the value of $\cos(\pi/4)$. Use the decimal format with six significant digits (apply rounding at each step). Calculate the truncation error.

Solution

The quantity $\pi/4$ to 6 significant figures with rounding is 0.785398. Using the first three terms in Eq. (1.20) and evaluating the terms also to 6 significant figures,

$$\begin{aligned} \cos(\pi/4) &= 1 - \frac{(0.785398)^2}{2!} + \frac{(0.785398)^4}{4!} = 1 - \frac{0.616850}{2} + \frac{0.380504}{24} \\ &= 1 - 0.308425 + 0.0158543 = 0.707429 \end{aligned}$$

In this case, the truncation error can be calculated because the “exact” value of $\cos(\pi/4)$ can be determined. Using MATLAB with format long, $\cos(\pi/4) = 0.707107$. Therefore the truncation error is simply $|0.707107 - 0.707429| = 0.000322$, or in other words, the true relative truncation error is $\left| \frac{0.707107 - 0.707429}{0.707107} \right| = 0.000455377$ or less than 0.046%.

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