Solution Manual

Ch_1

Section 1.2 **Dimensions, Dimensional Homogeneity, and Units**

Problem 1.2.1

The force, F, of the wind blowing against a building is given by $F = C_D \rho V^2 A/2$, where V is the wind speed, ρ the density of the air, A the cross-sectional area of the building, and C_D is a constant termed the drag coefficient. Determine the dimensions of the drag coefficient.

SOLUTION:

$$F = C_D \rho V^2 \frac{A}{2}$$

or

$$C_D = \frac{2F}{\rho V^2 A}, \text{ where}$$

$$F \doteq MLT^{-2},\ \rho \doteq ML^{-3},\ V \doteq LT^{-1},\ A \doteq L^2$$

Thus,

$$C_D \doteq \frac{\left(MLT^{-2}\right)}{\left[\left(ML^{-3}\right)\left(LT^{-1}\right)^2\left(L^2\right)\right]} = M^0 L^0 T^0$$

Hence, $\boxed{C_D \text{ is dimensionless}}$.

Problem 1.2.2

The Mach number is a dimensionless ratio of the velocity of an object in a fluid to the speed of sound in the fluid. For an airplane flying at velocity V in air at absolute temperature T, the Mach number Ma is,

$$\text{Ma} = \frac{V}{\sqrt{kRT}}$$

where k is a dimensionless constant and R is the specific gas constant for air. Show that Ma is dimensionless.

SOLUTION:

We denote the dimension of temperature by θ and use Newton's second law to get $F = \frac{ML}{T^2}$. Then

$$[\text{Ma}] = \frac{\left(\frac{L}{T}\right)}{\sqrt{(1)\left(\frac{FL}{M\theta}\right)\theta\left(\frac{ML}{T^2F}\right)}} = \frac{\left(\frac{L}{T}\right)}{\sqrt{\frac{L^2}{T^2}}}$$

or

$$\boxed{[\text{Ma}] = [1]}$$

Problem 1.2.3

Verify the dimensions, in both the FLT and the MLT systems, of the given quantities, which appear in Table 1.1:

(a) Volume, **(b)** acceleration, **(c)** mass, **(d)** moment of inertia (area), and **(e)** work.

SOLUTION:

(a) Volume $\doteq \boxed{L^3}$

(b) Acceleration = time rate of change of velocity $\doteq \frac{LT^{-1}}{T} \doteq \boxed{LT^{-2}}$

(c) Mass $\doteq \boxed{M}$

or with $F \doteq MLT^{-2}$

Mass $\doteq \boxed{FL^{-1}T^2}$

(d) Moment of inertia (area) = second moment of area $\doteq (L^2)(L^2) \doteq \boxed{L^4}$

(e) Work = force × distance $\doteq \boxed{FL}$

or with $F \doteq MLT^{-2}$

Work $\doteq \boxed{ML^2T^{-2}}$

Problem 1.2.4

Verify the dimensions, in both the FLT and the MLT systems, of the given quantities, which appear in Table 1.1:

(a) Angular velocity, **(b)** energy, **(c)** moment of inertia (area), **(d)** power, and **(e)** pressure.

SOLUTION:

(a) $\text{Angular velocity} = \dfrac{\text{angular displacement}}{\text{time}} \doteq \boxed{T^{-1}}$

(b) Energy ~ capacity of body to do work

Single work = force × distance → energy $\doteq \boxed{FL}$

or with $F \doteq MLT^{-2} \rightarrow \text{energy} \doteq \left(MLT^{-2}\right)(L) \doteq \boxed{ML^2T^{-2}}$

(c) $\text{Moment of inertia (area)} = \text{second moment of area} \doteq \left(L^2\right)\left(L^2\right) \doteq \boxed{L^4}$

(d) $\text{Power} = \text{rate of doing work} \doteq \dfrac{FL}{T} \doteq FLT^{-1} \doteq \left(MLT^{-2}\right)(L)\left(T^{-1}\right) \doteq \boxed{ML^2T^{-3}}$

(e) $\text{Pressure} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq FL^{-2} \doteq \left(MLT^{-2}\right)\left(L^{-2}\right) \doteq \boxed{ML^{-1}T^{-2}}$

Problem 1.2.5

Verify the dimensions, in both the FLT system and the MLT system, of the given quantities, which appear in Table 1.1:

(a) Frequency, **(b)** stress, **(c)** strain, **(d)** torque, and **(e)** work.

SOLUTION:

(a) $\text{Frequency} = \dfrac{\text{cycles}}{\text{time}} \doteq \boxed{T^{-1}}$

(b) $\text{Stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \boxed{FL^{-2}}$

Since $F \doteq MLT^{-2}$

$\text{Stress} \doteq \dfrac{MLT^{-2}}{L^2} \doteq \boxed{ML^{-1}T^{-2}}$

(c) $\text{Strain} = \dfrac{\text{change in length}}{\text{length}} \doteq \dfrac{L}{L} \doteq \boxed{L^0 \ (\text{dimensionless})}$

(d) $\text{Torque} = \text{force} \times \text{distance} \doteq FL \doteq \left(MLT^{-2}\right)(L) \doteq \boxed{ML^2T^{-2}}$

(e) $\text{Work} = \text{force} \times \text{distance} \doteq FL \doteq \left(MLT^{-2}\right)(L) \doteq \boxed{ML^2T^{-2}}$

Problem 1.2.6

If u is velocity, x is length, and t is time, what are the dimensions (in the MLT system) of **(a)** $\partial u/\partial t$, **(b)** $\partial^2 u/\partial x \partial t$, and **(c)** $\int (\partial u/\partial t)dx$?

SOLUTION:

(a) $\dfrac{\partial u}{\partial t} \doteq \dfrac{LT^{-1}}{T} \doteq \boxed{LT^{-2}}$

(b) $\dfrac{\partial^2 u}{\partial x \partial t} \doteq \dfrac{LT^{-1}}{(L)(T)} \doteq \boxed{T^{-2}}$

(c) $\int \dfrac{\partial u}{\partial t}\partial x \doteq \dfrac{\left(LT^{-1}\right)}{T}(L) \doteq \boxed{L^2T^{-2}}$

Problem 1.2.7

Verify the dimensions, in both the FLT system and the MLT system, of the given quantities, which appear in Table 1.1:

(a) Acceleration, **(b)** stress, **(c)** moment of a force, **(d)** volume, and **(e)** work.

SOLUTION:

(a) $\text{acceleration} = \dfrac{\text{velocity}}{\text{time}} \doteq \dfrac{L}{T^2} \doteq \boxed{LT^{-2}}$

(b) $\text{stress} = \dfrac{\text{force}}{\text{area}} \doteq \dfrac{F}{L^2} \doteq \boxed{FL^{-2}}$

Since $F \doteq MLT^{-2}$,

$\text{stress} \doteq \dfrac{MLT^{-2}}{L^2} \doteq \boxed{ML^{-1}T^{-2}}$

(c) $\text{moment of a force} = \text{force} \times \text{distance} \doteq FL \doteq \left(MLT^{-2}\right)L \doteq \boxed{ML^2T^{-2}}$

(d) $\text{volume} = (\text{length})^3 \doteq \boxed{L^3}$

(e) $\text{work} = \text{force} \times \text{distance} \doteq FL \doteq \left(MLT^{-2}\right)L \doteq \boxed{ML^2T^{-2}}$

Problem 1.2.8

If p is pressure, V is velocity, and ρ is fluid density, what are the dimensions (in the MLT system) of **(a)** p/ρ, **(b)** $pV\rho$, and **(c)** $p/\rho V^2$?

SOLUTION:

(a) $\dfrac{p}{\rho} \doteq \dfrac{FL^{-2}}{ML^{-3}} = \dfrac{MLT^{-2}L^{-2}}{ML^{-3}} = \dfrac{ML^{-1}T^{-2}}{ML^{-3}} \doteq \boxed{L^2T^{-2}}$

(b) $pV\rho \doteq \left(ML^{-1}T^{-2}\right)\left(LT^{-1}\right)\left(ML^{-3}\right) \doteq \boxed{M^2L^{-3}T^{-3}}$

(c) $\dfrac{p}{\rho V^2} \doteq \dfrac{ML^{-1}T^{-2}}{\left(ML^{-3}\right)\left(LT^{-1}\right)^2} \doteq \boxed{M^0L^0T^0\,(\text{dimensionless})}$

Problem 1.2.9

If P is force and x is length, what are the dimensions (in the FLT system) of **(a)** dP/dx, **(b)** d^3P/dx^3, and **(c)** $\int P\,dx$?

SOLUTION:

(a) $\dfrac{dP}{dx} \doteq \dfrac{F}{L} \doteq \boxed{FL^{-1}}$

(b) $\dfrac{d^3P}{dx^3} \doteq \dfrac{F}{L^3} \doteq \boxed{FL^{-3}}$

(c) $\int P dx \doteq \boxed{FL}$.

Problem 1.2.10

If V is velocity, ℓ is length, and ν is a fluid property (the kinematic viscosity) having dimensions of L^2T^{-1}, which of the given combinations are dimensionless: **(a)** $V\ell\nu$, **(b)** $V\ell/\nu$, **(c)** $V^2\nu$, and **(d)** $V/\ell\nu$?

SOLUTION:

(a) $V\ell\nu \doteq (LT^{-1})(L)(L^2T^{-1}) \doteq \boxed{L^4T^{-2}\,(\text{not dimensionless})}$

(b) $\dfrac{V\ell}{\nu} \doteq \dfrac{(LT^{-1})(L)}{(L^2T^{-1})} \doteq \boxed{L^0T^0\,(\text{dimensionless})}$

(c) $V^2\nu \doteq (LT^{-1})^2(L^2T^{-1}) \doteq \boxed{L^4T^{-3}\,(\text{not dimensionless})}$

(d) $\dfrac{V}{\ell\nu} \doteq \dfrac{(LT^{-1})}{(L)(L^2T^{-1})} \doteq \boxed{L^{-2}\,(\text{not dimensionless})}$

Problem 1.2.11

The momentum flux (discussed in Chapter 5) is given by the product $\dot{m}V$, where $\dot{m}$ is mass flowrate and V is velocity. If mass flowrate is given in units of mass per unit time, show that the momentum flux can be expressed in units of force.

SOLUTION:

$$[\dot{m}V] \doteq \left(\frac{M}{T}\right)\left(\frac{L}{T}\right) \doteq M\frac{L}{T^2}\left[\frac{FT^2}{ML}\right] \doteq \boxed{F}$$

where $\dfrac{1}{g_c} \doteq \left[\dfrac{FT^2}{ML}\right]$ comes from Newton's Second Law.

Problem 1.2.12

An equation for the frictional pressure loss Δp (mm H_2O) in a circular duct of inside diameter d(mm) and length L(m) for air flowing with velocity V (m/min) is

$$\Delta p = 0.027\left(\frac{L}{d^{1.22}}\right)\left(\frac{V}{V_0}\right)^{1.82}$$

Where V_0 is a reference velocity equal to 305 m/min. Find the units of the "constant" 0.027.

SOLUTION:

Solving for the constant gives

$$0.027 = \frac{\Delta p_L}{\left(\dfrac{L}{D^{1.22}}\right)\left(\dfrac{V}{V_0}\right)^{1.82}}$$

The units give

$$[0.027] \doteq \frac{\text{n H}_2\text{O)}}{\left(\frac{\text{m}}{\text{mm}^{1.22}}\right)\left(\frac{\frac{\text{m}}{\text{min}}}{\frac{\text{m}}{\text{min}}}\right)^{1.82}}$$

$$[0.027] \doteq \boxed{\frac{\text{O mm}^{1.22}}{\text{m}}}$$

Problem 1.2.13

The volume rate of flow, Q, through a pipe containing a slowly moving liquid is given by the equation

$$Q = \frac{\pi R^4 \Delta p}{8\mu\ell}$$

where R is the pipe radius, Δp the pressure drop along the pipe, μ is a fluid property called viscosity $\left(FL^{-2}T\right)$, and ℓ is the length of pipe. What are the dimensions of the constant $\pi/8$? Would you classify this equation as a general homogeneous equation? Explain.

SOLUTION:

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\frac{\left[L^4\right]\left[FL^{-2}\right]}{\left[FL^{-2}T\right]\left[L\right]}$$

$$\left[L^3T^{-1}\right] \doteq \left[\frac{\pi}{8}\right]\left[L^3T^{-1}\right]$$

$$\boxed{\text{The constant } \frac{\pi}{8} \text{ is dimensionless.}}$$

Yes. This is a general homogeneous equation because it is valid in any consistent units system.

Problem 1.2.14

Show that each term in the given equation has units of N/m^3. Consider u as velocity, y as length, x as length, p as pressure, and μ as absolute viscosity.

$$0 = -\frac{\partial p}{\partial x} + \mu\frac{\partial^2 u}{\partial y^2}$$

SOLUTION:

$$\left[\frac{\partial p}{\partial x}\right] = \frac{[\text{N/m}^2]}{[\text{m}]} \quad \text{or} \quad \boxed{\left[\frac{\partial p}{\partial x}\right] = [\text{N/m}^3]}$$

and

$$\left[\mu\frac{\partial^2 u}{\partial y^2}\right] = [\text{N}\cdot\text{s/m}^2]\frac{[\text{m}]}{[\text{m}^2]} \quad \text{or} \quad \boxed{\left[\mu\frac{\partial^2 u}{\partial y^2}\right] = [\text{N/m}^3]}$$

Problem 1.2.15

The pressure difference, Δp, across a partial blockage in an artery (called a stenosis) is approximated by the equation

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1 \right)^2 \rho V^2$$

where V is the blood velocity, μ is the blood viscosity $(FL^{-2}T)$, ρ is the blood density (ML^{-3}), D is the artery diameter, A_0 is the area of the unobstructed artery, and A_1 is the area of the stenosis. Determine the dimensions of the constants K_v and K_u. Would this equation be valid in any system of units?

SOLUTION:

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left[\frac{A_0}{A_1} - 1 \right]^2 \rho V^2$$

$$FL^{-2} \doteq [K_v] \frac{FT}{L^2} \frac{L}{T} \frac{1}{L} + [K_u] \left(\frac{L^2}{L^2} - 1 \right)^2 \left(\frac{FT^2}{L} \frac{1}{L^3} \right) \left(\frac{L}{T} \right)^2$$

$$FL^{-2} \doteq [K_v] \left(FL^{-2} \right) + [K_u] \left(FL^{-2} \right)$$

$\boxed{K_v \text{ and } K_u \text{ are dimensionless}}$ because each term in the equation must have the same dimensions. $\boxed{\text{Yes}}$, the equation would be valid in any consistent system of units.

Problem 1.2.16

Assume that the speed of sound, c, in a fluid depends on an elastic modulus, E_v, with dimensions FL^{-2}, and the fluid density, ρ, in the form $c = (E_v)^a (\rho)^b$. If this is to be a dimensionally homogeneous equation, what are the values for a and b? Is your result consistent with the standard formula for the speed of sound? (See the equation 1.19.)

SOLUTION:

Substituting $[c] \doteq LT^{-1}$, $[E_v] \doteq FL^{-2}$, $[\rho] \doteq FL^{-4}T^2$ into the equation provided yields:

$$\left[LT^{-1}\right] = \left[\left(FL^{-2}\right)^a\right]\left[\left(FL^{-4}T^2\right)^b\right] = F^{a+b}L^{-2a-4b}T^{2b}$$

Dimensional homogeneity requires that the exponent of each dimension on both sides of the equal sign be the same.

F: 0 = a + b
L: 1 = −2a − 4b
T: −1 = 2b

Therefore,

T: −1 = 2b → b = −1/2
F: a = −b → a = 1/2
L: 1 = −2a−4b = −2(1/2) −4(−1/2) = 1✓

$$\boxed{a = \frac{1}{2};\quad b = -\frac{1}{2}}$$

Yes, this is consistent with the standard formula for the speed of sound.

