# Chapter 2

1. For a particle Newton’s second law says vector(F) = m vector(a) = m[(d^2 x)/(d t^2) i-hat + (d^2 y)/(d t^2) j-hat + (d^2 z)/(d t^2)k-hat].

Take the second derivative of each of the expressions in Equation (2.1): (d^2 x^(prime))/(d t^2) = (d^2 x)/(d t^2) (d^2 y^(prime))/(d t^2) = (d^2 y)/(d t^2) (d^2 z^(prime))/(d t^2) = (d^2 z)/(d t^2). Substitution into the previous equation gives vector(F) = m vector(a) = m[(d^2 x^(prime))/(d t^2) i-hat + (d^2 y^(prime))/(d t^2) j-hat + (d^2 z^(prime))/(d t^2) k-hat] = vector(F^prime) .

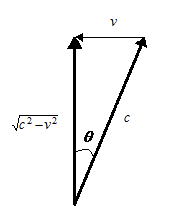
1. From Equation (2.1) vector(p) = m[(d x)/(d t) i-hat + (d y)/(d t) j-hat + (d z)/(d t) k-hat].

In a Galilean transformation (d x^(prime))/(d t) = (d x)/(d t) minus v (d y^(prime))/(d t) = (d y)/(d t) (d z^(prime))/(d t) = (d z)/(d t).

Substitution into Equation (2.1) gives vector(p) = m[(d x^(prime))/(d t) + v] i-hat + (d y^(prime))/(d t) j-hat + (d z^(prime))/(d t) k-hat != vector(p ^prime).

However, because vector(p ^prime) = m[(d x^(prime))/(d t) i-hat + (d y^(prime))/(d t) j-hat + (d z^(prime))/(d t) k-hat] the same form is clearly retained, given the velocity transformation(d x^(prime))/(d t) = (d x)/(d t) minus v.

1. Using the vector triangle shown, the speed of light coming toward the mirror is sqrt((c^2) minus (v^2)) and the same on the return trip. Therefore the total time is t_2 = (distance)/(speed) = (2 O_2)/(sqrt(c^2 minus v^2)). Notice that sin (theta) = (v/c), so theta = sin^(negative 1) (v/c).



1. As in Problem 3, sin (theta) = (v_1) / (v_2), so theta = sin^(negative 1) (v_1 / v_2) = sin^(negative 1) [(0.350 m / s)/(1.25 m / s)] = 16.3 degrees and v = sqrt((v_2^2) minus (v_1^2)) = sqrt((1.25 m / s)^2 minus (0.35 m / s)^2) = 1.20 m / s.
2. When the apparatus is rotated by 90°, the situation is equivalent, except that we have effectively interchanged O_1 and O_2. Interchanging O_1 and O_2 in Equation (2.3) leads to Equation (2.4).
3. Let *n* = the number of fringes shifted; then n = (Delta d)/(lamda). Because Delta d = c(Delta t^(prime) minus Delta t), we have n = (c(Delta t^(prime) minus Delta t))/(lamda) = (v^2 (O_1 + O_2))/(c^2 lamda). Solving for *v* and noting that O_1 + O_2 = 22 m, v = c sqrt((n lamda)/(O_1 + O_2)) = (3.00 * (10^8) m / s) sqrt(((0.005) (589 * 10^(negative 9) m))/(22 m)) = 3.47 km / s .
4. Letting O_1 right-arrow O_1 sqrt(1 minus (beta^2))(where beta = (v / c)) the text equation (not currently numbered) for t_1 becomes

t_1 = (2 l_1 sqrt(1 minus beta^2))/(c(1 minus beta)) = (2 l_1)/(c sqrt(1 minus beta^2))

which is identical to t_2when O_1 = O_2 so Delta t = 0as required.

1. Since the Lorentz transformations depend on *c* (and the fact that *c* is the same constant for all inertial frames), different values of *c* would necessarily lead two observers to different conclusions about the order or positions of two spacetime events, in violation of postulate 1.
2. Let an observer in K send a light signal along the + *x*-axis with speed *c*. According to the Galilean transformations, an observer in K^prime measures the speed of the signal to be (d x^(prime))/(d t) = (d x)/(d t) minus v = c minus v. Therefore the speed of light cannot be constant under the Galilean transformations.
3. From the Principle of Relativity, we know the correct transformation must be of the form (assuming y = y^primeand z = z^prime):

x = a x^(prime) + b t^(prime); x^prime = a x minus b t.

The spherical wave front equations (2.9a) and (2.9b) give us:

c t = (a c + b) t^(prime); c t^(prime) = (a c minus b) t.

Solve the second wave front equation for (t^prime)and substitute into the first:

c t = [((a c + b) (a c minus b) t)/c] or (c^2) = (a c + b) (a c minus b) = (a^2) (c^2) minus (b^2) .

