Chapter 2

2.1-1 The step function $u(t)$ is a power signal because its energy is infinite, i.e., $E_u = \int_{-\infty}^{\infty} u^2(t)\,dt = \infty$.

2.1-2 Let us denote the signal in question by $g(t)$ and its energy by E_g.

(a),(b) For parts **(a)** and **(b)**, we write

$$E_g = \int_0^{2\pi} \sin^2 t\,dt = \frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt = \pi + 0 = \pi$$

(c)

$$E_g = \int_{2\pi}^{4\pi} \sin^2 t\,dt = \frac{1}{2}\int_{2\pi}^{4\pi} dt - \frac{1}{2}\int_{2\pi}^{4\pi} \cos 2t\,dt = \pi + 0 = \pi$$

(d)

$$E_g = \int_0^{2\pi} (2\sin t)^2\,dt = 4\left[\frac{1}{2}\int_0^{2\pi} dt - \frac{1}{2}\int_0^{2\pi} \cos 2t\,dt\right] = 4[\pi + 0] = 4\pi$$

Sign change and time shift do not affect the signal energy. Doubling the signal quadruples its energy. In the same way, we can show that the energy of $kg(t)$ is $k^2 E_g$.

2.1-3 Both $\varphi(t)$ and $w_0(t)$ are periodic.

The average power of $\varphi(t)$ is $P_g = \frac{1}{T}\int_0^T \varphi^2(t)\,dt = \frac{1}{\pi}\int_0^{\pi}\left(e^{-t/2}\right)^2\,dt = \frac{1-e^{-\pi}}{\pi}$.

The average power of $w_0(t)$ is $P_g = \frac{1}{T_0}\int_o^{T_0} w_o^2(t)\,dt = \frac{1}{T_0}\int_0^{T_0} 1\cdot dt = 1$.

2.1-4

(a) Since $x(t)$ is a real signal, $E_x = \int_0^2 x^2(t)\,dt$.

Solving for Fig. S2.1-4(a), we have

$E_x = \int_0^2 (1)^2\,dt = 2,\quad E_y = \int_0^1 (1)^2\,dt + \int_1^2 (-1)^2\,dt = 2$

$E_{x+y} = \int_0^1 (2)^2\,dt = 4,\quad E_{x-y} = \int_1^2 (2)^2\,dt = 4$

Therefore, $E_{x\pm y} = E_x + E_y$.

Solving for Fig. S2.1-4(b), we have

$E_x = \int_0^{\pi} (1)^2\,dt + \int_{\pi}^{2\pi} (-1)^2\,dt = 2\pi,\quad E_y = \int_0^{\pi/2} (1)^2\,dt + \int_{\pi/2}^{\pi} (-1)^2\,dt + \int_{\pi}^{3\pi/2} (1)^2\,dt + \int_{3\pi/2}^{2\pi} (-1)^2\,dt = 2\pi$

$E_{x+y} = \int_0^{\pi/2} (2)^2\,dt + \int_{\pi/2}^{3\pi/2} (0)^2\,dt + \int_{3\pi/2}^{2\pi} (-2)^2\,dt = 4\pi$

$E_{x-y} = \int_0^{\pi/2} (0)^2\,dt + \int_{\pi/2}^{\pi} (2)^2\,dt + \int_{\pi/2}^{3\pi/2} (-2)^2\,dt + \int_{3\pi/2}^{2\pi} (0)^2\,dt = 4\pi$

Therefore, $E_{x\pm y} = E_x + E_y$.

(b) $E_x = \int_0^{\pi/4} (1)^2\,dt + \int_{\pi/4}^{\pi} (-1)^2\,dt = \pi,\quad E_y = \int_0^{\pi} (1)^2\,dt = \pi$

$E_{x+y} = \int_0^{\pi/4} (2)^2\,dt + \int_{\pi/4}^{\pi} (0)^2\,dt = \pi,\quad E_{x-y} = \int_0^{\pi/4} (0)^2\,dt + \int_{\pi/4}^{\pi} (-2)^2\,dt = 3\pi$

Therefore, $E_{x\pm y} \neq E_x + E_y$, and $E_{\hat{x}\pm\hat{y}} = E_{\hat{x}} \pm E_{\hat{y}}$ are not true in general.