Problem 1.2.17

A formula to estimate the volume rate of flow, *Q*, flowing over a dam of length, *B*, is given by the equation

$$Q = 3.09\, BH^{\frac{3}{2}}$$

where *H* is the depth of the water above the top of the dam (called the head). This formula gives *Q* in m^3/s when *B* and *H* are in meter. Is the constant, 3.09, dimensionless? Would this equation be valid if units other than meter and seconds were used?

SOLUTION:

$$Q = 3.09\, BH^{\frac{3}{2}}$$

$$[L^3T^{-1}] \doteq \quad L][L]^{\frac{3}{2}}$$

$$[L^3T^{-1}] \doteq \quad L]^{\frac{5}{2}}$$

Since each term in the equation must have the same dimensions the constant 3.09 must have dimensions of $L^{1/2}\,T^{-1}$ and is, therefore, not dimensionless. $\boxed{\text{No}}$. Since the constant has dimensions its value will change with a change in units. $\boxed{\text{No}}$.

Problem 1.2.18

A commercial advertisement shows a pearl falling in a bottle of shampoo. If the diameter D of the pearl is quite small and the shampoo is sufficiently viscous, the drag $\mathscr{D}$ on the pearl is given by Stokes's law,

$\mathscr{D} = 3\pi\mu V D$

where V is the speed of the pearl and μ is the fluid viscosity. Show that the term on the right side of Stokes's law has units of force.

SOLUTION:

$$[\mathscr{D}] = [3\pi\mu V D] \doteq \left(\frac{M}{LT}\right)\left(\frac{L}{T}\right)L = M\frac{L}{T^2} = M\underbrace{\left[\frac{FT^2}{ML}\right]}_{\frac{1}{g_c}}\frac{L}{T^2} = \boxed{F}$$

where $\frac{1}{g_c} = \left[\frac{FT^2}{ML}\right]$ comes from Newton's Second Law.

Problem 1.2.20

Express the given quantities in SI units: **(a)** 10.2 in./min, **(b)** 4.81 slugs, **(c)** 3.02 lb, **(d)** 73.1 ft/s^2, and **(e)** 0.0234 $\text{lb}\cdot\text{s}/\text{ft}^2$.

SOLUTION:

(a) $$10.2\,\frac{\text{in.}}{\text{min}} = \left(10.2\,\frac{\text{in.}}{\text{min}}\right)\left(2.540\times10^{-2}\,\frac{\text{m}}{\text{in.}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right) = 4.32\times10^{-3}\,\frac{\text{m}}{\text{s}} = \boxed{4.32\,\frac{\text{mm}}{\text{s}}}$$

(b) $$4.81\,\text{slugs} = (4.81\,\text{slugs})\left(1.459\times10\,\frac{\text{kg}}{\text{slug}}\right) = \boxed{70.2\,\text{kg}}$$

(c) $$3.02\,\text{lb} = (3.02\,\text{lb})\left(4.448\,\frac{\text{N}}{\text{lb}}\right) = \boxed{13.4\,\text{N}}$$

(d) $$73.1\,\frac{\text{ft}}{\text{s}^2} = \left(73.1\,\frac{\text{ft}}{\text{s}^2}\right)\left(3.048\times10^{-1}\,\frac{\frac{\text{m}}{\text{s}^2}}{\frac{\text{ft}}{\text{s}^2}}\right) = \boxed{22.3\,\frac{\text{m}}{\text{s}^2}}$$

(e) $$0.0234\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2} = \left(0.0234\,\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\right)\left(4.788\times10\,\frac{\frac{\text{N}\cdot\text{s}}{\text{m}^2}}{\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}\right) = \boxed{1.12\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

Problem 1.2.21

Express the given quantities in SI units: **(a)** 160 acres, **(b)** 15 gallons (US), **(c)** 240 miles, **(d)** 79.1 hp, and **(e)** 60.3 °F.

SOLUTION:

(a) $160\text{ acres} = (160\text{ acres})\left(4.356\times10^4\ \frac{\text{ft}^2}{\text{acre}}\right)\left(9.290\times10^{-2}\ \frac{\text{m}^2}{\text{ft}^2}\right) = \boxed{6.47\times10^5\ \text{m}^2}$

(b) $15\text{ gallons} = (15\text{ gal})\left(3.785\ \frac{\text{liters}}{\text{gal}}\right)\left(10^{-3}\ \frac{\text{m}^3}{\text{liter}}\right) = \boxed{56.8\times10^{-2}\ \text{m}^3}$

(c) $240\text{ mi} = (240\text{ mi})\left(5280\ \frac{\text{ft}}{\text{mi}}\right)\left(3.048\times10^{-1}\ \frac{\text{m}}{\text{ft}}\right) = \boxed{3.86\times10^5\ \text{m}}$

(d) $79.1\text{ hp} = (79.1\text{ hp})\left(550\ \frac{\frac{\text{ft}\cdot\text{lb}}{\text{s}}}{\text{hp}}\right)\left(1.356\ \frac{\text{J}}{\text{ft}\cdot\text{lb}}\right) = 5.90\times10^4\ \frac{\text{J}}{\text{s}}\times\frac{1\text{ W}}{1\frac{\text{J}}{\text{s}}} = \boxed{5.90\times10^4\ \text{W}}$

(e) Relationship between units of temperature:

$$\text{K} = {}^\circ\text{C} + 273 = \frac{5}{9}\left({}^\circ\text{F} - 32\right) + 273$$

$$\frac{5}{9}\left(60.3\ {}^\circ\text{F} - 32\right) + 273 = \boxed{289\ \text{K}}$$

Problem 1.2.22

Water flows from a large drainage pipe at a rate of 1200 gal/min. What is the volume rate of flow in **(a)** m^3/s, **(b)** liters/min, and **(c)** ft^3/s?

SOLUTION:

(a) $\text{Flowrate} = \left(1200\ \frac{\text{gal}}{\text{min}}\right)\left(6.309\times10^{-5}\ \frac{\frac{\text{m}^3}{\text{s}}}{\frac{\text{gal}}{\text{min}}}\right) = \boxed{7.57\times10^{-2}\ \frac{\text{m}^3}{\text{s}}}$

(b) Since 1 liter $= 10^{-3}\ \text{m}^3$,

$$\text{Flowrate} = \left(7.57\times10^{-2}\ \frac{\text{m}^3}{\text{s}}\right)\left(\frac{10^3\text{ liters}}{\text{m}^3}\right)\left(\frac{60\text{ s}}{\text{min}}\right) = \boxed{4540\ \frac{\text{liters}}{\text{min}}}$$

(c) $\text{Flowrate} = \left(7.57\times10^{-2}\ \frac{\text{m}^3}{\text{s}}\right)\left(3.531\times10\ \frac{\frac{\text{ft}^3}{\text{s}}}{\frac{\text{m}^3}{\text{s}}}\right) = \boxed{2.67\ \frac{\text{ft}^3}{\text{s}}}$

Problem 1.2.23

The universal gas constant R_0 is equal to $49{,}700\ \text{ft}^2/(\text{s}^2\cdot{}^\circ\text{R})$, or $8310\ \text{m}^2/(\text{s}^2\cdot\text{K})$. Show that these two magnitudes are equal.

SOLUTION:

$$R_0 = \left(\frac{8310\,\text{m}^2}{\text{s}^2\cdot\text{K}}\right)\left(\frac{3.281\,\text{ft}}{1\,\text{m}}\right)^2\left(\frac{(5/9)\,\text{K}}{1\,{}^\circ\text{R}}\right) = \boxed{49{,}700\ \frac{\text{ft}^2}{(\text{s}^2\cdot{}^\circ\text{R})}}$$

Problem 1.2.24

Dimensionless combinations of quantities (commonly called dimensionless parameters) play an important role in fluid mechanics. Make up five possible dimensionless parameters by using combinations of some of the quantities listed in Table 1.1.

SOLUTION:

Some possible examples:

$$\frac{\text{acceleration}\times\text{time}}{\text{velocity}} \doteq \frac{(LT^{-2})(T)}{(LT^{-1})} \doteq \boxed{L^0T^0}$$

$$\text{frequency}\times\text{time} \doteq (T^{-1})(T) \doteq \boxed{T^0}$$

$$\frac{(\text{velocity})^2}{\text{length}\times\text{acceleration}} \doteq \frac{(LT^{-1})^2}{(L)(LT^{-2})} \doteq \boxed{L^0T^0}$$

$$\frac{\text{force}\times\text{time}}{\text{momentum}} \doteq \frac{(F)(T)}{(MLT^{-1})} \doteq \frac{(F)(T)}{(FT^2L^{-1})(LT^{-1})} \doteq \boxed{F^0L^0T^0}$$

$$\frac{\text{density}\times\text{velocity}\times\text{length}}{\text{dynamic viscosity}} \doteq \frac{(ML^{-3})(LT^{-1})(L)}{ML^{-1}T^{-1}} \doteq \boxed{M^0L^0T^0}$$

Problem 1.2.25

An important dimensionless parameter in certain types of fluid flow problems is the *Froude number* defined as $V/\sqrt{g\ell}$, where V is velocity, g is the acceleration of gravity, and ℓ is length. Determine the value of the Froude number for $V = 10 \text{ ft/s}$, $g = 32.2 \text{ ft/s}^2$, and $\ell = 2 \text{ ft}$. Recalculate the Froude number using SI units for V, g, and ℓ. Explain the significance of the results of these calculations.

SOLUTION:

In BG units,

$$\frac{V}{\sqrt{g\ell}} = \frac{10 \ \frac{\text{ft}}{\text{s}}}{\sqrt{\left(32.2 \frac{\text{ft}}{\text{s}^2}\right)(2 \text{ ft})}} = \boxed{1.25}$$

In SI units:

$$V = \left(10 \frac{\text{ft}}{\text{s}}\right)\left(0.3048 \frac{\text{m}}{\text{ft}}\right) = 3.05 \frac{\text{m}}{\text{s}}$$

$$g = 9.81 \ \frac{\text{m}}{\text{s}^2}$$

$$\ell = (2 \text{ ft})\left(0.3048 \frac{\text{m}}{\text{ft}}\right) = 0.610 \text{ m}$$

Thus,

$$\frac{V}{\sqrt{g\ell}} = \frac{3.05 \ \frac{\text{m}}{\text{s}}}{\sqrt{\left(9.81 \ \frac{\text{m}}{\text{s}^2}\right)(0.610 \text{ m})}} = \boxed{1.25}$$

The value of a dimensionless parameter is independent of the unit system.

Section 1.4 **Measures of Fluid Mass and Weight**

Problem 1.4.2

A tank contains 500 kg of a liquid whose specific gravity is 2. Determine the volume of the liquid in the tank.

SOLUTION:

$$m = \rho V = SG\rho_{H_2O}V$$

Thus,

$$V = \frac{m}{\left(SG\rho_{H_2O}\right)} = \frac{500 \text{ kg}}{\left((2)\left(999 \frac{\text{kg}}{\text{m}^3}\right)\right)} = \boxed{0.250 \text{ m}^3}$$

Problem 1.4.3

A stick of butter at 25°C measures 31.75 mm × 31.75 mm × 118.11 mm and weighs 113.4 g. Find its specific weight.

SOLUTION:

$$\gamma = \frac{W}{\forall} = \frac{(113.4\text{ g})\left(\dfrac{1\text{ kg}}{1000\text{ g}}\right)}{(31.75\text{ mm})^2(118.11\text{ mm})\left(\dfrac{1\text{ m}}{1000\text{ mm}}\right)^3} = 952.44\text{ kg/m}^3$$

Problem 1.4.4

Clouds can weigh thousands of pounds due to their liquid water content. Often this content is measured in grams per cubic meter $\left(\text{g/m}^3\right)$. Assume that a cumulus cloud occupies a volume of 1 cubic kilometer, and its liquid water content is $0.2\ \text{g/m}^3$. **(a)** What is the volume of this cloud in cubic miles? **(b)** How much does the water in the cloud weigh in pounds?

SOLUTION:

(a) $\text{Volume} = (1\text{ km})^3\left(\dfrac{1000\text{ m}}{1\text{ km}}\right)^3 = 10^9\text{ m}^3\left(\dfrac{3.281\text{ ft}}{1\text{ m}}\times\dfrac{1\text{ m}}{5280\text{ ft}}\right)^3 = \boxed{0.240\text{ mi}^3}$

(b) $W = \gamma \times \text{Volume}$

$$\gamma = \rho g = \left(0.2\frac{\text{g}}{\text{m}^3}\right)\left(\frac{1\text{ kg}}{1000\text{ g}}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\text{ N}\cdot\text{s}^2}{1\text{ kg}\cdot\text{m}}\right) = 1.962\times10^{-3}\frac{\text{N}}{\text{m}^3}$$

$$W = \left(1.962\times10^{-3}\frac{\text{N}}{\text{m}^2}\right)\left(10^9\text{ m}^3\right)\left(2.248\times10^{-1}\frac{\text{lb}}{\text{N}}\right) = \boxed{4.4\times10^5\text{ lb}}$$

Problem 1.4.5

A certain object weighs 300 N at the Earth's surface. Determine the mass of the object (in kilograms) and its weight (in Newtons) when located on a planet with an acceleration of gravity equal to 1.22 m/s^2.

SOLUTION:

$$\text{Mass} = \frac{\text{weight}}{g} = \frac{300\text{ N}}{9.81\dfrac{\text{m}}{\text{s}^2}} = \boxed{30.6\text{ kg}}$$

For $g = 1.22\text{ m/s}^2$

$$\text{Weight} = (30.6\text{ kg})(1.22\text{ m/s}^2)\left(\frac{1\text{ N}\cdot\text{s}^2}{1\text{ kg}\cdot\text{m}}\right) = \boxed{37.3\text{ N}}$$

Problem 1.4.6

The density of a certain type of jet fuel is $775\ \text{kg/m}^3$. Determine its specific gravity and specific weight.

SOLUTION:

$$SG = \frac{\rho}{\rho_{H_2O@4\,°C}} = \frac{775\,\dfrac{\text{kg}}{\text{m}^3}}{1000\,\dfrac{\text{kg}}{\text{m}^3}} = \boxed{0.775}$$

$$\gamma = \rho g = \left(775\,\frac{\text{kg}}{\text{m}^3}\right)\left(9.81\,\frac{\text{m}}{\text{s}^2}\right) = \boxed{7.60\,\frac{\text{kN}}{\text{m}^3}}$$

Problem 1.4.7

At $4\,°\text{C}$ a mixture of automobile antifreeze (50% water and 50% ethylene glycol by volume) has a density of $1064\,\text{kg/m}^3$. If the water density is $1000\,\text{kg/m}^3$, find the density of the ethylene glycol.

SOLUTION:

$$S = \frac{(\rho_{\text{mixture}})_{4°\text{C}}}{(\rho_{\text{water}})_{4°\text{C}}} = \frac{\left(\dfrac{m_{eg}+m_w}{\forall}\right)_{4°\text{C}}}{\left(\dfrac{m_w}{\forall}\right)_{4°\text{C}}} = \frac{m_{eg}+m_w}{m_w}$$

where m_w is the mass of the pure water in volume $\forall$ at $4\,°\text{C}$.
Then,

$$S = \frac{\rho_{eg}(0.5\forall)+\rho_w(0.5\forall)}{\rho_w\forall} = 0.5\left(\frac{\rho_{eg}+\rho_w}{\rho_w}\right)$$

The problem statement gives

$$S = \frac{\rho_{\text{mixture}}}{\rho_w} = \frac{1064\,\dfrac{\text{kg}}{\text{m}^3}}{1000\,\dfrac{\text{kg}}{\text{m}^3}} = 1.064 \ \rightarrow\ 1.064 = 0.5\left(\frac{\rho_{eg}+\rho_w}{\rho_w}\right) \ \rightarrow\ \rho_{eg} = \rho_w\left(\frac{1.064}{0.5}-1\right)$$

$$= \left(1000\,\frac{\text{kg}}{\text{m}^3}\right)\left(\frac{1.064}{0.5}-1\right)\boxed{\rho_{eg} = 1130\,\frac{\text{kg}}{\text{m}^3}}$$

DISCUSSION: If the mixture were at some temperature T, then for equal volumes of mixture and $4\,°\text{C}$ water,

$$S = \frac{(m_{eg}+m_w)_T}{(m_w)_{4°\text{C}}} = \frac{0.5(\rho_{eg}+\rho_w)_T}{(\rho_w)_{4°\text{C}}}$$

Problem 1.4.8

A *hydrometer* is used to measure the specific gravity of liquids (See Vedio V2.8). For a certain liquid, a hydrometer reading indicates a specific gravity of 1.15. What is the liquid's density and specific weight? Express your answer in SI units.