Now *v* is the speed of the origin of the x^prime-axis. We can find that speed by setting (x^prime) = 0which gives 0 = a x minus b t, or v = (x / t) = (b / a), or equivalently *b* = *av*. Substituting this into the equation above for (c^2) yields (c^2) = (a^2) (c^2) minus (a^2) (v^2) = (a^2) ((c^2) minus (v^2)). Solving for *a*: a = 1/(sqrt(1 minus v^2 / c^2)) = gamma .

This expression, along with *b = av*, can be substituted into the original expressions for *x* and (x^prime)to obtain:

x = gamma ((x^prime) + v t^(prime)); (x^prime) = gamma (x minus v t)

which in turn can be solved for *t* and (t^prime) to complete the transformation.

1. When v < < cwe find 1 minus (beta^2) right-arrow 1, so:

(x^prime) = (x minus beta c t)/(sqrt(1 minus beta^2)) right-arrow x minus beta c t = x minus v t;

(t^prime) = ((t minus beta x / c)/(sqrt(1 minus beta^2))) right-arrow t minus (beta x / c) approximately t;

x = ((x^prime) + beta c t^(prime))/(sqrt(1 minus beta^2)) right-arrow (x^prime) + beta c t^(prime) = (x^prime) + v (t^prime);

t = ((t^prime) + beta x^(prime) / c)/(sqrt(1 minus beta^2)) right-arrow t prime + beta x^(prime) / c approximately (t^prime).

1. (a) First we convert to SI units: 95 km/h = 26.39 m/s, so beta = (v / c) = (26.39 m / s) / (3.00 * (10^8) m / s) = 8.8 * 10^(negative 8)

(b) beta = (v / c) = (240 m / s) / (3.00 * (10^8) m / s) = 8.0 * 10^(negative 7)

(c) v = 2.3 v_(sound) = (2.3 * 330 m / s) so beta = (v / c) = (2.3 * 330 m / s) / (3.00 * (10^8) m / s) = 2.5 * 10^(negative 6)

(d) Converting to SI units, 27,000 km/h = 7500 m/s, so beta = (v / c) = (7500 m / s) / (3.00 * (10^8) m / s) = 2.5 * 10^(negative 5)

(e) (25 cm)/(2 ns) = 1.25 * (10^8) m / sso beta = (v / c) = (1.25 * (10^8) m / s) / (3.00 * (10^8) m / s) = 0.42

(f) (1 * 10^(negative 14) m) / (0.35 * 10^(negative 22) s) = 2.857 * (10^8) m / s, so beta = (v / c) = (2.857 * (10^8) m / s) / (3.00 * (10^8) m / s) = 0.95

1. From the Lorentz transformationsDelta t^(prime) = gamma [Delta t minus v Delta x / (c^2)]. But Delta t^(prime) = 0in this case, so solving for *v* we find v = (c^2) Delta t / Delta x. Inserting the values Delta t = t_2 minus t_1 = (negative a) / 2 cand Delta x = x_2 minus x_1 = a, we find v = (c^2 (negative a / 2 c))/a = (negative c) / 2. We conclude that the frame (K^prime)travels at a speed *c*/2 in the negative x-direction. Note that there is no motion in the transverse direction.
2. Try settingDelta x^(prime) = 0 = gamma (Delta x minus v Delta t). Thus 0 = Delta x minus v Delta t = (a + v a) / 2 c. Solving for *v* we find v = negative 2 c, which is impossible. There is no such frame (K^prime).
3. For the smaller values of β we use the binomial expansion gamma = (1 minus beta^2)^(negative 1 / 2) approximately (1 + (beta^2)) / 2.
4. gamma approximately (1 + (beta^2)) / 2 = 1 + 3.87 * 10^(negative 15)
5. gamma approximately (1 + (beta^2)) / 2 = 1 + 3.2 * 10^(negative 13)
6. gamma approximately (1 + (beta^2)) / 2 = 1 + 3.1 * 10^(negative 12)
7. gamma approximately (1 + (beta^2)) / 2 = 1 + 3.1 * 10^(negative 10)
8. gamma = (1 minus (beta^2))^(negative 1 / 2) = (1 minus (0.42^2))^(negative 1 / 2) = 1.10
9. gamma = (1 minus (beta^2))^(negative 1 / 2) = (1 minus (0.95^2))^(negative 1 / 2) = 3.20
10. There is no motion in the transverse direction, so y = z = 3.5m.

gamma = 1/(sqrt(1 minus beta^2)) = 1/(sqrt(1 minus 0.8^2)) = (5 / 3)

x = gamma ((x^prime) + v t^(prime)) = (5/3) (2 m + 0.8 c(0)) = (10 / 3) m

t = gamma (t^prime + v x^(prime) / (c^2)) = (5/3) (0 + (0.8 c) (2 m) / (c^2)) = 8.9 * 10^(negative 9) s

