

**1.19** Consider the function  $f(x) = \frac{\sqrt{4+x}-2}{x}$ .

- (a) Use the decimal format with six significant digits (apply rounding at each step) to calculate (using a calculator)  $f(x)$  for  $x = 0.001$ .
- (b) Use MATLAB (`format long`) to calculate the value of  $f(x)$ . Consider this to be the true value, and calculate the true relative error due to rounding in the value of  $f(x)$  that was obtained in part (a).
- (c) Multiply  $f(x)$  by  $\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$  to obtain a form of  $f(x)$  that is less prone to rounding errors. With the new form, use the decimal format with six significant digits (apply rounding at each step) to calculate (using a calculator)  $f(x)$  for  $x = 0.001$ . Compare the value with the values in parts (a) and (b).

**Solution**

(a) To six significant figures,  $f(x) = \frac{2.00025 - 2}{0.001} = \frac{0.00025}{0.001} = 0.25$

(b) Using the format long in MATLAB:

```
>> format long
>> x=0.001;
>> f=(sqrt(4+x)-2)/x
f =
    0.249984376953005
```

The true relative error, according to Eq. (1.17) is:

$$\text{TrueRelativeError} = \left| \frac{0.249984376953005 - 0.25}{0.249984376953005} \right| = 0.000062496$$

(c) Multiplying  $f(x)$  by  $\frac{\sqrt{4+x}+2}{\sqrt{4+x}+2}$  yields:

$$f(x) = \frac{\sqrt{4+x}-2}{x} \left( \frac{\sqrt{4+x}+2}{\sqrt{4+x}+2} \right) = \frac{1}{\sqrt{4+x}+2}$$

Using the values to 6 significant figures from part (a),

$$f(0.001) = \frac{1}{\sqrt{4.001}+2} = \frac{1}{2.00025+2} = 0.249984$$

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The true relative error is now:

$$TrueRelativeError = \left| \frac{0.249984376953005 - 0.249984}{0.249984376953005} \right| = 1.504 \times 10^{-6}$$

Clearly, the above form for  $f(x)$  is more accurate than the form used in part (a) and the form given in the problem statement.