SOLUTION:

$$SG = \frac{\rho}{\rho_{H_2O\,@4\,°C}}$$

$$1.15 = \frac{\rho}{1000\,\frac{kg}{m^3}}$$

$$\rho = (1.15)\left(1000\ \frac{kg}{m^3}\right) = \boxed{1150\,\frac{kg}{m^3}}$$

$$\gamma = \rho g = \left(1150\ \frac{kg}{m^3}\right)\left(9.81\,\frac{m}{s^2}\right)\left(\frac{1\ N\cdot s^2}{1\ kg\cdot m}\right) = \boxed{11.3\,\frac{kN}{m^3}}$$

Problem 1.4.9

The information on a can of pop indicates that the can contains 355 mL. The mass of a full can of pop is 0.369 kg, while an empty can weighs 0.153 N. Determine the specific weight, density, and specific gravity of the pop and compare your results with the corresponding values for water at 20 °C. Express your results in SI units.

SOLUTION:

$$\gamma = \frac{\text{weight of fluid}}{\text{volume of fluid}}$$

$$\text{Total weight} = \text{mass} \times g = (0.369\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}}\right) = 3.62\ \text{N}$$

$$\text{Weight of can} = 0.153\ \text{N}$$

$$\text{Volume of fluid} = \left(355\times10^{-3}\ \text{liter}\right)\left(\frac{1\ \text{m}^3}{1000\ \text{liter}}\right) = 355\times10^{-6}\ \text{m}^3$$

Therefore,

$$\gamma = \frac{3.62\ \text{N} - 0.153\ \text{N}}{355\times10^{-6}\ \text{m}^3} = \boxed{9770\ \frac{\text{N}}{\text{m}^3}}$$

$$\rho = \frac{\gamma}{g} = \frac{9770\ \frac{\text{N}}{\text{m}^3}}{9.81\ \frac{\text{m}}{\text{s}^2}} = 996\ \frac{\text{N}\cdot\text{s}^2}{\text{m}^4}\left(\frac{1\ \text{kg}\cdot\text{m}}{1\ \text{N}\cdot\text{s}^2}\right) = \boxed{996\ \frac{\text{kg}}{\text{m}^3}}$$

$$SG = \frac{\rho}{\rho_{\text{H}_2\text{O}\,@4\,°\text{C}}} = \frac{996\ \frac{\text{kg}}{\text{m}^3}}{1000\ \frac{\text{kg}}{\text{m}^3}} = \boxed{0.996}$$

For water at 20 °C (see Table B.2 Physical Properties of Water [SI Units])

$$\gamma_{\text{H}_2\text{O}} = 9789\ \frac{\text{N}}{\text{m}^3}; \quad \rho_{\text{H}_2\text{O}} = 998.2\ \frac{\text{kg}}{\text{m}^3}; \quad SG = 0.9982$$

A comparison of these values for water with those for the pop shows that the specific weight, density, and specific gravity of the pop are all slightly lower than the corresponding values for water.

Problem 1.4.10

The variation in the density of water, ρ, with temperature, T, in the range $20\ °C \le T \le 50\ °C$, is given in the table here.

Density (kg/m³)	998.2	997.1	995.7	994.1	992.2	990.2	988.1
Temperature (°C)	20	25	30	35	40	45	50

Use these data to determine an empirical equation of the form $\rho = c_1 + c_2 T + c_3 T^2$, which can be used to predict the density over the range indicated. Compare the predicted values with the data given. What is the density of water at 42.1 °C?

SOLUTION:

Fit the data to a second order polynomial using a standard curve-fitting program such as found in Excel. Thus,

$$\rho = \boxed{1001 - 0.0533T - 0.0041T^2}$$

As shown in the table here, ρ (predicted) from Eq. 1 is in good agreement with ρ (given).

T (°C)	ρ-predict (kg/m^3)	ρ-data (kg/m^3)
20	998.3	998.2
25	997.1	997.1
30	995.7	995.7
35	994.1	994.1
40	992.3	992.2
45	990.3	990.2
50	988.1	988.1

At T = 42.1 °C: $\rho = 1001 - 0.0533(42.1\ °C) - 0.0041(42.1\ °C)^2 = \boxed{991.5\ \dfrac{kg}{m^3}}$

Problem 1.4.11

If one cup of cream having a density of 1005 kg/m^3 is turned into three cups of whipped cream, determine the specific gravity and specific weight of the whipped cream.

SOLUTION:

Mass of cream, $m = \left(1005 \frac{kg}{m^3}\right) \times \left(V_{cup}\right)$, where V = volume

Noting that the mass is the same in liquid and in "whipped" form,

$$\rho_{\substack{whipped\\cream}} = \frac{m_{\substack{whipped\\cream}}}{V_{3\,cups}} = \frac{\left(1005 \frac{kg}{m^3}\right) V_{cup}}{V_{3\,cups}} = \frac{1005 \frac{kg}{m^3}}{3} = 335 \frac{kg}{m^3}$$

$$SG = \frac{\rho_{\substack{whipped\\cream}}}{\rho_{H_2O @ 4°C}} = \frac{335 \frac{kg}{m^3}}{1000 \frac{kg}{m^3}} = \boxed{0.335}$$

$$\gamma_{\substack{whipped\\cream}} = \rho_{\substack{whipped\\cream}} \times g = \left(335 \frac{kg}{m^3}\right)\left(9.81 \frac{m}{s^2}\right)\left(\frac{1\,N \cdot s^2}{1\,kg \cdot m}\right) = \boxed{3290 \frac{N}{m^3}}$$

Section 1.5 **Ideal Gas Law**

Problem 1.5.1

Nitrogen is compressed to a density of 4 kg/m^3 under an absolute pressure of 400 kPa. Determine the temperature in degrees Celsius.

SOLUTION:

$$T = \frac{p}{\rho R} = \frac{400 \times 10^3 \frac{N}{m^2}}{\left(4 \frac{kg}{m^3}\right)\left(296.8 \frac{J}{kg \cdot K}\right)\left(\frac{1 \frac{N \cdot m}{s}}{1\,J}\right)} = 337\text{ K}$$

$$T_C = T_K - 273 = 337\text{ K} - 273 = \boxed{64\ °C}$$

Problem 1.5.2

The temperature and pressure at the surface of Mars during a Martian spring day were determined to be $-50\ ^\circ\text{C}$ and $900\ \text{Pa}$, respectively. **(a)** Determine the density of the Martian atmosphere for these conditions if the gas constant for the Martian atmosphere is assumed to be equivalent to that of carbon dioxide. **(b)** Compare the answer from part (a) with the density of the Earth's atmosphere during a spring day when the temperature is $18\ ^\circ\text{C}$ and the pressure $101.6\ \text{kPa (abs)}$.

SOLUTION:

(a) $$\rho_{\text{Mars}} = \frac{p}{RT} = \frac{900\ \dfrac{\text{N}}{\text{m}^2}}{\left(188.9\ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(\dfrac{1\ \frac{\text{N}\cdot\text{m}}{\text{s}}}{1\ \text{J}}\right)\left[\left(-50\ ^\circ\text{C}+273\right)\text{K}\right]} = \boxed{0.0214\ \frac{\text{kg}}{\text{m}^3}}$$

(b) $$\rho_{\text{Earth}} = \frac{p}{RT} = \frac{101.6\times10^3\ \dfrac{\text{N}}{\text{m}^2}}{\left(286.9\ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(\dfrac{1\ \frac{\text{N}\cdot\text{m}}{\text{s}}}{1\ \text{J}}\right)\left[\left(18\ ^\circ\text{C}+273\right)\text{K}\right]} = \boxed{1.22\ \frac{\text{kg}}{\text{m}^3}}$$

Thus, $$\frac{\rho_{\text{Mars}}}{\rho_{\text{Earth}}} = \frac{0.0214\ \dfrac{\text{kg}}{\text{m}^3}}{1.22\ \dfrac{\text{kg}}{\text{m}^3}} = 0.0175 = \boxed{1.75\%}$$

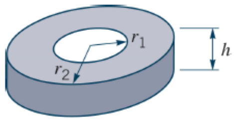
Problem 1.5.3

Assume that the air volume in a small automobile tire is constant and equal to the volume between two concentric cylinders 13 cm high with diameters of 33 cm and 52 cm. The air in the tire is initially at 25 °C and 202 kPa. Immediately after air is pumped into the tire, the temperature is 30 °C and the pressure is 303 kPa. What mass of air was added to the tire? What would be the air pressure after the air has cooled to a temperature of 0 °C?

SOLUTION:

The mass of air added to the tire is the difference of the final mass of air m_f and the initial mass m_i. Assuming air is an ideal gas,

$$m_f - m_i = \left(\frac{p}{RT}\forall\right)_f - \left(\frac{p}{RT}\forall\right)_i = \frac{\forall}{R}\left(\frac{p_f}{T_f} - \frac{p_i}{T_i}\right)$$



Now

$$\forall = \pi\left(r_2^2 - r_1^2\right)h = \pi\left[(26\text{ cm})^2 - (16.5\text{ cm})^2\right](13\text{ cm})\left(\frac{1\text{ m}}{100\text{ cm}}\right)^3$$

$$= 0.0165\text{ m}^3$$

$$m_f - m_i = \frac{\left(0.0165\text{ m}^3\right)}{\left(287.0\frac{\text{N}\cdot\text{m}}{\text{kg}\cdot\text{K}}\right)}\left[\frac{303\text{ kPa}}{(273+30)\text{ K}} - \frac{202\text{ kPa}}{(273+25)\text{ K}}\right]\left(\frac{1000\frac{\text{N}}{\text{m}^2}}{\text{kPa}}\right)$$

$$\boxed{m_f - m_i = 0.0185\text{ kg}}$$

Now consider the cooling process. The initial state will be 30 °C and 303 kPa. The final state will be 0 °C and p_f. Applying the ideal gas law to both states gives

$$\left(\frac{p}{RT}\forall\right)_i = \left(\frac{p}{RT}\forall\right)_f$$

Since $\forall_f = \forall_i$

$$p_f = p_i\left(\frac{T_f}{T_i}\right) = (303\text{ kPa})\left(\frac{273+0}{273+30}\right) = \boxed{p_f = 273\text{ kPa}}$$

Problem 1.5.4

A compressed air tank contains 5 kg of air at a temperature of 80 °C. A gage on the tank reads 300 kPa. Determine the volume of the tank.

SOLUTION:

$$\text{Volume} = \frac{\text{mass}}{\rho}$$

$$\rho = \frac{p}{RT} = \frac{(300+101)\times 10^3 \ \frac{\text{N}}{\text{m}^3}}{\left(286.9\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(\frac{1\ \frac{\text{N}\cdot\text{m}}{\text{s}}}{1\ \text{J}}\right)\left[(80\ °\text{C}+273)\,\text{K}\right]} = 3.96\ \frac{\text{kg}}{\text{m}^3}$$

$$\text{Volume} = \frac{5\ \text{kg}}{3.96\ \frac{\text{kg}}{\text{m}^3}} = \boxed{1.26\ \text{m}^3}$$

Problem 1.5.5

A rigid tank contains air at pressure of 620.5 kPa and a temperature of 15.5 °C. By how much will the pressure increase as the temperature is increased to 43.3 °C?

SOLUTION:

$p = \rho RT$

For a rigid closed tank, the air mass and volume are constant so ρ = constant. Thus, from the equation here (with R constant)

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \textbf{(1)}$$

where $p_1 = 620.5\ \text{kPa}, T_1 = 15.5\ °\text{C} + 273 = 288.5\ \text{K}$ and $T_2 = 43.3\ °\text{C} + 273 = 316.3\ \text{K}$

From Eq. 1

$$p_2 = \frac{T_2}{T_1} p_1 = \left(\frac{316.3\ \text{K}}{288.5\ \text{K}}\right)(620.5\ \text{kPa}) = \boxed{680.3\ \text{kPa}}$$

Problem 1.5.6

The density of oxygen contained in a tank is 2.0 kg/m^3 when the temperature is 25 °C. Determine the gage pressure of the gas if the atmospheric pressure is 97 kPa.

SOLUTION:

$$p = \rho RT = \left(2.0\ \frac{\text{kg}}{\text{m}^3}\right)\left(259.8\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left[(25\ °\text{C}+273)\ \text{K}\right] = 155\ \text{kPa (abs)}$$

$$p(\text{gage}) = P_{\text{abs}} - P_{\text{atm}} = 155\ \text{kPa} - 97\ \text{kPa} = \boxed{58\ \text{kPa}}$$

Problem 1.5.7

Develop a computer program for calculating the density of an ideal gas when the gas pressure in Pascals (abs), the temperature in degrees Celsius, and the gas constant in $\text{J/kg}\cdot\text{K}$ are specified. Plot the density of helium as a function of temperature from 0 °C to 200 °C and pressures of 50, 100, 150, and 200 kPa (abs).

SOLUTION:

For an ideal gas

$$p = \rho \text{R} T$$

so that

$$\rho = \frac{p}{\text{R}T}$$

where p is the absolute pressure, R is the gas constant, and T is the absolute temperature. Thus, if the temperature is in °C then

$$T = °\text{C} + 273.15 \text{ K}$$

A spreadsheet (Excel) program for calculating ρ is given here.

This program calculates the density of an ideal gas when the absolute pressure in Pascal, the temperature in degrees C, and the gas constant in J/kg·K are specified. To use, replace current values with desired values of temperature, pressure, and gas constant.

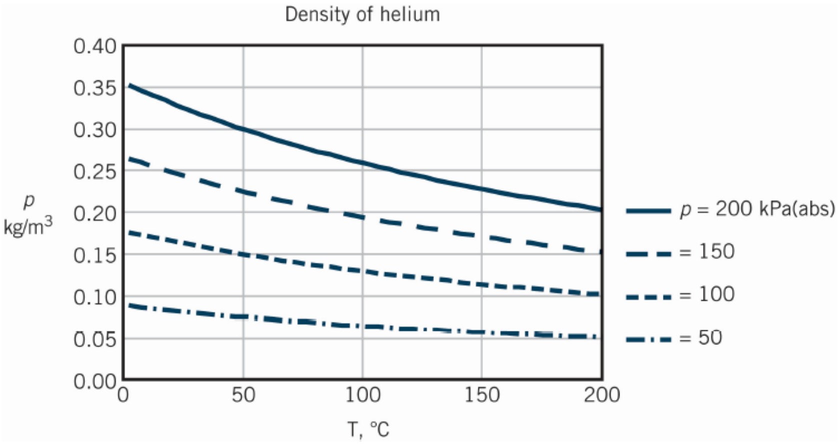
A	B	C	D	
Pressure	Temperature	Gas constant	Density	
Pa	°C	J/kg·K	kg/m³	
1.01E+05	15	286.9	1.22	

Formula:
=A10/((B10+273.15)*C10)

Example: Calculate ρ for $p = 200$ kPa, temperature = 20 °C, and $R = 287 \dfrac{\text{J}}{\text{kg}\cdot\text{K}}$

This program calculates the density of an ideal gas when the absolute pressure in Pascal, the temperature in degrees Celsius, and the gas constant in J/kg·K are specified. To use, replace current values with desired values of temperature, pressure, and gas constant.			
A	B	C	D
Pressure	Temperature	Gas constant	Density
Pa	°C	J/kg·K	kg/m³
2.00E+05	20	287	2.38

The density of helium is plotted in the graph here.