1. (a) t = (sqrt(x^2 + y^2 + z^2))/c = (sqrt((3 m)^2 + (5 m)^2 + (10 m)^2))/(3.00 * (10^8) m / s) = 3.86 * 10^(negative 8) s

(b) With beta = 0.8we find gamma = (5 / 3). Then (y^prime) = y = 5m, (z^prime) = z = 10m,(x^prime) = gamma (x minus v t) = (5/3) [3 m minus (2.40 * (10^8) m / s) (3.86 * 10^(negative 8) s)] = negative 10.4 m

(t^prime) = gamma (t minus v x / c^2) = (5/3) [(3.86 * 10^(negative 8) s) minus (2.40 * (10^8) m / s) (3 m) / (3.00 * (10^8) m / s)^2] = 51.0 n s

(c) (sqrt(x^(prime 2) + y^(prime 2) + z^(prime 2)))/(t^(prime)) = (sqrt((negative 10.4 m)^2 + (5 m)^2 + (10 m)^2))/(51.0 * 10^(negative 9) s) = 2.994 * (10^8) m / s which equals *c* to within rounding errors.

1. At the point of reflection the light has traveled a distance L + v Delta t_1 = c Delta t_1. On the return trip it travels L minus v Delta t_2 = c Delta t_2. Then the total time is Delta t = Delta t_1 + Delta t_2 = (2 L c)/(c^2 minus v^2) = (2 L / c)/(1 minus (v^2) / (c^2)). But from time dilation we know (with Delta t^(prime) = proper time = 2 (L_0 / c)) that Delta t = gamma Delta t^(prime) = (2 L_0 / c)/(sqrt(1 minus v^2 / c^2)). Comparing these two results for Delta t we get (2 L / c)/ (1 minus (v^2)/(c^2)) = (2 L_0 / c)/(sqrt(1 minus v^2 / c^2))which reduces to L = L_0 sqrt((1 minus v^2) / c^2) = (L_0)/(gamma). This is Equation (2.21).
2. (a) With a contraction of 1%, (L / L_0) = 0.99 = sqrt((1 minus v^2) / c^2). Thus 1 minus (beta^2) = (0.99)^2 = 0.9801 . Solving for beta, we find beta = 0.14or v = 0.14 c .

(b) The time for the trip in the Earth-based frame is Delta t = (d/v) = (5.00 * (10^6) m)/(0.14 * 3.00 * (10^8) m / s) = 1.19 * 10^(negative 1) s. With the relativistic factor gamma = 1.01(corresponding to a 1% shortening of the ship’s length), the elapsed time on the rocket ship is 1% less than the Earth-based time, or a difference of (0.01) 1.2 * 10^(negative 1) s = 1.2 * 10^(negative 3) s .

1. The round-trip distance is *d* = 40 ly. Assume the same constant speed v = beta cfor the entire round trip. In the rocket’s reference frame the distance is only (d^prime) = d sqrt(1 minus (beta^2)) . Then in the rocket’s frame of reference v = (distance)/(time) = (d^prime)/(40 y) = (40 l y sqrt(1 minus (beta^2)))/(40 y) = c sqrt(1 minus beta^2) . Rearranging beta = (v/c) = sqrt(1 minus beta^2). Solving for betawe find beta = sqrt(0.5), or v = sqrt(0.5) c approximately 0.71 c. To find the elapsed time *t* on Earth, we know (t^prime) = 40 y, so t = gamma t^(prime) = 1/(sqrt(1 minus beta^2)) 40 y = 56.6 y .
2. In the muon’s frame T_0 = 2.2 mu s . In the lab frame the time is longer; see Equation (2.19): (T^prime) = gamma T_0. In the lab the distance traveled is 9.5 cm = sqrt(T^(prime)) = v gamma T_0 = beta c gamma T_0, since v = beta c. Therefore beta = (9.5 cm(sqrt(1 minus beta^2)))/(c T_0), so beta = (v/c) = (9.5 cm(sqrt(1 minus beta^2)))/(c (2.2 mu s)). Now all quantities are known except β. Solving for β we find beta = 1.4 * 10^(negative 4) or v = 1.4 * 10^(negative 4) c.
3. Converting the speed to m/s we find 25,000 mi/h = 11,176 m/s. From tables the distance is 3.84 * (10^8) m. In the earth’s frame of reference the time is the distance divided by speed, or t = (d/v) = (3.84 * 10^8 m)/(11,176 m / s) = 34,359 s. In the astronauts’ frame the time elapsed is(t^prime) = (t / gamma) = t sqrt(1 minus beta^2). The time difference is Delta t = t minus t^(prime) = t minus t sqrt(1 minus beta^2) = t[1 minus sqrt(1 minus beta^2)]. Evaluating numerically Delta t = 34,359 s [1 minus sqrt(1 minus ((11,176 m / s)/(3.00 * (10^8) m / s))^2)] = 2.4 * 10^(negative 5) s .
4. (T^prime) = gamma T_0, so we know that gamma = (5 / 3) = 1/(sqrt(1 minus v^2 / c^2)). Solving for *v* we find v = (4 c / 5) .
5. L = (L_0) / gammaso clearly gamma = 2in this case. Thus 2 = 1/(sqrt(1 minus v^2 / c^2)) and solving for *v* we find v = (sqrt(3)c)/2 .
6. The clocks’ rates differ by a factor ofgamma = 1 / (sqrt(1 minus v^2 / c^2)). Because beta is very small we will use the binomial theorem approximation gamma approximately (1 + beta^2) / 2. Then the time difference is Delta t = t minus t^(prime) = t minus gamma t = t(gamma minus 1). Using gamma minus 1 approximately (beta^2) / 2and the fact that the time for the trip equals distance divided by speed,