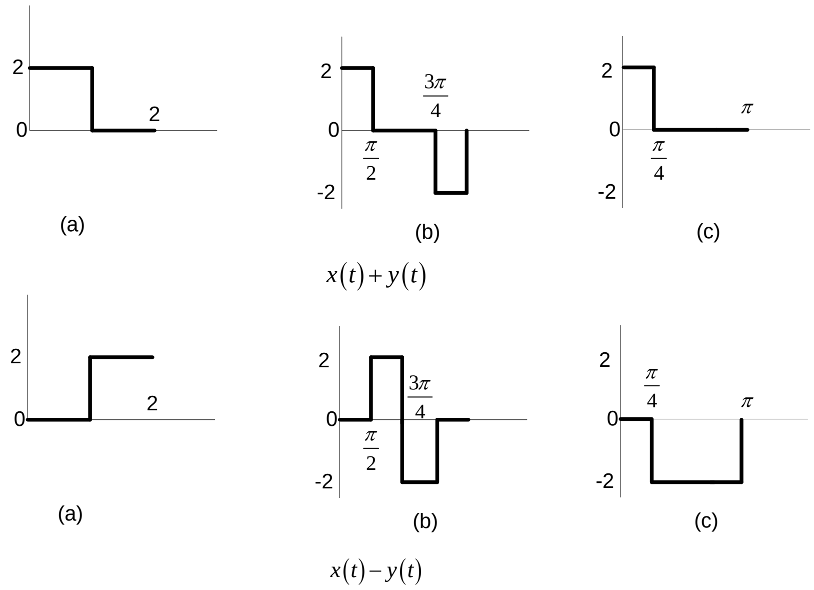

Fig. S2.1-4

2.1-5

$$
\begin{aligned}
P_g &= \frac{1}{4}\int_{-2}^{2}(t^3)^2\,dt = 64/7 \\
\textbf{(a) } P_{-g(t)} &= \frac{1}{4}\int_{-2}^{2}(-t^3)^2\,dt = 64/7 \\
\textbf{(b) } P_{1.5g(t)} &= \frac{1}{4}\int_{-2}^{2}(1.5t^3)^2\,dt = 144/7 \\
\textbf{(c) } P_{g(-t)} &= \frac{1}{4}\int_{-2}^{2}(-t^3)^2\,dt = 64/7 \\
\textbf{(d) } P_{g(1.5t)} &= \frac{3}{8}\int_{-4/3}^{4/3}((1.5t)^3)^2\,dt = 64/7
\end{aligned}
$$

Comment: Changing the sign of a signal does not affect its power. Multiplication of a signal by a constant c increases the power by a factor of c^2. Time-scaling of a signal does not change its power, though the signal period changes.

2.1-6

$$
\begin{aligned}
P_g &= \frac{1}{T_0}\int_0^{T_0} C^2\cos^2(\omega_0 t+\theta)\,dt = \frac{C^2}{2T_0}\int_0^{T_0}\left[1+\cos(2\omega_0 t+2\theta)\right]\,dt \\
&= \frac{C^2}{2T_0}\left[\int_0^{T_0} dt + \int_0^{T_0}\cos(2\omega_0 t+2\theta)\,dt\right] = \frac{C^2}{2T_0}\left[T_0+0\right] = \frac{C^2}{2}
\end{aligned}
$$

2.1-7 If $\omega_1 = \omega_2$, then

$$
\begin{aligned}
g^2(t) &= \left(C_1\cos(\omega_1 t+\theta_1) + C_2\cos(\omega_1 t+\theta_2)\right)^2 \\
&= C_1^2\cos^2(\omega_1 t+\theta_1)\ +\ C_2^2\cos^2(\omega_1 t+\theta_2)\ +\ 2C_1C_2\cos(\omega_1 t+\theta_1)\cos(\omega_1 t+\theta_2)
\end{aligned}
$$

$$
\begin{aligned}
P_g &= \lim_{T_0\to\infty} \quad \frac{1}{T_0}\int_0^{T_0} \left(C_1\cos(\omega_1 t+\theta_1)+C_2\cos(\omega_1 t+\theta_2)\right)^2\, dt \\
&= \frac{C_1^2}{2}+\frac{C_2^2}{2}+\lim_{T\to\infty} 2C_1C_2\frac{1}{T_0}\int_0^{T_0} \cos(\omega_1 t+\theta_1)\cos(\omega_1 t+\theta_2)\, dt \\
&= \frac{C_1^2}{2}+\frac{C_2^2}{2}+\lim_{T\to\infty} 2C_1C_2\frac{1}{T_0}\int_0^{T_0} \frac{1}{2}\big[\cos(2\omega_1 t+\theta_1+\theta_2)+\cos(\theta_1-\theta_2)\big]\, dt \\
&= \frac{C_1^2}{2}+\frac{C_2^2}{2}+0+\frac{2C_1C_2}{2}\cos(\theta_1-\theta_2) \\
&= \frac{C_1^2+C_1^2+2C_1C_2\cos(\theta_1-\theta_2)}{2}
\end{aligned}
$$

2.1-8

$$
\begin{aligned}
P_g &= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} g(t)g^*(t)\, dt \\
&= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}\sum_{r=m}^{n} D_k {D^*}_r e^{j(\omega_k-\omega_r)t}\, dt \\
&= \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}\sum_{r=m,r\neq k}^{n} D_k {D^*}_r e^{j(\omega_k-\omega_r)t}\, dt + \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}|D_k|^2\, dt
\end{aligned}
$$

The integrals of the cross-product terms (when $k \neq r$) are finite because the integrands (functions to be integrated) are periodic signals (made up of sinusoids). These terms, when divided by $T \to \infty$, yield zero. The remaining terms ($k = r$) yield

$$
P_g = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}\sum_{k=m}^{n}|D_k|^2\, dt = \sum_{k=m}^{n}|D_k|^2
$$

2.1-9

(a) From Eq. (2.5a), the power of a signal of amplitude C is $P_g = \frac{C^2}{2}$, regardless of phase and frequency; therefore, $P_g = 25/2$; the rms value is $\sqrt{P_g} = 5/\sqrt{2}$.