Section 1.6 **Viscosity**

Problem 1.6.2

For flowing water, what is the magnitude of the velocity gradient needed to produce a shear stress of $1.0\ \frac{\text{N}}{\text{m}^2}$?

SOLUTION:

$$\tau = \mu \frac{du}{dy} \text{ where } \mu = 1.12 \times 10^{-3}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2} \text{ and } \tau = 1.0\ \frac{\text{N}}{\text{m}^2}$$

Thus,

$$\frac{du}{dy} = \frac{\tau}{\mu} = \frac{1.0\ \frac{\text{N}}{\text{m}^2}}{1.12 \times 10^{-3}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}} = \boxed{893\ \frac{1}{\text{s}}}$$

Problem 1.6.3

Make use of the data in Appendix B to determine the dynamic viscosity of glycerin at 85 °F. Express your answer in both SI and BG units.

SOLUTION:

$$T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(85\ ^\circ\text{F} - 32) = 29.4\ ^\circ\text{C}$$

From the figure in Appendix B:

$$\mu(\text{glycerin at } 85\ ^\circ\text{F }(29.4\ ^\circ\text{C})) \approx \boxed{0.6\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}\ (\text{SI units})}$$

$$\mu \approx \left(0.6\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(2.089\times10^{-2}\ \frac{\frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}{\frac{\text{N}\cdot\text{s}}{\text{m}^2}}\right) \approx \boxed{1.3\times10^{-2}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}\ (\text{BG units})}$$

Problem 1.6.4

One type of *capillary-tube viscometer* is shown in adjacent Fig. P1.6.4. For this device the liquid to be tested is drawn into the tube to a level above the top etched line. The time is then obtained for the liquid to drain to the bottom etched line. The kinematic viscosity, ν, in m^2/s is then obtained from the equation $\nu = KR^4 t$, where K is a constant, R is the radius of the capillary tube in mm, and t is the drain time in seconds. When glycerin at $20\ °C$ is used as a calibration fluid in a particular viscometer, the drain time is $1430\ s$. When a liquid having a density of $970\ kg/m^3$ is tested in the same viscometer the drain time is $900\ s$. What is the dynamic viscosity of this liquid?

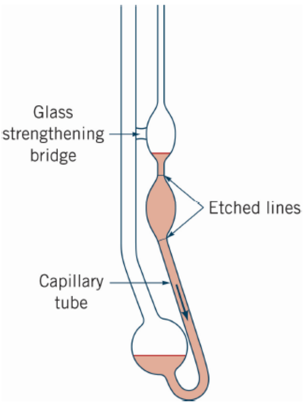


Figure P1.6.4

SOLUTION:

$\nu = KR^4 t$

For glycerin @ 20 °C $\nu = 1.19\times10^{-3}\ \dfrac{m^2}{s^2} = \left(KR^4\right)(1430\ s)$

$KR^4 = 8.32\times10^{-7}\ \dfrac{m^2}{s^2}$

For unknown liquid with $t = 900\ s$

$$\nu = \left(8.32\times10^{-7}\ \frac{m^2}{s^2}\right)(900\ s) = 7.49\times10^{-4}\ \frac{m^2}{s^2}$$

By definition: $\nu \equiv \dfrac{\mu}{\rho} \rightarrow \quad \mu = \left(970\ \dfrac{kg}{m^3}\right)\left(7.49\times10^{-4}\ \dfrac{m^2}{s}\right) = 0.727\ \dfrac{kg}{m\cdot s}\times\dfrac{1\ N\cdot s^2}{1\ kg\cdot m}$

$$\boxed{\mu = 0.727\ \frac{N\cdot s}{m^2}}$$

Problem 1.6.5

The viscosity of a soft drink was determined by using a capillary tube viscometer similar to that shown in Fig. P1.6.4 and **Video V1.5**. For this device, the kinematic viscosity, ν, is directly proportional to the time, *t*, that it takes for a given amount of liquid to flow through a small capillary tube. That is, $\nu = K\,t$. The following data were obtained from regular pop and diet pop. The corresponding measured specific gravities are also given. Based on these data, by what percent is the absolute viscosity, μ, of regular pop greater than that of diet pop?

	Regular pop	Diet pop
t(s)	377.8	300.3
SG	1.044	1.003

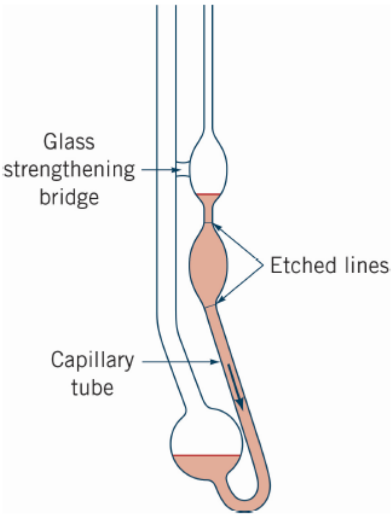


Figure P1.6.4

SOLUTION:

$$\%\text{ greater} = \left[\frac{\mu_{\text{reg}} - \mu_{\text{diet}}}{\mu_{\text{diet}}}\right]\times 100 = \left[\frac{\mu_{\text{reg}}}{\mu_{\text{diet}}} - 1\right]\times 100$$

By definition $\nu \equiv \dfrac{\mu}{\rho}$, and $\rho \equiv (SG)\,\rho_{\text{H}_2\text{O @4 °C}}$.

Given $\nu = kt$:

$$\%\text{ greater} = \left[\frac{(\nu\rho)_{\text{reg}}}{(\nu\rho)_{\text{diet}}} - 1\right]\times 100 = \left[\frac{(t\times SG)_{\text{reg}}}{(t\times SG)_{\text{diet}}} - 1\right]\times 100 = \left[\frac{(377.8\text{ s})(1.044)}{(300.3\text{ s})(1.003)} - 1\right]\times 100$$

$$\boxed{\%\text{ greater} = 31.0\%}$$

Problem 1.6.6

The viscosity of a certain fluid is 5×10^{-4} poise. Determine its viscosity in both SI and BG units.

SOLUTION:

From Appendix E, $1 \text{ poise} = 10^{-1}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}$

Thus, $\mu = \left(5\times10^{-4}\ \text{poise}\right)\left(\dfrac{10^{-1}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}}{1\ \text{poise}}\right) = \boxed{5\times10^{-5}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}}$

From Table 1.4 Conversion Factors from SI Units to BG and EE Units (end paper)

$$\mu = \left(5\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(\frac{2.089\times10^{-2}\ \dfrac{\text{lb}\cdot\text{s}}{\text{ft}^2}}{1\dfrac{\text{N}\cdot\text{s}}{\text{m}^2}}\right) = \boxed{10.4\times10^{-7}\ \frac{\text{lb}\cdot\text{s}}{\text{ft}^2}}$$

Problem 1.6.7

The kinematic viscosity and specific gravity of a liquid are $3.5\times10^{-4}\ \dfrac{\text{m}^2}{\text{s}}$ and 0.79, respectively. What is the dynamic viscosity of the liquid in SI units?

SOLUTION:

$\mu = \nu\rho$

$\rho = (SG)\left(\rho_{\text{H}_2\text{O @ 4 °C}}\right)$

$$\mu = \left(3.5\times10^{-4}\ \frac{\text{m}^2}{\text{s}}\right)\left(0.79\times10^{3}\ \frac{\text{kg}}{\text{m}^3}\right) = 0.277\ \frac{\text{kg}}{\text{m}\cdot\text{s}} \times \frac{1\,\text{N}\cdot\text{s}^2}{1\,\text{kg}\cdot\text{m}} = \boxed{0.277\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

Problem 1.6.8

A liquid has a specific weight of 9268 N/m^3 and a dynamic viscosity of 131.6 N·s/m^2. Determine its kinematic viscosity.

SOLUTION:

By definition: $\nu \equiv \dfrac{\mu}{\rho}$, and $\rho \equiv \dfrac{\gamma}{g}$

$$\nu = \frac{\mu g}{\gamma} = \frac{(131.6\ \text{N}\cdot\text{s/m}^2)(9.81\ \text{m/s}^2)}{9268\ \text{N/m}^3} = \boxed{0.139\ \text{m}^2/\text{s}}$$

Problem 1.6.9

The kinematic viscosity of oxygen at 20 °C and a pressure of 150 kPa (abs) is 0.104 stokes. Determine the dynamic viscosity of oxygen at this temperature and pressure.

SOLUTION:

$$\nu \equiv \frac{\mu}{\rho}$$

$$\rho = \frac{p}{RT} = \frac{150\times 10^3 \ \dfrac{\text{N}}{\text{m}^2}}{\left(259.8 \ \dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)\left(\dfrac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)\left[(20\ ^\circ\text{C}+273)\ \text{K}\right]} = 1.97 \ \frac{\text{kg}}{\text{m}^3}$$

$$\nu = 0.104 \text{ stokes} = 0.104 \ \frac{\text{cm}^2}{\text{s}}$$

$$\mu = \nu\rho = \left(0.104 \ \frac{\text{cm}^2}{\text{s}}\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right)^2\left(1.97 \ \frac{\text{kg}}{\text{m}^3}\right) = 2.05\times 10^{-5} \ \frac{\text{kg}}{\text{m}\cdot\text{s}} \times \frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}} = \boxed{2.05\times 10^{-5} \ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

Problem 1.6.10

Calculate the Reynolds numbers for the flow of water and for air through a 4-mm-diameter tube, if the mean velocity is 3 m/s and the temperature is 30°C in both cases (see Example 1.4). Assume the air is at standard atmospheric pressure.

SOLUTION:

For water at 30 °C (from Table B.2 Physical Properties of Water [SI Units]):

$$\rho = 995.7 \frac{\text{kg}}{\text{m}^3}$$

$$\mu = 7.975\times 10^{-4} \ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{\left(995.7 \ \dfrac{\text{kg}}{\text{m}^3}\right)\left(3\dfrac{\text{m}}{\text{s}}\right)(0.004\ \text{m})}{7.975\times 10^{-4} \ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}} \times \frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}} = \boxed{15{,}000}$$

For air at 30 °C (from the Physical Properties of Air at Standard Atmospheric Pressure [SI Units]):

$$\rho = 1.165 \ \frac{\text{kg}}{\text{m}^3}$$

$$\mu = 1.86\times 10^{-5} \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{\left(1.165 \ \dfrac{\text{kg}}{\text{m}^3}\right)\left(3\dfrac{\text{m}}{\text{s}}\right)(0.004\ \text{m})}{1.86\times 10^{-5} \ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}} \times \frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}} = \boxed{752}$$

Problem 1.6.11

SAE 30 oil at 15.6 °C flows through a 0.05 m-diameter pipe with a mean velocity of 1.5 m/s. Determine the value of the Reynolds number (see Example 1.4).

SOLUTION:

$\rho = 912.2 \text{ kg/m}^3$

$\mu = 0.383 \text{ N·s/m}^2$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(912.2 \text{ kg/m}^3)(1.5 \text{ m/s})(0.05 \text{ m})}{0.383 \text{ N·s/m}^2} = \boxed{179}$$

Problem 1.6.12

For air at standard atmospheric pressure the values of the constants that appear in the Sutherland equation (Eq. 1.10) are $C = 1.458 \times 10^{-6}$ kg/(m · s · $K^{1/2}$) and $S = 110.4$ K . Use these values to predict the viscosity of air at 10°C and 90°C and compare with values given in Table B.4 in Appendix B.

SOLUTION:

$$\mu = \frac{CT^{\frac{3}{2}}}{T+S} = \frac{\left(1.458\times10^{-6}\ \frac{\text{kg}}{\text{m}\cdot\text{s}\cdot\text{K}^{1/2}}\right)T^{3/2}}{T+110.4\text{ K}}$$

For $T = 10\ ^\circ\text{C} \rightarrow T = 10 + 273.15\text{ K} = 283.15\text{ K}$

$$\mu = \frac{\left(1.458\times10^{-6}\ \frac{\text{kg}}{\text{m}\cdot\text{s}\cdot\text{K}^{1/2}}\right)(283.15\text{ K})^{\frac{3}{2}}}{283.15\text{ K} + 110.4\text{ K}}$$

$$\mu = 1.765\times10^{-5}\ \frac{\text{kg}}{\text{m}\cdot\text{s}}\text{K}^{3/2-1/2-1} \times \frac{1\text{ N}\cdot\text{s}^2}{1\text{ kg}\cdot\text{m}} = \boxed{1.765\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

From the table, $\mu = 1.76\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$

For $T = 90\ ^\circ\text{C} = 90\ ^\circ\text{C} + 273.15 = 363.15\text{ K}$,

$$\mu = \frac{\left(1.458\times10^{-6}\right)(363.15)^{3/2}}{363.15+110.4} = \boxed{2.13\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

From the table, $\mu = 2.14\times10^{-5}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$

Problem 1.6.13

Use the values of viscosity of air given in Table B.4 at temperatures of 0, 20, 40, 60, 80, and 100 °C to determine the constants C and S that appear in the Sutherland equation (Eq. 1.10). Compare your results with the values given in Problem 1.6.14. *Hint:* Rewrite the equation in the form

$$\frac{T^{3/2}}{\mu} = \left(\frac{1}{C}\right)T + \frac{S}{C}$$

and plot $T^{3/2}/\mu$ vs. T. From the slope and intercept of this curve, C and S can be obtained.

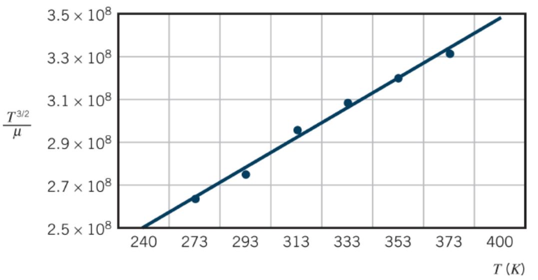
SOLUTION:

Equation $\mu = \dfrac{CT^{3/2}}{T+S}$ can be written in the form $\dfrac{T^{\frac{3}{2}}}{\mu} = \left(\dfrac{1}{C}\right)T + \dfrac{S}{C}$ **(1)**

Entering the specified temperatures and the corresponding viscosities from the table yields the 1st and 3rd columns in the following table. The 2nd column is a conversion of the temperatures to an absolute scale and the 4th column contains the values of the LHS of Eq. (1) for these temperatures and viscosities.

T (°C)	T (K)	$\mu\left(\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)$	$\frac{T^{3/2}}{\mu}\left[\frac{K^{3/2}}{\left(\frac{\text{kg}}{\text{m}\cdot\text{s}}\right)}\right]$
0	273.15	1.71×10^{-5}	2.640×10^{8}
20	293.15	1.82×10^{-5}	2.758×10^{8}
40	313.15	1.87×10^{-5}	2.963×10^{8}
60	333.15	1.97×10^{-5}	3.087×10^{8}
80	353.15	2.07×10^{-5}	3.206×10^{8}
100	373.15	2.17×10^{-5}	3.322×10^{8}

Plotting $\dfrac{T^{3/2}}{\mu}$ vs. T yields:



A polynomial of order one, which is a straight line, would be a reasonable fit for the data. Using Excel to determine the constants for a fit of the data to a straight line given by

$y = bx + a$

where $x = T$, $y = \dfrac{T^{3/2}}{\mu}$, $b = \dfrac{1}{C}$, and $a = \dfrac{S}{C}$.

yields: $y = 6.969\times10^{5}x + 7.441\times10^{7}$.

Therefore: $\dfrac{1}{C} = b = 6.969\times10^{5}$

$$\boxed{C = 1.43\times10^{-6}\ \frac{\text{kg}}{\left(\text{m}\cdot\text{s}\cdot\text{K}^{3/2}\right)}}$$

And $\dfrac{S}{C} = a = 7.441\times10^{7}$

$$\boxed{S = 107\ \text{K}}$$

These values for C and S are in good agreement with values given in Problem 1.6.14.