Delta t = t(beta^2 / 2) = (8 * (10^6) m)/(375 m / s) (((375 m / s)/(3.00 * (10^8) m / s))^2)/2

Delta t = 1.67 * 10^(negative 8) s = 16.7 n s .

1. (a) (L^prime) = L / gamma = L sqrt(1 minus (v^2) / (c^2)) = (3.58 * (10^4) km) sqrt(1 minus 0.94^2) = 1.22 * (10^4) km

(b) Earth’s frame: t = (L / v) = (3.58 * (10^7) m)/((0.94) (3.00 * (10^8) m / s)) = 0.127 s

Golf ball’s frame: (t^prime) = t / gamma = 0.127 s sqrt(1 minus 0.94^2) = 0.0433 s

1. Spacetime invariant (see Section 2.9): (c^2) Delta (t^2) minus Delta (x^2) = (c^2) Delta t^(prime 2) minus Delta x^(prime 2). We know Delta x = 4 km, Delta t = 0, and Delta x^(prime) = 5km. Thus Delta t^(prime 2) = (Delta x^(prime 2) minus Delta x^2)/(c^2) = ((5000 m)^2 minus (4000 m)^2)/((3.00 * (10^8) m / s)^2) = 1.0 * 10^(negative 10) (s^2)and Delta t^(prime) = 1.0 * 10^(negative 5) s.
2. (a) Converting *v* = 120 km/h = 33.3 m/s. Now with *c* = 100 m/s, we have beta = (v / c) = 0.333 and gamma = 1/(sqrt(1 minus beta^2)) = 1/(sqrt(1 minus 0.333^2)) = 1.061. We conclude that the moving person ages 6.1% slower.

(b) (L^prime) = (L / gamma) = (1 m) / (1.061) = 0.942 m .

1. Converting *v* = 300 km/h = 83.3 m/s. Now with *c* =100 m/s, we have beta = (v / c) = 0.833 and gamma = 1/(sqrt(1 minus beta^2)) = 1/(sqrt(1 minus 0.833^2)) = 1.81 . So the length is L = (L_0 / gamma) = (40 / 1.81) = 22.1 m.
2. Let subscript 1 refer to firing and subscript 2 to striking the target. Therefore we can see that x_1 = 1m, x_2 = 121m, and t_1 = 3ns. t_2 = t_1 + (distance)/(speed) = 3 n s + (120 m)/(0.98 c) = 3 n s + 408 n s = 411 n s .

To find the four primed quantities we can use the Lorentz transformations with the known values of x_1, x_2, t_1, and t_2. Note that with v = 0.8 c, gamma = sqrt((1 minus v^2) / (c^2)) = (5 / 3).

(t_1^prime) = gamma ((t_1 minus v x_1) / (c^2)) = 0.56 n s
(t_2^prime) = gamma ((t_2 minus v x_2) /(c^2)) = 147 n s
(x_1^prime) = gamma (x_1 minus v t_1) = 0.47 m
(x_2^prime) = gamma (x_2 minus v t_2) = 37.3 m

1. Start from the formula for velocity addition, Equation (2.23a): u_x = ((u_x^prime) + v)/(1 + (v u_x^prime / c^2)).

(a) u_x = (0.62 c + 0.84 c)/(1 + (0.62 c) (0.84 c) / c^2) = (1.46 c)/(1.52) = 0.96 c

(b) u_x = (negative 0.62 c + 0.84 c)/(1 + (negative 0.62 c) (0.84 c) / c^2) = (0.22 c)/(0.48) = 0.46 c

1. Velocity addition, Equation (2.24): (u_x^prime) = (u_x minus v)/(1 minus (v u_x / c^2)) with v = negative 0.8 cand u_x = 0.8 c .

