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Light‒Matter Interaction

## Optical Fields

1.1.1 A given set of vector and scalar potentials,  and , defines a unique set of  and  fields through (1.8) and (1.9):

 and .

For any set of  and  that are related to  and  through the relations given in (1.12) with a smooth scalar function  of space and time, we find that



and



Therefore,  and  also define the same set of  and  fields.

1.1.2 According to Problem 1.1.1, the  and  fields remain unchanged a gauge transformation of  and  as defined in (1.12) because

 and ,

as expressed in (1.13) and (1.14). Therefore, Maxwell’s equations are invariant under a gauge transformation of  and  as defined in (1.12) for any smooth scalar function  of space and time.

1.1.3 In the Coulomb gauge,  according to (1.15). Then, (1.11) directly reduces to the relation given in (1.17):

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By expressing the current density vector in terms of the sum of a longitudinal component and a transverse component as  with  and , we can express the continuing equation given in (1.7) as

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From the above two relations, we find



By using the above relation and the relation that , we find that (1.10) reduces to the relation given in (1.16):

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1.1.4 Maxwell’s equations and the continuity equation are:

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From Subsection 1.1.3, we know that , , and  all change sign under spatial inversion but not under time reversal. We also know that , , and  all change sign under time reversal but not under spatial inversion. The current density vector  changes sign under either spatial inversion or time reversal, but  never changes sign under spatial inversion or time reversal.

(a) By taking spatial inversion, the equations become:

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Each equation returns to its original form after the signs are cleared up. Hence, Maxwell’s equations and the continuity equation are invariant under spatial inversion.

(b) Taking time reversal, the equations become:

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Each equation returns to its original form after the signs are cleared up. Hence, Maxwell’s equations and the continuity equation are invariant under time reversal.

(c) Taking both spatial inversion and time reversal, the equations become:

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Note the sign of  changes twice in this situation because it changes under both spatial inversion and time reversal. Each equation returns to its original form after the signs are cleared up. Hence, Maxwell’s equations and the continuity equation are invariant under simultaneous spatial inversion and time reversal.

## Interaction Hamiltonians

## Transformation of Hamiltonians

## Multipole Expansion

## Density Matrix Formulation

1.5.1 From (1.111), we have

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By taking the Hermitian adjoint on this equation, we get

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Then, by using the definition of the density matrix that , as given in (1.113), we find that

 

Therefore, we obtain the equation of motion for the density matrix given in (1.117):



1.5.2 From (1.132), we have

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For an off-diagonal element , with , this equation leads to the relation:

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Then, by using (1.134) and (1.136), we obtain the relation given in (1.138):



This equation can be integrated by first multiplying both sides with , followed by some rearrangements of the terms, to obtain (1.139):



## Electric Polarization

## Electric Dipole Approximation

1.7.1 For a two-level system, there are only two energy levels,  and , with  such that . Under the electric dipole interaction,  from (1.151). Thus,  and  from (1.152). For the thermal relaxation rates, we take  for the thermal population relaxation from the upper energy level to the lower energy level  and  for the thermal population excitation from the lower energy level  to the upper energy level .

For the diagonal elements,  and , we then find (1.153) and (1.154) from (1.127):





For the diagonal elements,  and , we find (1.155) from (1.128), and (1.156) from (1.114):





1.7.2 From (1.157), we find the real part of :



By using the fact that , we find by using (1.145) for a two-level system that



By comparing this relation with (1.159), we find the electric susceptibility tensor  as given in (1.160):

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1.7.3 From (1.161), we find the real part of :



By using the fact that  and by using (1.146), we find the linear polarization for a two-level system:



By comparing this relation with (1.163), we find the linear electric susceptibility tensor  as given in (1.164):

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## Rotating-Wave Approximation