Problem 1.6.14

Use the value of the viscosity of water given in Table B.2 at temperatures of 0, 20, 40, 60, 80, and 100 °C to determine the constants D and B that appear in Andrade's equation (Eq. 1.11). Calculate the value of the viscosity at 50 °C and compare with the value given in Table B.2. *Hint:* Rewrite the equation in the form

$$\ln \mu = (B)\frac{1}{T} + \ln D$$

and plot $\ln \mu$ vs. $1/T$. From the slope and intercept of this curve, B and D can be obtained. If a nonlinear curve-fitting program is available, the constants can be obtained directly from Eq. 1.11 without rewriting the equation.

SOLUTION:

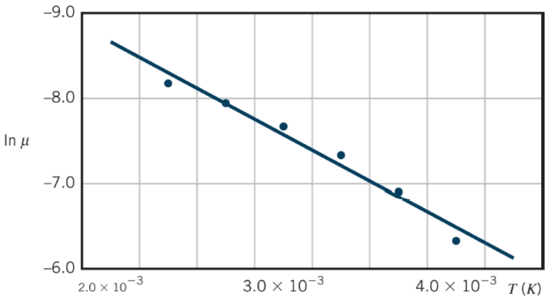
Equation $\mu = De^{B/T}$ can be written in the form

$$\ln \mu = (B)\frac{1}{T} + \ln D \qquad \textbf{(1)}$$

and with data from the table in the problem:

T (°C)	T (K)	$\frac{1}{T\text{ (K)}}$	$\mu \left(\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)$	$\ln \mu$
0	273.15	3.661×10^{-3}	1.787×10^{-3}	−6.327
20	293.15	3.411×10^{-3}	1.002×10^{-3}	−6.906
40	313.15	3.193×10^{-3}	6.529×10^{-4}	−7.334
60	333.15	3.002×10^{-3}	4.665×10^{-4}	−7.670
80	353.15	2.832×10^{-3}	3.547×10^{-4}	−7.944
100	373.15	2.680×10^{-3}	2.818×10^{-4}	−8.174

A plot of $\ln \mu$ vs. $\frac{1}{T}$ is shown here:



Although there appears to be a slight curvature to the data in the semi-log plot, it also appears to be reasonably well approximated by a straight line as would be expected for data that follows an exponential law. Using an exponential law ($y = ae^{bx}$) fit in Excel, which is the same as fitting a straight-line on a semi-log plot, yields:

$$\boxed{D = a = 1.767\times10^{-6}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

and

$$\boxed{B = b = 1.870\times10^{3}\ \text{K}}$$

so that

$$\mu = 1.767\times10^{-6}e^{\frac{1870}{T}}$$

At 50 °C (323.15 K), $\mu = 1.767\times10^{-6}e^{\frac{1870}{323.15}} = \boxed{5.76\times10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$

From the table in the problem, $\mu = 5.468\times10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$

Problem 1.6.15

For a parallel plate arrangement of the type shown in Fig. P1.6.15 it is found that when the distance between plates is 2 mm, a shearing stress of 150 Pa develops at the upper plate when it is pulled at a velocity of 1 m/s. Determine the viscosity of the fluid between the plates. Express your answer in SI units.

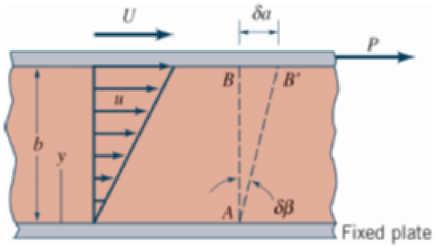


Figure P1.6.15

SOLUTION:

$$\tau = \mu \frac{du}{dy} = \mu \frac{U}{b}$$

$$\mu = \frac{\tau}{\left(\frac{U}{b}\right)} = \frac{150 \ \frac{\text{N}}{\text{m}^2}}{\left(\frac{1 \ \frac{\text{m}}{\text{s}}}{0.002 \text{ m}}\right)} = \boxed{0.300 \ \frac{\text{N} \cdot \text{s}}{\text{m}^2}}$$

Problem 1.6.16

Two flat plates are oriented parallel above a fixed lower plate as shown in Fig. P1.6.16. The top plate, located a distance b above the fixed plate, is pulled along with speed V. The other thin plate is located a distance cb, where $0 < c < 1$, above the fixed plate. This plate moves with speed V_1, which is determined by the viscous shear forces imposed on it by the fluids on its top and bottom. The fluid on the top is twice as viscous as that on the bottom. Plot the ratio V_1/V as a function of c for $0 < c < 1$.

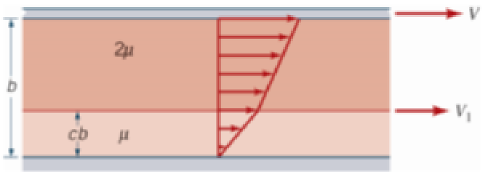


Figure P1.6.16

SOLUTION:

For constant speed, V_1, of the middle plate, the net force on the plate is 0. Hence, $F_{top} = F_{bottom}$, where $F = \tau A$. Thus, the shear stress on the top and bottom of the plate must be equal.

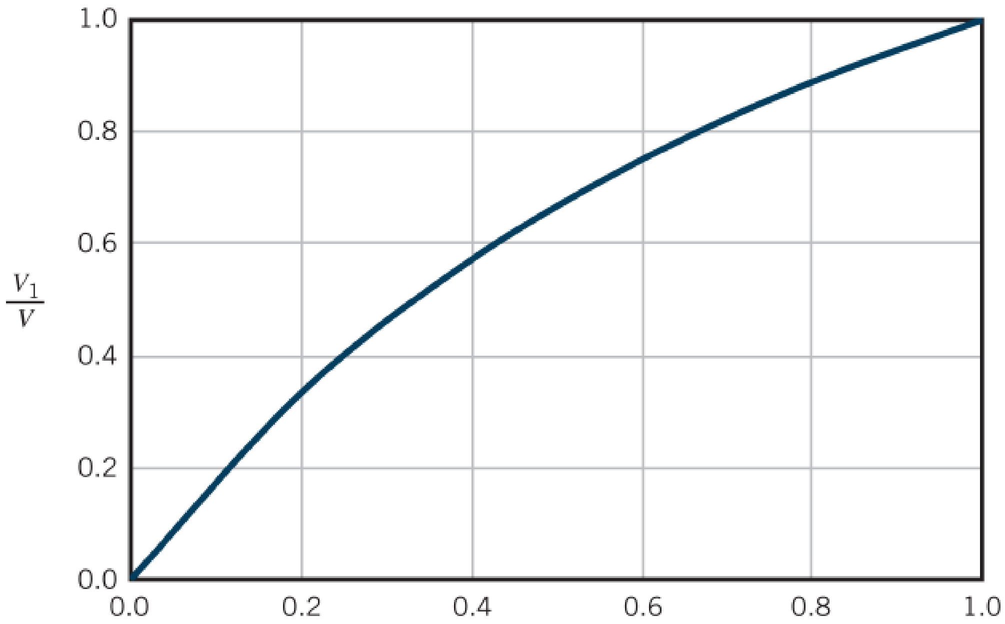
$\tau_{top} = \tau_{bottom}$ where $\tau = \mu \dfrac{du}{dy}$ **(1)**

For the bottom fluid $\dfrac{du}{dy} = \dfrac{V_1}{cb}$, while for the top fluid $\dfrac{du}{dy} = \dfrac{(V - V_1)}{b - cb}$.

Hence, from Eq. 1, $(2\mu)\dfrac{(V - V_1)}{b(1-c)} = (\mu)\dfrac{V_1}{cb}$

$$2cV - 2cV_1 = V_1 - cV_1 \rightarrow \frac{V_1}{V} = \boxed{\frac{2c}{c+1}}$$

NOTE: $c = 0 \rightarrow \dfrac{V_1}{V} = 0$, $c = \dfrac{1}{2} \rightarrow \dfrac{V_1}{V} = \dfrac{2}{3}$, $c = 1 \rightarrow \dfrac{V_1}{V} = 1$



Problem 1.6.17

Three large plates are separated by thin layers of ethylene glycol and water, as shown in Fig. P1.6.17. The top plate moves to the right at 2 m/s. At what speed and in what direction must the bottom plate be moved to hold the center plate stationary?

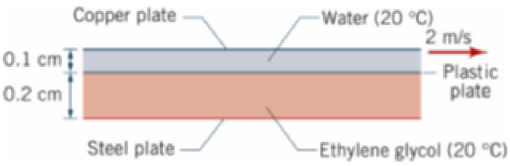
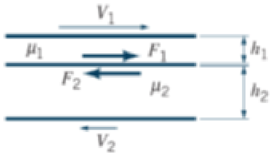


Figure P1.6.17

SOLUTION:

The center plate is stationary if $F_1 = F_2$ (see image). Assuming Newtonian fluids and thin layers,



$$F = \mu\left(\frac{du}{dy}\right)_{\substack{\text{center}\\\text{plate}}} \cong \mu\frac{V}{h}$$

so

$$\mu_1\frac{V_1}{h_1} = \mu_2\frac{V_2}{h_2}$$

or

$$V_2 = \left(\frac{\mu_1}{\mu_2}\right)\left(\frac{h_2}{h_1}\right)V_1 = \left(\frac{\mu_w}{\mu_{eg}}\right)\left(\frac{h_{eg}}{h_w}\right)V_1$$

From the liquid properties table: $\mu_{eg} = 1.99\times10^{-2}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}$ and $\mu_w = 1.00\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}$

$$V_2 = \left(\frac{1.00\times10^{-3}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}{1.99\times10^{-2}\,\frac{\text{N}\cdot\text{s}}{\text{m}^2}}\right)\left(\frac{0.2\,\text{cm}}{0.1\,\text{cm}}\right)\left(2\,\frac{\text{m}}{\text{s}}\right)$$

$$\boxed{V_2 = 0.201\,\frac{\text{m}}{\text{s}},\ \text{left}}$$

Problem 1.6.18

There are many fluids that exhibit non-Newtonian behavior (see, for example, **Video V1.6**). For a given fluid, the distinction between Newtonian and non-Newtonian behavior is usually based on measurements of shear stress and rate of shearing strain. Assume that the viscosity of blood is to be determined by measurements of shear stress, τ, and rate of shearing strain, du/dy, obtained from a small blood sample tested in a suitable viscometer. Based on the data provided in the table, determine if the blood is a Newtonian or non-Newtonian fluid. Explain how you arrived at your answer.

$\tau\,(\text{N/m}^2)$	0.04	0.06	0.12	0.18	0.30	0.52	1.12	2.10
$du/dy\,(\text{s}^{-1})$	2.25	4.50	11.25	22.5	45.0	90.0	225	450

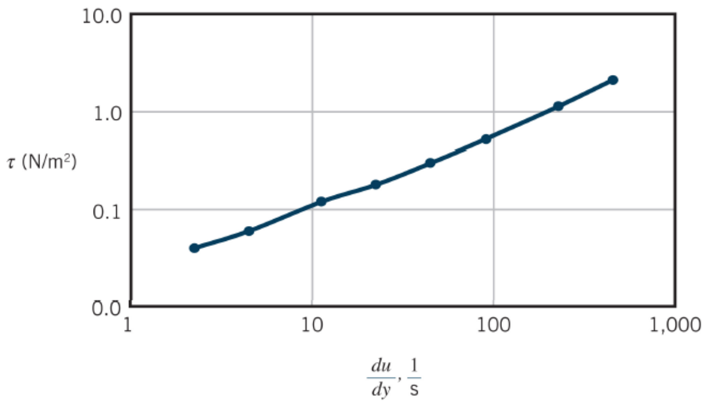
SOLUTION:

For a Newtonian fluid the ratio of τ to $\dfrac{du}{dy}$ is a constant. For the data given:

$\dfrac{\tau}{du/dy}\,(\text{N}\cdot\text{s/m}^2)$	0.0178	0.0133	0.0107	0.0080	0.0067	0.0058	0.0050	0.0047

The ratio is not a constant but decreases as the rate of shearing strain increases. Thus, this fluid (blood) is a $\boxed{\text{non-Newtonian}}$ fluid.

NOTE: The behavior of many non-Newtonian fluids can be well approximated by a power-law relationship. If that is true for this fluid, on a log–log plot the relationship between shear stress and strain rate should be a straight line.



It appears that for this sample, the blood indeed is well represented by a power law.

Problem 1.6.19

The sled shown in Fig. P1.6.19 slides along on a thin horizontal layer of water between the ice and the runners. The horizontal force that the water puts on the runners is equal to 5.34 N when the sled's speed is 15.24 m/s. The total area of both runners in contact with the water is 0.0074 m^2, and the viscosity of the water is 1.67×10^{-3} N·s/m². Determine the thickness of the water layer under the runners. Assume a linear velocity distribution in the water layer.

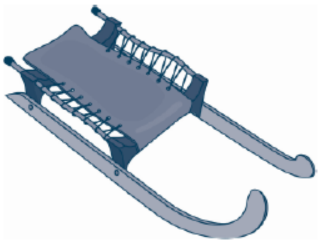


Figure P1.6.19

SOLUTION:

F (force) = τA

$$\tau = \mu \frac{dv}{dy} = \mu \frac{V}{d} \quad \text{where } d = \text{thickness of water layer.}$$

Thus,

$$F = \mu \frac{V}{d} A$$

and

$$d = \frac{\mu V A}{F} = \frac{(1.67 \times 10^{-3}\ \mathrm{N \cdot s/m^2})(15.24\ \mathrm{m/s})(0.0074\ \mathrm{m^2})}{5.34\ \mathrm{N}} = \boxed{3.5 \times 10^{-5}\ \mathrm{m}}$$

Problem 1.6.20

A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in Fig. P1.6.20. The lubricant that fills the 0.3-mm gap between the shaft and bearing is oil having a kinematic viscosity of $8.0\times10^{-4}\ \text{m}^2/\text{s}$ and a specific gravity of 0.91. Determine the force P required to pull the shaft at a velocity of $3\ \text{m/s}$. Assume the velocity distribution in the gap is linear.

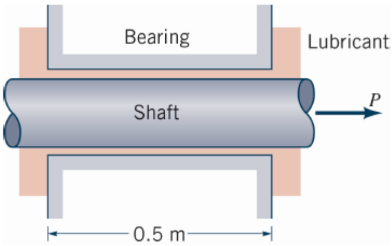
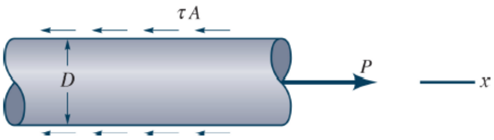


Figure P1.6.20

SOLUTION:



$$\sum F_x = 0$$

Thus,

$$P = \tau A$$

where $A = \pi D \times (\text{shaft length in bearing}) = \pi D \ell$

and $\tau = \mu \dfrac{(\text{velocity of shaft})}{(\text{gap width})} = \mu \dfrac{V}{b}$

so that

$$P = \left(\mu \frac{V}{b}\right)(\pi D \ell) = \left(\nu\rho \frac{V}{b}\right)(\pi D \ell)$$

Since $\mu = \nu\rho = \nu(SG)\left(\rho_{H_2O\,@\,4\,^\circ C}\right)$

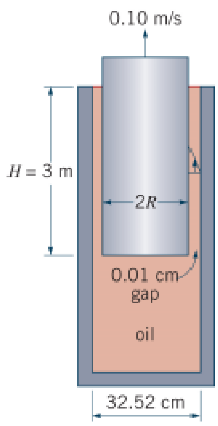
$$P = \frac{\left(8.0\times10^{-4}\ \frac{\text{m}^2}{\text{s}}\right)\left(0.91\times10^{3}\ \frac{\text{kg}}{\text{m}^3}\right)\left(3\frac{\text{m}}{\text{s}}\right)(\pi)(0.025\ \text{m})(0.5\ \text{m})}{(0.0003\ \text{m})}$$

$$P = 286\frac{\text{kg}\cdot\text{m}}{\text{s}^2}\times\frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}} = \boxed{286\ \text{N}}$$

Problem 1.6.21

A hydraulic lift in a service station has a 32.50-cm- diameter ram that slides in a 32.52-cm-diameter cylinder. The annular space is filled with SAE 10 oil at 20 °C. The ram is traveling upward at the rate of 0.10 m/s. Find the frictional force when 3.0 m of the ram is engaged in the cylinder.