(u_x^prime) = (0.8 c minus (negative 0.8 c))/(1 minus (negative 0.8 c) (0.8 c) / c^2) = (1.6 c)/(1.64) = 0.976 c

1. Conversion: 110 km/h = 30.556 m/s and 140 km/h = 38.889 m/s. Let u_x = 30.556 m/s and v = negative 38.889m/s. Our premise is that c = 100m/s. Then by velocity addition, (u_x^prime) = (u_x minus v)/(1 minus (v u_x / c^2)) = (30.556 m / s minus (negative 38.889 m / s))/(1 minus (negative 38.889 m / s) (30.556 m / s) / (100 m / s)^2) = 62.1 m / s . By symmetry each observer sees the other one traveling at the same speed.
2. From Example 2.5 we have u = (c/n) [(1 + n v / c)/(1 + v / n c)]. For light traveling in opposite directions Delta u = (c/n) [(1 + n v / c)/(1 + v / n c) minus (1 minus n v / c)/(1 minus v / n c)]. Because (v / c) is very small, use the binomial expansion: (1 + n v / c)/(1 + v / n c) = (1 + n v / c) (1 + v / n c)^(negative 1) approximately (1 + n v / c) (1 minus v / n c) approximately 1 + (n v / c) minus (v / n c), where we have dropped terms of order (v^2) / (c^2) . Similarly (1 minus n v / c)/(1 minus v / n c) approximately 1 minus (n v / c) + (v / n c) . Thus Delta u approximately (c/n) [(1 + n v / c minus v / n c) minus (1 minus n v / c + v / n c)] = (2 v)/n(1 minus 1 / n) = 2 v(1 minus 1 / n^2) . Evaluating numerically we find Delta u approximately 2 (5 m / s) (1 minus 1/(1.33^2)) = 4.35 m / s .
3. Clearly the speed of B is just 0.60 c. To find the speed of C use u_x = 0.60 cand v = negative 0.60 c: (u_x^prime) = (u_x minus v)/(1 minus (v u_x / c^2)) = (0.60 c minus (negative 0.60 c))/(1 minus (negative 0.60 c) (0.60 c) / c^2) = 0.88 c .
4. We can ignore the 400 km, which is small compared with the Earth-to-moon distance 3.84 * (10^8)m. The rotation rate is omega = 2 pi rad * 100 s^(negative 1) = 2 pi * (10^2) rad/s. Then the speed across the moon’s surface is v = omega R = (2 pi * (10^2) rad / s) (3.84 * (10^8) m) = 2.41 * (10^11) m / s .
5. Classical: t = (4205 m)/(0.98 c) = 1.43 * 10^(negative 5) s. Then N = N_0 exp[(negative (ln 2) t)/(t_(1 / 2))] = 14.6 or about 15 muons.

Relativistic: (t^prime) = (t / gamma) = (1.43 * 10^(negative 5) s)/5 = 2.86 * 10^(negative 6) s so N = N_0 exp[(negative (ln 2) t)/(t_(1 / 2))] = 2710 muons . Because of the exponential nature of the decay curve, a factor of five (shorter) in time results in many more muons surviving.

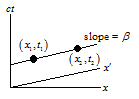
1. The circumference of the fixed point’s rotational path is 2 pi R_(E) cos(39 degrees), where R_(E) = Earth’s radius = 6378 km. Thus the circumference of the path is 31,143 km. The rotational speed of that point is v = (31,143 km) / 24 h = 1298 km / h = 360.5 m / s. The observatory clock runs slow by a factor of gamma = 1/(sqrt(1 minus beta^2)) approximately (1 + beta^2) / 2 = 1 + 7.22 * 10^(negative 13). In 41.2 h the observatory clock is slow by (41.2 h) (7.22 * 10^(negative 13)) = 2.9746 * 10^(negative 11) h = 107 ns. In 48.6 h it is slow by (48.6 h) (7.22 * 10^(negative 13)) = 3.5089 * 10^(negative 11) h = 126 ns. The Eastward-moving clock has a ground speed of 31,143 km/41.2 h = 755.9 km/h = 210.0 m/s and thus has a net speed of 210.0 m/s + 360.5 m/s = 570.5 m/s. For this clock gamma = 1/(sqrt(1 minus beta^2)) approximately (1 + beta^2) / 2 = 1 + 1.81 * 10^(negative 12) and in 41.2 hours it runs slow by (41.2 h) (1.81 * 10^(negative 12)) = 7.4572 * 10^(negative 11) h = 268 ns. The Westward-moving clock has a ground speed of 31,143 km/48.6 h = 640.8 km/h = 178.0 m/s and thus has a net speed of 360.5 m/s − 178.0 m/s = 182.5 m/s. For this clock gamma = 1/(sqrt(1 minus beta^2)) approximately (1 + beta^2) / 2 = 1 + 1.85 * 10^(negative 13) and in 48.6 hours it runs slow by (48.6 h) (1.85 * 10^(negative 13)) = 8.991 * 10^(negative 12) h = 32 ns. So our prediction is that the Eastward-moving clock is off by 107 ns – 269 ns = negative 162ns, while the Westward-moving clock is off by 126 ns − 32 ns = 94 ns. These results are correct for special relativity but do not reconcile with those in the table in the text, because general relativistic effects are of the same order of magnitude.
2. The derivations of Equations (2.31) and (2.32) in the beginning of Section 2.10 will suffice. Mary receives signals at a rate (f^prime) for (t_1^prime)and a rate f^prime primefor (t_2^prime). Frank receives signals at a rate (f^prime) for t_1 and a rate f^prime primefor t_2.
3. T = t_1 + t_2 = (L/v) + (L/c) + (L/v) minus (L/c) = (2 L)/v; Frank sends signals at rate *f*, so Mary receives f T = (2 f L) / v signals.

(T^prime) = (t_1^prime) + (t_2^prime) = (2 L)/(gamma v); Mary sends signals at rate *f* , so Frank receives f T^(prime) = (2 f L) / gamma v signals.

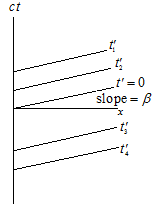
1. (s^2) = (x^2) + (y^2) + (z^2) minus (c^2) (t^2); Using the Lorentz transformation

(s^2) = gamma^2 (x^prime + v t^(prime))^2 + y^(prime 2) + z^(prime 2) minus (c^2) (gamma^2) (t prime + v x^(prime) / c^2)^2
= x^(prime 2) (gamma^2) (1 minus v^2 / c^2) + y^(prime 2) + z^(prime 2) minus (c^2) t^(prime 2) (gamma^2) (1 minus v^2 / c^2)
= x^(prime 2) + y^(prime 2) + z^(prime 2) minus (c^2) t^(prime 2) = s^(prime 2)

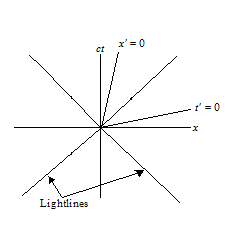
1. For a timelike interval Delta (s^2) < 0so Delta (x^2) < (c^2) Delta (t^2). We will prove by contradiction. Suppose that there is a frame (K^prime)is which the two events were simultaneous, so that Delta t^(prime) = 0. Then by the spacetime invariant Delta (x^2) minus (c^2) Delta (t^2) = Delta x^(prime 2) minus (c^2) Delta t^(prime 2) = Delta x^(prime 2). But because Delta (x^2) < (c^2) Delta (t^2), this implies Delta x^(prime 2) < 0 which is impossible because Delta (x^prime)is real.
2. As in Problem 42, we know that for a spacelike interval Delta (s^2) > 0so Delta (x^2) > (c^2) Delta (t^2). Then in a frame (K^prime) in which the two events occur in the same place, Delta x^(prime) = 0and Delta (x^2) minus (c^2) Delta (t^2) = Delta x^(prime 2) minus (c^2) Delta t^(prime 2) = negative (c^2) Delta t^(prime 2). But because Delta (x^2) > (c^2) Delta (t^2)we have (c^2) Delta t^(prime 2) < 0, which is impossible because Delta (t^prime)is real.
3. In order for two events to be simultaneous in (K^prime), the two events must lie along the (x^prime)axis, or along a line parallel to the (x^prime)axis. The slope of the (x^prime)axis is beta = (v / c), so (v / c) = slope = (c Delta t)/(Delta x). Solving for *v*, we findv = (c^2) (Delta t / Delta x). Since the slope of the (x^prime)axis must be less than one, we see that Delta x > c Delta t so (s^2) = Delta (x^2) minus (c^2) Delta (t^2) > 0 is required.



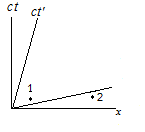
1. parts (a) and (b) To find the equation of the line use the Lorentz transformation. With (t^prime) = 0we have (t^prime) = 0 = gamma (t minus v x / c^2)or, rearranging, c t = (v x / c) = beta x. Thus the graph of *ct* vs. *x* is a straight line with a slope beta. (c) Now with (t^prime)constant, the Lorentz transformation gives (t^prime) = gamma (t minus v x / c^2). Again we solve for *ct* : c t = beta x + c t^(prime) / gamma = beta x + constant. This line is parallel to the (t^prime) = 0 line we found earlier but shifted by the constant. (d) Here both the (x^prime) and c (t^prime) axes are shifted from their normal (*x*, *ct*) orientation and they are not perpendicular. The angle between the axes decreases as *β* increases.