SOLUTION:



Modeling the oil as a Newtonian fluid:

$$\tau = \mu \frac{du}{dy}$$

linear velocity profile across gap.

$$\frac{du}{dy} = \frac{\left(0.10\frac{\text{m}}{\text{s}}\right)}{(0.01\text{ cm})\left(\frac{\text{m}}{100\text{ cm}}\right)} = 1000\frac{1}{\text{s}}.$$

$\mu = 0.123\,\text{Pa}\cdot\text{s}$ at 20 °C

$$\tau = \left(0.123\frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(1000\frac{1}{\text{s}}\right) = 123\frac{\text{N}}{\text{m}^2}$$

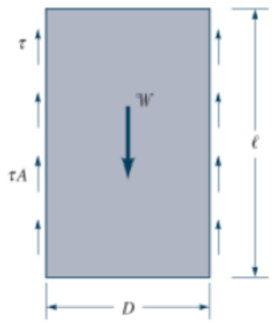
Therefore, $F = \tau A = \tau(2\pi RH) = \left(123\frac{\text{N}}{\text{m}^2}\right)2\pi(16.25\text{ cm})(3\text{ m})\left(\frac{1\text{ m}}{100\text{ cm}}\right)$

$\boxed{F = 377\text{ N}}$

Problem 1.6.22

A piston having a diameter of 0.14 m and a length of 0.24 m slides downward with a velocity V through a vertical pipe. The downward motion is resisted by an oil film between the piston and the pipe wall. The film thickness is 5.08×10^{-5} m, and the cylinder weighs 2.225 N. Estimate V if the oil viscosity is 0.766 $\text{N}\cdot\text{s/m}^2$. Assume the velocity distribution in the gap is linear.

SOLUTION:



Constant velocity $\rightarrow \sum F_{\text{vertical}} = 0$

$$W = \tau A = \tau\pi D\ell$$

Linear velocity profile → Newtonian fluid → $\tau = \mu\frac{du}{dy} = \mu\frac{(\text{velocity})}{(\text{film thickness})} = \mu\frac{V}{\delta}$

Substitution yields:

$$W = \left(\mu\frac{V}{\delta}\right)(\pi D\ell$$

$$V = \frac{W\delta}{\pi D\ell} \qquad \frac{(2.225\text{ N})(5.08\times10^{-5}\text{ m})}{14\text{ m})(0.24\text{ m})(0.766\text{ N}\cdot\text{s/m}^2)} = \boxed{0.0014\text{ m/s}}$$

Problem 1.6.23

A 10-kg block slides down a smooth inclined surface as shown in Fig. P1.6.23. Determine the terminal velocity of the block if the 0.1-mm gap between the block and the surface contains SAE 30 oil at 15.5°C. Assume the velocity distribution in the gap is linear, and the area of the block in contact with the oil is $0.1\ \text{m}^2$.

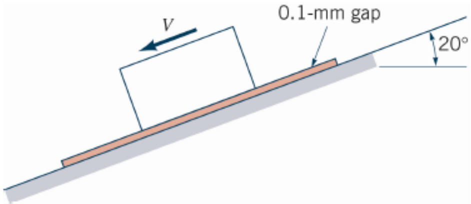
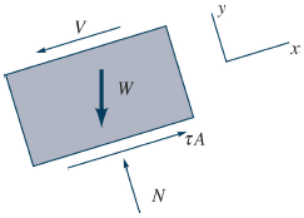


Figure P1.6.23

SOLUTION:

Draw free-body diagram to help resolve forces:



Constant velocity $\rightarrow \sum F_x = 0$

$$W \sin\theta = \tau A$$

Linear velocity profile $\rightarrow$ Newtonian fluid $\rightarrow \tau = \mu \dfrac{du}{dy} = \mu \dfrac{(\text{velocity})}{(\text{film thickness})} = \mu \dfrac{V}{b}$

where b is film thickness. Substitution yields:

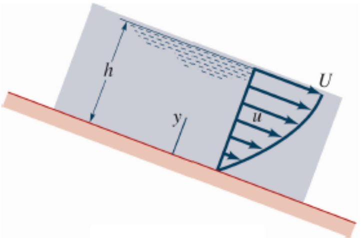
$$W \sin\theta = mg \sin\theta = \mu \frac{V}{b} A$$

$$V = \frac{bmg \sin\theta}{\mu A} = \frac{(0.0001\ \text{m})(10\ \text{kg})\left(9.81\ \frac{\text{m}}{\text{s}^2}\right)(\sin 20°)}{\left(0.38 \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)(0.1\ \text{m}^2)} \times \frac{1\ \text{N}\cdot\text{s}^2}{1\ \text{kg}\cdot\text{m}}$$

$$\boxed{V = 0.0883\ \frac{\text{m}}{\text{s}}}$$

Problem 1.6.24

A layer of water flows down an inclined fixed surface with the velocity profile shown in Fig. P1.6.24. Determine the magnitude and direction of the shearing stress that the water exerts on the fixed surface for $U = 2$ m/s and $h = 0.1$ m.



$$\frac{u}{U} = 2\frac{y}{h} - \frac{y^2}{h^2}$$

Figure P1.6.24

SOLUTION:

Enforcing the no-slip boundary condition at the solid surface:

$$\tau = \mu\frac{du}{dy} = \mu\frac{d}{dy}\left[U\left(2\frac{y}{h} - \frac{y^2}{h^2}\right)\right] = \mu U\left(\frac{2}{h} - \frac{2y}{h^2}\right)$$

Thus, at the fixed surface $(y = 0)$

$$\left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{2D}{h}$$

Thus,

$$\tau_{y=0} = \mu U\frac{2}{h} = \left(1.12\times10^{-3}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}\right)\left(2\ \frac{\text{m}}{\text{s}}\right)\frac{2}{0.1\ \text{m}}$$

$$= \boxed{4.48\times10^{-2}\ \frac{\text{N}}{\text{m}^2}\ \text{acting in direction of flow}}$$

Problem 1.6.25

The concentric cylinder viscometer shown in adjacent Fig. P1.6.25 has a cylinder height of 10.0 cm, a cylinder radius of 3.0 cm, and a uniform gap between the cylinder and the container (bottom and sides) of 0.10 cm. The pulley has a radius of 3.0 cm. Determine the weight required to produce a constant rotational speed of 30 rpm if the gap is filled with **(a)** water, **(b)** gasoline, **(c)** glycerin.

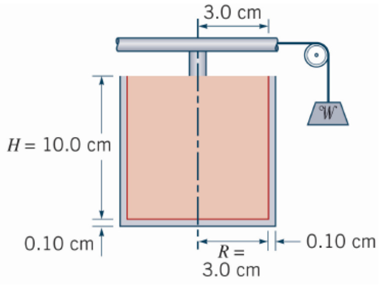
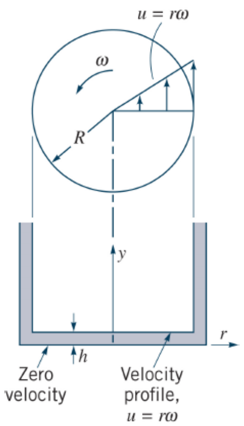


Figure P1.6.25

SOLUTION:

Resisting torque is due to shear stress acting on cylinder surfaces. Assuming a linear velocity profile across the narrow gaps, the torque on the cylinder wall is



$$T_1 = (\tau A)R = \left(\mu \frac{du}{dy}\bigg|_w\right)(\pi DH)R = \mu\left(\frac{\omega R}{h}\right)(2\pi RH)R = \frac{2\pi\mu\omega HR^3}{h}$$

The velocity at the cylinder bottom is a function of radial position. The infinitesimal torque acting on an annular ring of differential width is

$$dT_2 = (\tau dA)r = \left(\mu\frac{\omega r}{h}\right)(2\pi r dr)r = \frac{2\pi\mu\omega}{h}r^3 dr$$

$$T_2 = \frac{2\pi\mu\omega}{h}\int_0^R r^3 dr = \frac{\pi\mu\omega}{2h}R^4$$

Total torque:

$$T = T_1 + T_2 = 2\pi\mu\omega R^3\left(\frac{H}{h}\right) + \frac{\pi\mu\omega}{2h}R^4 = \frac{\pi\mu\omega R^4}{h}\left(\frac{2H}{R}+\frac{1}{2}\right)$$

Neglecting friction: $WR_p = T \rightarrow W = \dfrac{\pi\mu\omega R^4}{hR_p}\left(\dfrac{2H}{R}+\dfrac{1}{2}\right)$

$$W = \frac{\pi\mu\left(30\frac{\text{rev}}{\text{min}}\times\frac{2\pi\text{ rad}}{1\text{ rev}}\times\frac{1\text{ min}}{60\text{ s}}\right)(3\text{ cm})^{4\!\!\!/\,2}}{(0.1\text{ cm})(3\text{ cm})}\left(\frac{2(10\text{ cm})}{3\text{ cm}}+\frac{1}{2}\right)\left(\frac{1\text{ m}}{100\text{ cm}}\right)^2 = \left(1.91\frac{\text{m}^2}{\text{s}}\right)\mu$$

(a) Water $\rightarrow W = \left(1.910\dfrac{\text{m}^2}{\text{s}}\right)\left(1.12\times10^{-3}\dfrac{\text{N}\cdot\text{s}}{\text{m}^2}\right) = \boxed{2.14\times10^{-3}\text{ N}}$

(b) Gasoline $\rightarrow W = \left(1.910\dfrac{\text{m}^2}{\text{s}}\right)\left(3.10\times10^{-4}\dfrac{\text{N}\cdot\text{s}}{\text{m}^2}\right) = \boxed{5.92\times10^{-4}\text{ N}}$

(c) Glycerine $\rightarrow W = \left(1.910\dfrac{\text{m}^2}{\text{s}}\right)\left(1.50\dfrac{\text{N}\cdot\text{s}}{\text{m}^2}\right) = \boxed{2.87\text{ N}}$

Problem 1.6.26

A 0.3 m diameter circular plate is placed over a fixed bottom plate with a 0.0025 m gap between the two plates filled with glycerin as shown in Fig. P1.6.26. Determine the torque required to rotate the circular plate slowly at 2 rpm. Assume that the velocity distribution in the gap is linear and that the shear stress on the edge of the rotating plate is negligible.

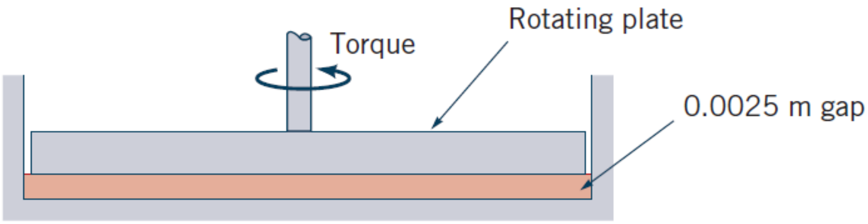
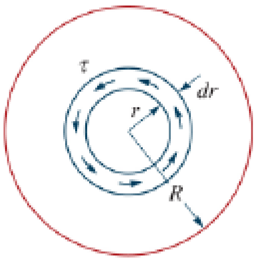


Figure P1.6.26

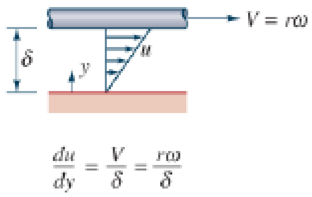
SOLUTION:

As shown, considering an annular ring of differential width $dT = r\tau dA = r\tau 2\pi r dr$

Integration yields: $T = 2\pi \int_0^R r^2 \tau dr$



Stresses acting on bottom of plate



Velocity distribution

For the annular strip: $\tau = \mu \dfrac{r\omega}{\delta}$

Thus, $T = \dfrac{2\pi\mu\omega}{\delta} \int_0^R r^3 dr = \dfrac{2\pi\mu\omega}{\delta}\left(\dfrac{R^4}{4}\right)$

Using the data specified:

$$T = \frac{2\pi(1.5\ \text{N}\cdot\text{s/m}^2)\left(2\dfrac{\text{rev}}{\text{min}}\right)\left(2\pi\dfrac{\text{rad}}{\text{rev}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)(0.15\ \text{m})^4}{(0.0025\ \text{m})(4)}$$

$\boxed{T = 0.016\ \text{N}\cdot\text{m}}$

Problem 1.6.28

Some measurements on a blood sample at 37 °C indicate a shearing stress of $0.52\ \text{N/m}^2$ for a corresponding rate of shearing strain of $200\ \text{s}^{-1}$. Determine the apparent viscosity of the blood and compare it with the viscosity of water at the same temperature.

SOLUTION:

$$\tau = \mu \frac{du}{dy} = \mu\dot{\gamma}$$

$$\mu_{\text{blood}} = \frac{\tau}{\dot{\gamma}} = \frac{0.52\dfrac{\text{N}}{\text{m}^2}}{200\dfrac{1}{\text{s}}} = \boxed{26.0\times10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}}$$

From Table B.2 Physical Properties of Water (SI Units)

$$@30\ ^\circ\text{C}\ \ \mu_{\text{H}_2\text{O}} = 7.975\times10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

$$@40\ ^\circ\text{C}\ \ \mu_{\text{H}_2\text{O}} = 6.529\times10^{-4}\ \frac{\text{N}\cdot\text{s}}{\text{m}^2}$$

Thus, with linear interpolation, $\mu_{\text{H}_2\text{O}}(37\ ^\circ\text{C}) = 6.96\times10^{-4}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}$

and

$$\frac{\mu_{\text{blood}}}{\mu_{\text{H}_2\text{O}}} = \frac{26.0\times10^{-4}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}}{6.96\times10^{-4}\ \dfrac{\text{N}\cdot\text{s}}{\text{m}^2}} = \boxed{3.74}$$

Section 1.7 **Compressibility of Fluids**

Problem 1.7.2

A sound wave is observed to travel through a liquid with a speed of $1500\ \text{m/s}$. The specific gravity of the liquid is 1.5. Determine the bulk modulus for this fluid.

SOLUTION:

$c = \sqrt{\dfrac{E_N}{\rho}}$, where $\rho = SG\rho_{\text{H}_2\text{O}}$ and $SG = 1.5$

Thus,

$$E_N = c^2\rho = c^2 SG\rho_{\text{H}_2\text{O}} = \left(1500\frac{\text{m}}{\text{s}}\right)^2 (1.5)\left(999\frac{\text{kg}}{\text{m}^3}\right) = 3.37\times10^9\ \frac{\text{kg}\cdot\text{m}}{\text{s}^2\text{m}^2}$$

or

$$\boxed{E_N = 3.37\times10^9\ \frac{\text{N}}{\text{m}^2}}$$

Problem 1.7.3

A rigid-walled cubical container is completely filled with water at 4.45 °C and sealed. The water is then heated to 37.78 °C. Determine the pressure that develops in the container when the water reaches this higher temperature. Assume that the volume of the container remains constant and the value of the bulk modulus of the water remains constant and equal to 2068 MPa.

SOLUTION:

Since the water mass remains constant,

$$\rho_{4.45^\circ}\forall = \rho_{37.78^\circ}(\forall + \Delta\forall)$$

where $\forall$ is volume and $\Delta\forall$ is change in volume if water were unconstrained during heating.