1. The diagram is shown here. Note that there is only one worldline for light, and it bisects both the *x*, *ct* axes and the (x^prime), c (t^prime)axes. The (x^prime)and c (t^prime)axes are not perpendicular. This can be seen as a result of the Lorentz transformations, since (x^prime) = 0defines the c (t^prime)axis and (t^prime) = 0defines the (x^prime)axis.



1. The diagram shows that the events A and B that occur at the same time in K occur at different times in (K^prime).



1. The Doppler shift gives lamda = lamda_0 sqrt((1 minus beta)/(1 + beta)). With numerical values lamda_0 = 650 nm and lamda = 540nm, solving this equation for betagives beta = 0.183. The astronaut’s speed is v = beta c = 5.50 * (10^7)m/s. In addition to a red light violation, the astronaut gets a speeding ticket.
2. According to the fixed source (K) the signal and receiver move at speeds *c* and *v*, respectively, in opposite directions, so their relative speed is c + v. The time interval between receipt of signals is Delta t = lamda / (c + v) = (1 / f_0). By time dilation Delta t^(prime) = (Delta t)/(gamma) = (lamda)/(gamma (c + v)). Using lamda = (c / v_0) and gamma = 1/(sqrt(1 minus v^2 / c^2)) we find Delta t^(prime) = (c sqrt(1 minus v^2 / c^2))/(f_0 (c + v)) = (sqrt(1 minus beta^2))/(f_0 (1 + beta))and (f^prime) = 1/(Delta t^(prime)) = (f_0 (1 + beta))/(sqrt(1 minus beta^2)) = f_0 sqrt((1 + beta)/(1 minus beta)).
3. For a fixed source and moving receiver, the length of the wave train is c T + v T . Since *n* waves are emitted during time *T*, lamda = (c T + v T)/nand the frequency f = (c / lamda)isf = (c n)/(c T + v T). As in the text n = f_0 (T_0^prime) and (T_0^prime) = (T / gamma) . Therefore f = (c f_0 T / gamma)/(c T + v T) = (f_0 sqrt(1 minus beta^2))/(1 + beta) = f_0 sqrt((1 minus beta)/(1 + beta)).
4. f = f_0 sqrt((1 minus beta)/(1 + beta)) = (1400 kz) sqrt((1 minus 0.95)/(1 + 0.95)) = 224 kHz

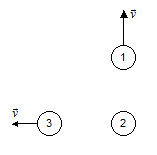
An illustration shows a small circle #2 representing a fixed van at the center and two small circles #1 and #3 representing two moving vans to the right and left of #2, respectively. The van #1 moves to the right with speed v-bar represented by a right arrow labeled v-bar. The van #3 moves to the left with speed v-bar represented by a left arrow labeled v-bar.

1. The Doppler shift function (f^prime) = f_0 sqrt((1 minus beta)/(1 + beta))

is the rate at which #1 and #2 receive signals from each other and the rate at which #2 and #3 receive signals from each other. But for signals between #1 and #3 the rate is (f^prime prime) = (f^prime) sqrt((1 minus beta)/(1 + beta)) = f_0 (1 minus beta)/(1 + beta).

1. The Doppler shift function (f^prime) = f_0 sqrt((1 minus beta)/(1 + beta)) is the rate at which #1 and #2 receive signals

from each other and the rate at which #2 and #3 receive signals from each other. As for #1 and #3 we will assume that these plumbing vans are non-relativistic (v < < c). Otherwise it would be necessary to use the velocity addition law and apply the transverse Doppler shift. From the figure we see that (f^prime) = 1/(t_0 + (t_2 minus t_1)). Now f_0 = (1 / t_0) and t_2 minus t_1 = (2 x)/c = (2 v t_0 cos (theta))/c. With an angle of 45 degrees, cos(45) = 1 / sqrt(2) and (f^prime) = 1/(1 / f_0 + (2 v cos theta) / c f_0) = (f_0)/(1 + (2 v cos theta) / c) = (f_0)/(1 + sqrt(2) v / c).



1. The Doppler shift to higher wavelengths is (with lamda_0 = 589 nm) lamda = 700 nm = lamda_0 sqrt((1 + beta)/(1 minus beta)). Solving for beta we find beta = 0.171. Then t = (v/a) = ((0.171) (3.00 * (10^8) m / s))/(29.4 m / s^2) = 1.75 * (10^6) swhich is 20.25 days. One problem with this analysis is that we have only computed the time as measured by Earth. We are not prepared to handle the non-inertial frame of the spaceship.
2. Let the instantaneous momentum be in the *x*-direction and the force be in the *y*-direction. Then d vector(p) = vector(F) d t = gamma m d vector(v) and d vector(v) is also in the *y*-direction. So we have vector(F) = gamma m(d vector(v))/(d t) = gamma m vector(a).
3. The magnitude of the centripetal force is gamma m a = gamma m(v^2)/rfor circular motion. For a charged particle F = q v B, so q v B = gamma m(v^2)/ror, rearranging q B r = gamma m v = p. Therefore

r = p/(q B) .