Thus, $\dfrac{\Delta\forall}{\forall} = \dfrac{\rho_{4.45^\circ}}{\rho_{37.78^\circ}} - 1$

From the table Physical Properties of Water (BG/EE Units)

$\rho_{4.45^\circ} = 1000\ \text{kg/m}^3$ and $\rho_{37.78^\circ} = 993\ \text{kg/m}^3$ so that

$$\frac{\Delta\forall}{\forall} = \frac{1000\ \text{kg/m}^3}{993\ \text{kg/m}^3} - 1 = 0.007$$

From the equation $E_v = -\dfrac{dp}{\dfrac{\Delta\forall}{\forall}}$ it follows with $d\forall \approx \Delta\forall$ and $dp \approx \Delta p$ that the change in pressure required to compress the water back to its original volume is

$$\Delta p = -(2068\ \text{MPa})(0.007)$$

$$= \boxed{14.47\ \text{MPa}}$$

Problem 1.7.4

Estimate the increase in pressure (in MPa) required to decrease a unit volume of mercury by 0.1%.

SOLUTION:

$$E_v = -\forall \frac{dp}{d\forall} \approx -\forall \frac{\Delta p}{\Delta\forall}$$

Thus,

$$\Delta p \approx -\frac{E_v \Delta\forall}{\forall} = -(28544\ \text{MPa})(-0.001)$$

$$\boxed{\Delta p \approx 28.544\ \text{MPa}}$$

Problem 1.7.5

A 1 m^3 volume of water is contained in a rigid container. Estimate the change in the volume of the water when a piston applies a pressure of 35 MPa.

SOLUTION:

$$E_v = -\forall \frac{dp}{d\forall} \approx -\forall \frac{\Delta p}{\Delta \forall}$$

Thus,

$$\Delta\forall \approx -\frac{\forall \Delta p}{E_v} = -\frac{(1\ \text{m}^3)\left(35\times10^6\ \frac{\text{N}}{\text{m}^2}\right)}{2.15\times10^9\ \frac{\text{N}}{\text{m}^2}} = -0.0163\ \text{m}^3$$

or $\boxed{\text{decrease in volume} \approx 0.0163\ \text{m}^3}$

Problem 1.7.6

Determine the speed of sound at 20 °C in **(a)** air, **(b)** helium, and **(c)** natural gas (methane). Express your answer in m/s.

SOLUTION:

$$c = \sqrt{kRT}$$

With $T = 20\ °\text{C} + 273 = 293\ \text{K}$:

(a) For air, $c = \sqrt{(1.40)\left(286.9\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(293\ \text{K})\left(\frac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)\left(\frac{1\ \text{kg}\cdot\text{m}}{1\ \text{N}\cdot\text{s}^2}\right)} = \boxed{343\frac{\text{m}}{\text{s}}}$

(b) For helium, $c = \sqrt{(1.66)\left(2077\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(293\ \text{K})\left(\frac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)\left(\frac{1\ \text{kg}\cdot\text{m}}{1\ \text{N}\cdot\text{s}^2}\right)} = \boxed{1010\frac{\text{m}}{\text{s}}}$

(c) For natural gas, $c = \sqrt{(1.31)\left(518.3\frac{\text{J}}{\text{kg}\cdot\text{K}}\right)(293\ \text{K})\left(\frac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)\left(\frac{1\ \text{kg}\cdot\text{m}}{1\ \text{N}\cdot\text{s}^2}\right)} = \boxed{446\frac{\text{m}}{\text{s}}}$

Problem 1.7.7

Air is enclosed by a rigid cylinder containing a piston. A pressure gage attached to the cylinder indicates an initial reading of 172.3 kPa. Determine the reading on the gage when the piston has compressed the air to one-third its original volume. Assume the compression process to be isothermal and the local atmospheric pressure to be 101.3 kPa.

SOLUTION:

For isothermal compression, $\frac{p}{\rho} = \text{constant} = \frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}$, where $i \sim$ initial state and $f \sim$ final state.

Thus, $p_f = \left(\frac{\rho_f}{\rho_i}\right) p_i$

Because the mass of air is constant: $\rho = \dfrac{\text{mass}}{\text{volume}} \rightarrow \dfrac{\rho_f}{\rho_i} = \dfrac{\text{initial volume}}{\text{final volume}} = 3$

Therefore,

$$p_f = (3)[(172.3 + 101.3)\text{kPa (abs)}] = 820.8 \text{ kPa (abs)}$$

or

$$p_f\,(\text{gage}) = (820.8 - 101.3)\text{kPa} = \boxed{719.5 \text{ kPa (gage)}}$$

Problem 1.7.8

Air is enclosed by a rigid cylinder containing a piston. A pressure gage attached to the cylinder indicates an initial reading of 172.3 kPa. Determine the reading on the gage when the piston has compressed the air to one-third its original volume. Assume the compression process take place without friction and without heat transfer (isentropic process) and the local atmospheric pressure to be 101.3 kPa.

SOLUTION:

For isentropic compression, $\dfrac{p}{\rho^k} = \text{constant} = \dfrac{p_i}{\rho_i^k} = \dfrac{p_f}{\rho_f^k}$, where $i \sim$ initial state and $f \sim$ final state.

Thus $p_f = \left(\dfrac{\rho_f}{\rho_i}\right) p_i$

Because the amount of mass is constant: $\rho = \dfrac{\text{mass}}{\text{volume}} \rightarrow \dfrac{\rho_f}{\rho_i} = \dfrac{\text{initial volume}}{\text{final volume}} = 3$

Therefore,

$$p_f = (3)^{1.40}[(172.3 + 101.3)\text{kPa (abs)}] = 1273.7 \text{ kPa (abs)}$$

or

$$p_f\,(\text{gage}) = 1273.7 - 101.3 = \boxed{1172.4 \text{ kPa (gage)}}$$

Problem 1.7.9

Carbon dioxide at 30 °C and 300 kPa absolute pressure expands isothermally to an absolute pressure of 165 kPa. Determine the final density of the gas.

SOLUTION:

For isothermal compression, $\dfrac{p}{\rho} = \text{constant} = \dfrac{p_i}{\rho_i} = \dfrac{p_f}{\rho_f}$, where $i \sim$ initial state and $f \sim$ final state.

Thus, $\rho_f = \dfrac{p_f}{p_i}\rho_i$

Also, $$\rho_i = \frac{p_i}{RT_i} = \frac{300\times 10^3 \dfrac{\text{N}}{\text{m}^2}}{\left(188.9 \dfrac{\text{J}}{\text{kg}\cdot\text{K}} \times \dfrac{1\ \text{N}\cdot\text{m}}{1\ \text{J}}\right)\left[(30\ °\text{C} + 273)\ \text{K}\right]} = 5.24 \frac{\text{kg}}{\text{m}^3}$$

Therefore,

$$\rho_f = \left(\frac{165 \text{ kPa}}{300 \text{ kPa}}\right)\left(5.24 \frac{\text{kg}}{\text{m}^3}\right) = \boxed{2.88 \frac{\text{kg}}{\text{m}^3}}$$

Problem 1.7.10

Natural gas at 21°C and standard atmospheric pressure of 101.3 kPa (abs) is compressed isentropically to a new absolute pressure of 482.6 kPa. Determine the final density and temperature of the gas.

SOLUTION:

For isentropic compression, $\frac{p}{\rho^k} = \text{constant} = \frac{p_i}{\rho_i^k} = \frac{p_f}{\rho_f^k}$, where $i \sim$ initial state and $f \sim$ final state.

Therefore, $\rho_f = \left(\frac{p_f}{p_i}\right)^{\frac{1}{k}} \rho_i$

Also, $\rho_i = \frac{p_i}{RT_i} = \frac{(101.3\text{ kPa})}{(518.23\text{ J/kg}\cdot\text{K})[(21°\text{C}+273)\text{K}]} = 0.665\text{ kg/m}^3$

Therefore, $\rho_f = \left[\frac{482.6\text{ kPa}}{101.3\text{ kPa}}\right]^{\frac{1}{1.31}} (0.665\text{ kg/m}^3) = 2.19\text{ kg/m}^3$

Using the ideal gas model:

$$T_f = \frac{p_f}{\rho_f R} = \frac{(482.6\text{ kPa})}{(2.19\text{ kg/m}^3)(518.23\text{ J/kg}\cdot\text{K})} = 425\text{ K}$$

Problem 1.7.11

A compressed air tank in a service station has a volume of 0.283 m^3. It contains air at 21°C and 1034 kPa. How many tubeless tires can it fill to 308.2 kPa at 21°C if each tire has a volume of 0.0424 m^3 at 21°C because of heat transfer through the tank's large surface area.

SOLUTION:

Modeling the air as an ideal gas, the mass of air m_i that can be put into each tire is found from

$$m_{\text{tire}} = (p\forall)_f - (p\forall)_i = \left(\frac{p\forall}{RT}\right)_f - \left(\frac{p\forall}{RT}\right)_i = \frac{\forall}{RT}(p_f - p_i)$$

$$m_{\text{tire}} = \frac{(0.0424\text{ m}^3)(308.2 - 101.3)\text{kPa}}{(287\text{ J/kg}\cdot\text{K})(294\text{ K})} = 0.104\text{ kg}$$

Air in the tank can be put into the tires until the tank air pressure drops to 308.2 kPa absolute. The mass of air m_T that can be taken out of the tank and put into the tires is

$$m_T = (p\forall)_i - (p\forall)_f = \frac{\forall}{RT}(p_i - p_f)$$

$$= \frac{(0.283\text{ m}^3)(1034 - 308.2)\text{kPa}}{(287\text{ J/kg}\cdot\text{K})(294\text{ K})} = 2.434\text{ kg}$$

The number of tires that can be filled is

$$\text{No.} = \frac{2.434\text{ kg}}{0.104\text{ kg}} = 23.4 \quad \text{or} \quad \boxed{\text{No.} = 23\text{ tires}}$$

Problem 1.7.12

Oxygen at 30 °C and 300 kPa absolute pressure expands isothermally to an absolute pressure of 120 kPa. Determine the final density of the gas.

SOLUTION:

For isothermal expansion, $\frac{p}{\rho} = \text{constant}$ so that

$$\frac{p_i}{\rho_i} = \frac{p_f}{\rho_f}, \text{ where } i \sim \text{initial state and } f \sim \text{final state.}$$

Thus $p_f = \left(\frac{p_f}{\rho_i}\right) p_i$

Also, $\rho_i = \frac{p_i}{RT_i} = \frac{300 \times 10^3 \ \frac{\text{N}}{\text{m}^3}}{\left(259.8 \frac{\text{J}}{\text{kg} \cdot \text{K}}\right)\left[(30°\text{C} + 273)\text{K}\right]} = 3.81 \frac{\text{kg}}{\text{m}^3}$

So that $\rho_f = \left(\frac{120 \text{ kPa}}{300 \text{ kPa}}\right)\left(3.81 \frac{\text{kg}}{\text{m}^3}\right) = 1.52 \frac{\text{kg}}{\text{m}^3}$

Problem 1.7.13

Compare the isentropic bulk modulus of air at 101 kPa (abs) with that of water at the same pressure.
SOLUTION:

For air (Eq. 1.17),

$$E_V = kp = (1.40)(101 \times 10^3 \text{ Pa}) = 1.41 \times 10^5 \text{ Pa}$$

For water (Table 1.6)

$$E_V = 2.15 \times 10^9 \text{ Pa}$$

Thus, $\frac{E_V(\text{water})}{E_V(\text{air})} = \frac{2.15 \times 10^9 \text{ Pa}}{1.41 \times 10^5 \text{ Pa}} = 1.52 \times 10^4$

Problem 1.7.14
Often the assumption is made that the flow of a certain fluid can be considered as incompressible flow if the density of the fluid changes by less than 2%. If air is flowing through a tube such that the air pressure at one section is 62 kPa and at a downstream section it is 59.3 kPa at the same temperature, do you think that this flow could be considered an incompressible flow? Support your answer with the necessary calculations. Assume standard atmospheric pressure.
SOLUTION:
Modeling the air as an ideal gas undergoing an isothermal process:

$$p = \rho RT \rightarrow \frac{p_1}{\rho_1} = \frac{p_2}{\rho_2} \rightarrow \frac{\rho_2}{\rho_1} = \frac{p_2}{p_1}$$

% density change $= \frac{\rho_1 - \rho_2}{\rho_1} \times 100 = \left(1 - \frac{\rho_2}{\rho_1}\right) \times 100 = \left(1 - \frac{p_2}{p_1}\right) \times 100 = \left[1 - \frac{(59.3 + 101.3)\text{kPa}}{(62 + 101.3)\text{kPa}}\right] \times 100$
$= 1.69\ \% < 2\%$

Yes. This process is well modelled as an incompressible flow.

Problem 1.7.15
An important dimensionless parameter concerned with very high-speed flow is the *Mach number*, defined as V/c, where V is the speed of the object such as an airplane or projectile, and c is the speed of sound in the fluid surrounding the object. For a projectile traveling at 287.5 kmph through air at 10 °C and standard atmospheric pressure, what is the value of the Mach number?
SOLUTION:

Mach number $= \frac{V}{c}$

From the table of Physical Properties of Air at Standard Atmospheric Pressure (BG/EE Units)

$$c_{\text{air@10°C}} = 337 \text{ m/s}$$

$$\text{Mach number} = \frac{(1287.5 \text{ kmph})(1000 \text{ m/km})\left(\dfrac{1 \text{ hr}}{3600 \text{ s}}\right)}{337 \text{ m/s}} = \boxed{1.06}$$

Problem 1.7.16

The "power available in the wind" of velocity V through an area A is $\dot{V}$ $\quad_2\quad$ V^3,

where ρ is the air density (11.78 N/m^3). For an 29 kmph wind, find the wind area A that will supply a power of 2983 W.

SOLUTION:

Solving for the area A and using appropriate unit conversion factors:

$$A = \frac{2\dot{V}}{\rho V^3} - \frac{2(2983 \text{ W})}{(11.78 \text{ N/m}^3)(29 \text{ kmph})^3\left(1000 \dfrac{\text{m}}{\text{km}} \times \dfrac{1 \text{ hr}}{3600 \text{ s}}\right)^3} = 0.9688 \text{ m}\cdot\text{s}^2(9.81 \text{ m/s}^2)$$

$$\boxed{A = 9.5 \text{ m}^2}$$

Problem 1.7.17

Air enters the converging nozzle shown in Fig. P1.7.17 at $T_1 = 21°\text{C}$ and $V_1 = 15.24$ m/s. At the exit of the nozzle, V_2 is given by

$$V_2 = \sqrt{V_1^2 + 2c_p(T_1 - T_2)}$$

where $c_p = 1.005$ kJ/kg·K and T_2 is the air temperature at the exit of the nozzle. Find the temperature T_2 for which $V_2 = 304.8$ m/s

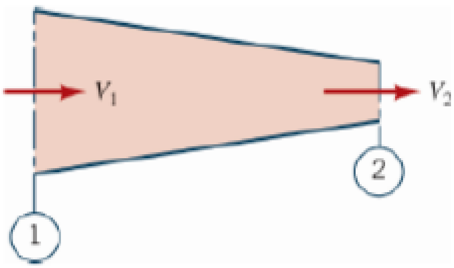


Figure P1.7.17

SOLUTION:

Solving for T_2 and inserting the values specified: q_c gives

$$T_2 = T_1 - \frac{V_2^2 - V_1^2}{2c_p} = 294 \text{ K} - \frac{(304.8 \text{ m/s})^2 - (15.24 \text{ m/s})^2}{2(1.005 \text{ kJ/kg}\cdot\text{K})}$$

$$\boxed{T_2 = 248 \text{ K}}$$

Problem 1.7.18

(See The Wide World of Fluids article titled "This water jet is a blast", section 1.7.1.) By what percent is the volume of water decreased if its pressure is increased to an equivalent to 3000 atmospheres (304 MPa)?