When the speed increases the momentum increases, and thus for a given value of *B* the radius must increase.

1. vector(p) = gamma m vector(v) = (m vector(v))/(sqrt(1 minus v^2 / c^2)) and vector(F) = (d vector(p))/(d t). The momentum is the product of two factors that contain the velocity, so we apply the product rule for derivatives:

vector(F) = m d/(d t) [(m vector(v))/(sqrt(1 minus v^2 / c^2))]
= m[(d vector(v) / d t)/(sqrt(1 minus v^2 / c^2)) + vector(v) d/(d t) (1/(sqrt(1 minus v^2 / c^2)))]
= gamma m vector(a) + m vector(v) (negative 1/2) (negative (2 v)/(c^2)) (gamma^3) (d v)/(d t)
= gamma m vector(a) + (gamma^3) m vector(a) ((v^2)/(c^2))
= (gamma^3) m vector(a) [1 minus (v^2)/(c^2) + (v^2)/(c^2)]
= (gamma^3) m vector(a)

1. From the preceding problem F = (gamma^3) m a. We have a = (10^19)m/s2 and m = 1.67 * 10^(negative 27) kg.
2. gamma = 1/(sqrt(1 minus v^2 / c^2)) = 1/(sqrt(1 minus 0.01^2)) = 1.00005
   F = (1.00005)^3 (1.67 * 10^(negative 27) kg) (10^19 m / s^2) = 1.67 * 10^(negative 8) N
3. As in (a) gamma = 1.005and F = 1.70 * 10^(negative 8) N
4. As in (a) gamma = 2.294and F = 2.02 * 10^(negative 7)N
5. As in (a) gamma = 7.0888and F = 5.95 * 10^(negative 6)N
6. p = gamma m v with gamma = 1/(sqrt(1 minus v^2 / c^2)) = 1/(sqrt(1 minus 0.92^2)) = 2.5516; m = p/(gamma v) = (10^(negative 16) kg * m / s)/((2.5516) (0.92) (3.00 * (10^8) m / s)) = 1.42 * 10^(negative 25) kg
7. The initial momentum isp_0 = gamma m v = 1/(sqrt(1 minus (0.5)^2)) m(0.5 c) = 0.57735 m c.

(a) (p / p_0) = 1.01

1.01 = (gamma m v)/(0.57735 m c)
gamma v = (1.01) (0.57735 c) = 0.58312 c

Substituting forgammaand solving for *v*, v = [1/((.58312 c)^2) + 1/(c^2)]^(negative 1 / 2) = 0.504 c .

(b) Similarly v = [1/((.63509 c)^2) + 1/(c^2)]^(negative 1 / 2) = 0.536 c

(c) Similarly v = [1/((1.1547 c)^2) + 1/(c^2)]^(negative 1 / 2) = 0.756 c

1. 6.3 GeV protons have K = 6.3 * (10^3)MeV and E = K + E_0 = 7238MeV. Then p = (sqrt(E^2 minus E_0^2))/c = 7177 MeV / c. Converting to SI units p = 7177 MeV / c((1.60 * 10^(negative 13) J)/(MeV)) (c/(3.00 * (10^8) m / s)) = 3.83 * 10^(negative 18) kg * m / s

From Problem 56 we have B = p/(q r) = (3.83 * 10^(negative 18) kg * m / s)/((1.60 * 10^(negative 19) C) (15.2 m)) = 1.57 T.

1. Initially Mary throws her ball with velocity (primes showing the measurements are in Mary’s frame): u_(M_x)^(prime) = 0 u_(M_y)^(prime) = negative u_0 . After the elastic collision, the signs on the above expressions are reversed, so the change in momentum as measured by Mary is Delta p_M^(prime) = (m u_0)/(sqrt(1 minus u_0^2 / c^2)) minus (negative m u_0)/(sqrt(1 minus u_0^2 / c^2)) = (2 m u_0)/(sqrt(1 minus u_0^2 / c^2)).

Now for Frank’s ball, we know u_(F_x) = 0and u_(F_y) = u_0. The velocity transformations give for Frank’s ball as measured by Mary: u_(F_x)^(prime) = negative v ; u_(F_y)^(prime) = u_0 sqrt(1 minus (v^2) / (c^2)).