SOLUTION:

$$E_V = -\frac{dp}{\dfrac{d\mathcal{V}}{\mathcal{V}}} \approx -\frac{\Delta p}{\dfrac{\Delta\mathcal{V}}{\mathcal{V}}}$$

$$\frac{\Delta\mathcal{V}}{\mathcal{V}} \approx -\frac{\Delta p}{E_V} = -\frac{304 \text{ MPa} - 0.101 \text{ MPa}}{2151.165 \text{ MPa}} = -0.141$$

Thus % decrease in volume = $\boxed{14\%}$

Section 1.8 **Vapor Pressure**

Problem 1.8.1

During a mountain climbing trip it is observed that the water used to cook a meal boils at 90 °C rather than the standard 100 °C at sea level. At what altitude are the climbers preparing their meal? (See Tables B.2 and C.2 for data needed to solve this problem.)

SOLUTION:

Water boils when the vapor pressure of the liquid is the same as atmospheric pressure.

From the water property table, at $T = 90\,°\mathrm{C}$, $p_v = 7.01\times10^4\ \frac{\mathrm{N}}{\mathrm{m}^2}(\mathrm{abs})$

From standard atmosphere table, $p = 7.01\times10^4\ \frac{\mathrm{N}}{\mathrm{m}^2}(\mathrm{abs}) \rightarrow \boxed{\text{altitude} = 3000\ \mathrm{m}}$

Problem 1.8.2

When a fluid flows through a sharp bend, low pressure may develop in localized regions of the bend. Estimate the minimum absolute pressure (in kPa) that can develop without causing cavitation if the fluid is water at 71°C.

SOLUTION:

Cavitation may occur when the local pressure equals the vapor pressure. For water at 71°C (from Table of Physical Properties of Water [BG/EE Units])

$$T = 71°C \rightarrow p_v = 32.7\ \text{kPa (abs)} \rightarrow \boxed{\text{minimum pressure} = 32.7\ \text{kPa (abs)}}$$

Problem 1.8.3

A partially filled closed tank contains ethyl alcohol at 20°C. If the air above the alcohol is evacuated, what is the minimum absolute pressure that develops in the evacuated space?

SOLUTION:

Minimum pressure = vapor pressure = $\boxed{5.86\ \text{kPa (abs)}}$

Problem 1.8.4

Estimate the minimum absolute pressure (in Pascals) that can be developed at the inlet of a pump to avoid cavitation if the fluid is carbon tetrachloride at 20 °C.

SOLUTION:

Cavitation may occur when the section pressure at the pump inlet equals the vapor pressure.

For carbon tetrachloride at 20 °C, $p_v = 13\ \mathrm{kPa\,(abs)}$.

Thus, minimum pressure $= \boxed{13\ \mathrm{kPa\,(abs)}}$.

Problem 1.8.5

When water at 70 °C flows through a converging section of pipe, the pressure decreases in the direction of flow. Estimate the minimum absolute pressure that can develop without causing cavitation. Express your answer in both BG and SI units.

SOLUTION:

Cavitation may occur in the converging section of pipe when the pressure equals the vapor pressure. From the Table of Physical Properties of Water (SI Units) for water at 70 °C, $p_v = 31.2 \text{ kPa (abs)}$.

Therefore,

$$\text{minimum pressure} = \boxed{31.2 \text{ kPa (abs)}} = \left(31.2\times10^3 \frac{\text{N}}{\text{m}^2}\right)\left(\frac{1.450\times10^{-4} \text{ psi}}{1 \frac{\text{N}}{\text{m}^2}}\right) = \boxed{4.52 \text{ psia}}$$

Problem 1.8.6

At what atmospheric pressure will water boil at 35 °C? Express your answer in both SI and BG units.

SOLUTION:

The vapor pressure of water at 35°C is 5.81 kPa (abs) (from Table of Physical Properties of Water [SI Units] using linear interpolation).

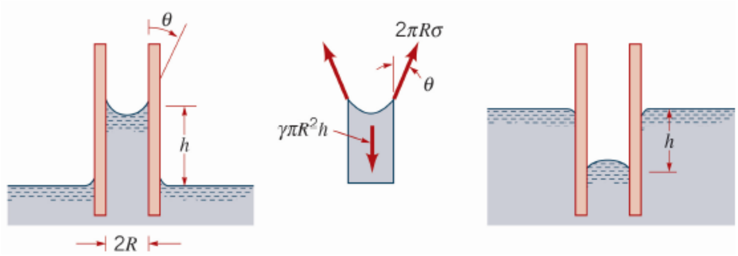
Thus, if water boils at this temperature the atmospheric pressure must be equal to $\boxed{5.81 \text{kPa (abs)}}$ in SI units. In BG units,

$$\left(5.81\times10^3 \frac{\text{N}}{\text{m}^2}\right)\left(\frac{1.450\times10^{-4} \text{ psi}}{1 \frac{\text{N}}{\text{m}^2}}\right) = \boxed{0.842 \text{ psi (abs)}}$$

Problem 1.9.2

When a 2-mm-diameter tube is inserted into a liquid in an open tank, the liquid is observed to rise 10 mm above the free surface of the liquid (see Video V1.10). The contact angle between the liquid and the tube is zero, and the specific weight of the liquid is 1.2×10^4 N/m³ . Determine the value of the surface tension for this liquid.

SOLUTION:



The action of surface tension in a tube inserted into a pool is to draw upward (or depress) the liquid in the tube at a distance $h = \dfrac{2\sigma\cos\theta}{\gamma R}$ with respect to the elevation of the surrounding free surface.

For the specified contact angle, $\theta = 0$:

$$\sigma = \frac{\gamma h R}{2\cos\theta} = \frac{1.2\times10^4 \dfrac{\text{N}}{\text{m}^3}\left(10\times10^{-3}\ \text{m}\right)\left(\dfrac{2\times10^{-3}\ \text{m}}{2}\right)}{2\cos 0} = \boxed{0.060\dfrac{\text{N}}{\text{m}}}$$

Problem 1.9.3

Small droplets of carbon tetrachloride at 20°C are formed with a spray nozzle. If the average diameter of the droplets is 200 μm, what is the difference in pressure between the inside and outside of the droplets?

SOLUTION:

From the force balance on a half-droplet presented in the chapter:

$$\Delta p = \frac{2\sigma}{R}$$

Looking in the properties table for carbon tetrachloride at 20°C, $\sigma = 2.69\times10^{-2}\dfrac{\text{N}}{\text{m}}$.

Substitution yields:

$$\Delta p = \frac{2\left(2.69\times10^{-2}\dfrac{\text{N}}{\text{m}}\right)}{100\times10^{-6}\ \text{m}} = \boxed{538\ \text{Pa}}$$

Problem 1.9.4

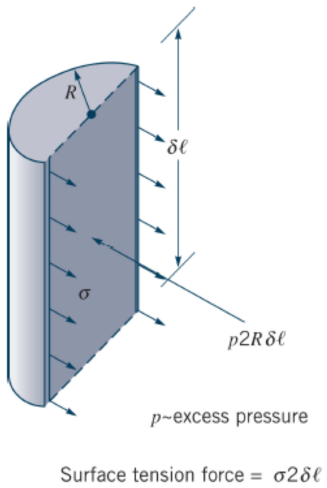
A 12-mm-diameter jet of water discharges vertically into the atmosphere. Due to surface tension, the pressure inside the jet will be slightly higher than the surrounding atmospheric pressure. Determine this difference in pressure.

SOLUTION:

Considering the free-body diagram of one half of a short length of jet, $\delta\ell$, equilibrium requires

$$p(2R\delta\ell) = \sigma(2\delta\ell)$$

$$p = \frac{\sigma}{R} = \frac{7.34\times10^{-2}\ \frac{\text{N}}{\text{m}}}{\frac{1}{2}\left(\frac{12}{1000}\ \text{m}\right)} = \boxed{12.2\ \text{Pa}}$$



Problem 1.9.5

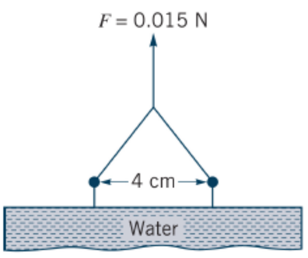
A method used to determine the surface tension of a liquid is to determine the force necessary to raise a wire ring through the air–liquid interface, as shown in the figure here. What is the value of the surface tension if a force of 0.015 N is required to raise a 4-cm-diameter ring? Consider the ring weightless, as a tensiometer (used to measure the surface tension) "zeroes" out the ring weight.

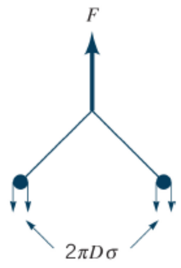
SOLUTION:

A free-body diagram of the ring and supporting wires is shown on the right and gives

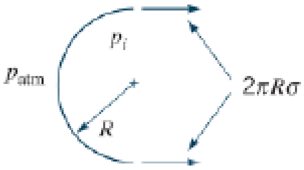
$$F = 2\pi D\sigma$$

$$\sigma = \frac{F}{2\pi D} = \frac{0.015\ \text{N}}{2\pi\left(\frac{4}{100}\ \text{m}\right)} = \boxed{5.97\times10^{-2}\ \frac{\text{N}}{\text{m}}}$$





Problem 1.9.6
Calculate the pressure difference between the inside and outside of a spherical water droplet having a diameter of 0.79 mm and a temperature of 10°C.
SOLUTION:
A force balance on the outside surface of the drop gives



$$\overset{\rightarrow}{+}\sum F = 0$$
$$p_{\text{atm}}\pi R^2 - p_1\pi R^2 + 2\pi R\sigma = 0$$
$$p_i - p_{\text{atm}} = \frac{2\sigma}{R}$$
For water at 10°C, σ = 0.074 N/m so
$$p_i - p_{\text{atm}} = \frac{2(0.074\text{ N/m})}{7.9\times10^{-4}\text{m}} = 187\text{ Pa}$$

Problem 1.9.7

As shown in Video V1.9, surface tension forces can be strong enough to allow a double-edge steel razor blade to "float" on water, but a single-edge blade will sink. Assume that the surface tension forces act at an angle θ relative to the water surface, as shown in **Fig. P1.9.7**. (a) The mass of the double-edge blade is 0.64×10^{-3} kg, and the total length of its sides is 206 mm. Determine the value of θ required to maintain equilibrium between the blade weight and the resultant surface tension force. (b) The mass of the single-edge blade is 2.61×10^{-3} kg, and the total length of its sides is 154 mm. Explain why this blade sinks. Support your answer with the necessary calculations.

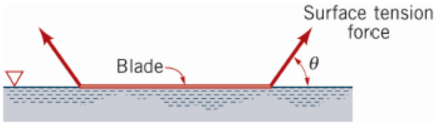
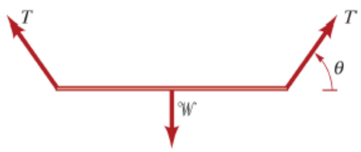


Figure P1.9.7

SOLUTION:



(a) Water $\rightarrow \sigma = 7.34\times10^{-2}\ \dfrac{\text{N}}{\text{m}}$

$$\sum F_{\text{vertical}} = T\sin\theta - W = 0$$

$$\sin\theta = \frac{W}{T} = \frac{mg}{\sigma L}$$

$$\theta = \sin^{-1}\left(\frac{(0.64\ \text{kg})\left(9.81\dfrac{\text{m}}{\text{s}^2}\right)}{\left(7.34\times10^{-2}\dfrac{\text{N}}{\text{m}}\right)(0.206\ \text{m})}\right)$$

$$\boxed{\theta = 0.415\ \text{radians} = 24.5^{\circ}}$$

(b) For the single-edge blade,

$$W = m_{\text{blade}}\times g = \left(2.61\times10^{-3}\ \text{kg}\right)\left(9.81\frac{\text{m}}{\text{s}^2}\right) = 0.0256\ \text{N}$$

and

$$T\sin\theta = (\sigma L)\sin\theta = \left(7.34\times10^{-2}\frac{\text{N}}{\text{m}}\right)(0.154\,\text{m})\sin\theta = (0.0113\ \text{N})\sin\theta$$

$$\text{For static equilibrium, } \sin\theta = \frac{0.0256\ \text{N}}{0.0113\ \text{N}} > 1, \quad \text{but } \sin\theta \le 1$$

$\rightarrow$ $\boxed{\text{single-edge blade cannot float}}$

Problem 1.9.8

Explain how sweat soldering of copper pipe works from a fluid mechanics viewpoint.

SOLUTION:

Solder for sweat soldering copper pipe is an alloy with a melting point below that of copper. The copper parts are typically heated using a gas torch to a temperature below the melting point of copper but above the melting point of the solder. When the solder is "touched" to the joint, it melts. To form a good quality joint between a copper pipe and fittings, or between fittings, capillary action must draw liquid solder into the small gap to between the two parts to fill the gap and the solder must bond with the copper surface.

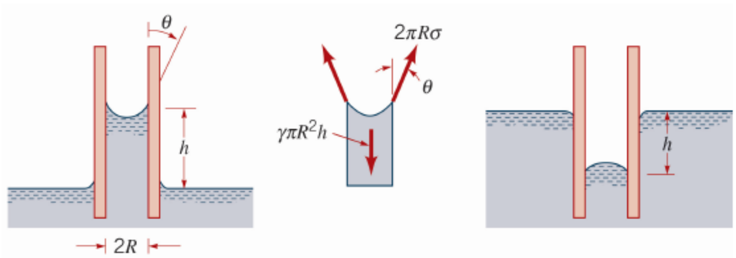
From "a fluid mechanics viewpoint," the flux used for sweat soldering of copper pipe reduces the surface tension of the liquefied solder, reducing the contact angle at the solder-copper interface, thereby producing a stronger capillary action that more effectively draws the liquid solder into the joint to fill it with solder.

From "a chemical and mechanical viewpoint," at the elevated temperatures occurring during the soldering process, oxides quickly form on the surface of copper and interfere with the bonding process. Therefore, even after mechanical cleaning of the parts, flux acts as a reducing agent to remove oxides from the surface of the copper, facilitating a stronger bond between the solder and the copper.

Problem 1.9.9

An open, clean glass tube, having a diameter of 3 mm, is inserted vertically into a dish of mercury at 20 °C (see Video V1.10). How far will the column of mercury in the tube be depressed?

SOLUTION:



The action of surface tension in a tube inserted into a pool is to draw upward (or depress) the liquid in the tube a distance $h = \dfrac{2\sigma\cos\theta}{\gamma R}$ with respect to the elevation of the surrounding free surface.

For the specified information,

$$h = \frac{2\left(4.66\times10^{-1}\,\dfrac{\text{N}}{\text{m}}\right)\cos130^\circ}{\left(133\times10^{3}\,\dfrac{\text{N}}{\text{m}^3}\right)(0.0015\,\text{m})} = -3.00\times10^{-3}\,\text{m}$$

Thus, column will be depressed $\boxed{3.00\text{ mm}}$

Problem 1.9.10

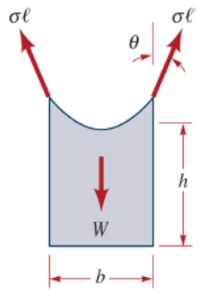
Two vertical, parallel, clean glass plates are spaced at a distance of 2 mm. If the plates are placed in water, how high will the water rise between the plates due to capillary action?

SOLUTION:

For equilibrium in the vertical direction,

$$W = \gamma h b \ell = 2(\sigma \ell \cos\theta)$$

$$h = \frac{2\sigma\cos\theta}{\gamma b}$$



(ℓ ~ width of plates)

Thus, (for $\theta = 0$)

$$h = \frac{2\left(7.34\times10^{-2}\ \frac{\text{N}}{\text{m}}\right)(1)}{\left(9.80\times10^{3}\ \frac{\text{N}}{\text{m}^3}\right)(0.002\ \text{m})} = 7.49\times10^{-3}\ \text{m} = \boxed{7.49\ \text{mm}}$$