(b) From Eq. (2.5b), the power of the sum of two sinusoids of different frequencies is the sum of the power of individual sinusoids, regardless of the phase, $\frac{C_1^2}{2}+\frac{C_2^2}{2}$, therefore, $P_g = 25/2 + 4/2 = 12.5 + 2 = 14.5$; the rms value is $\sqrt{P_g} = \sqrt{14.5}$.

(c) Same as **(b)**, $P_g = 25/2 + 4/2 = 12.5 + 2 = 14.5$; the rms value is $\sqrt{P_g} = \sqrt{14.5}$.

(d) $g(t) = 5\sin(55t)\sin(\pi t) = \frac{5(\cos(55t-\pi t)-\cos(55t+\pi t))}{2}$
Therefore, $P_g = 25/8 + 25/8 = 25/4$; the rms value is $\sqrt{P_g} = 5/2$.

(e) Given $g(t) = 10\sin(5t)\cos(10t)\,u(t)$. By definition,

$$
\begin{aligned}
P_g = &\quad \lim_{T\to\infty}\tfrac{1}{T}\int_0^{T/2} 100\sin^2(5t)\cos^2(10t)dt \\
&= \lim_{T\to\infty}\tfrac{100}{T}\int_0^{T/2}\left(\tfrac{(1-\cos(10t))}{2}\tfrac{(1+\cos(20t))}{2}\right) = \tfrac{100}{4T}\tfrac{T}{2} = 25/2.
\end{aligned}
$$

The rms value is $\sqrt{P_g} = 5/\sqrt{2}$.

(f) $|g(t)|^2 = \sin^2(\omega_0 t)$

Therefore, $P_g = 1/2 = 0.5$; the rms value is $\sqrt{P_g} = \sqrt{0.5}$.

2.1-10

(a) $P_g = \frac{1}{4}\int_{-2}^{2} 1^2 dt = 1$; the rms value is $\sqrt{P_g} = 1$.

(b) $P_g = \frac{1}{10\pi}\int_{-\pi}^{\pi} 1^2 dt = \frac{1}{5}$; the rms value is $\sqrt{P_g} = 1/\sqrt{5}$.

(c) $P_g = \frac{1}{2\pi}\int_{0}^{2\pi} \left(\frac{t}{2\pi}\right)^2 dt = \frac{1}{3}$; the rms value is $\sqrt{P_g} = 1/\sqrt{3}$.

(d) $P_g = \frac{1}{\pi}\int_{-\pi/4}^{\pi/4} \left(\frac{4t}{\pi}\right)^2 dt = \frac{1}{6}$; the rms value is $\sqrt{P_g} = 1/\sqrt{6}$.

(e) $P_g = \frac{1}{3}\int_{0}^{1} t^2 dt = \frac{1}{9}$; the rms value is $\sqrt{P_g} = 1/3$.

(f) $P_g = \frac{1}{6}\left(\int_{-2}^{-1}(t+2)^2 dt + \int_{-1}^{1} dt + \int_{1}^{2}(t-2)^2 dt\right) = \frac{4}{9}$; the rms value is $\sqrt{P_g} = 2/3$.

2.2-1 If a is real, then both $E_g = \int_{-\infty}^{\infty} |e^{-at}|^2 \cdot dt = \infty$ and $P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |e^{-at}|^2 \cdot dt = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} e^{-2at} \cdot dt = \infty$.

If a is purely imaginary, $a = i\alpha$; then, $g(t) = e^{-i\alpha t}$ and $|g(t)|^2 = 1$. $P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} 1 \cdot dt = \lim_{T\to\infty} \frac{1}{T} T = 1$. It is not an energy signal since $E_g = \int_{-\infty}^{\infty} |g(t)|^2 \cdot dt = \infty$. Hence it is a power signal.

2.2-2

(a) $P_g = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} t^4\, dt = \infty$. Hence, it is not a power signal.

(b) $E_g = \int_{-\infty}^{\infty} t^2\, dt = \infty$. Hence, it is not an energy signal.

2.3-1

$$g_2(t) = g(t-1) + g_1(t-1), \quad g_3(t) = g(t-1) + g_1(t+1), \quad g_4(t) = g(t-0.5) + g_1(t+0.5)$$

The signal $g_5(t)$ can be obtained by (i) delaying $g(t)$ by 1 second (replace t with $t-1$), (ii) then time-expanding by a factor 2 (replace t with $t/2$), (iii) then multiplying by 1.5. Thus $g_5(t) = 1.5g(\frac{t}{2} - 1)$.

2.3-2 See Fig. S2.3-2.

2.3-3

(a) See Fig. S2.3-3.

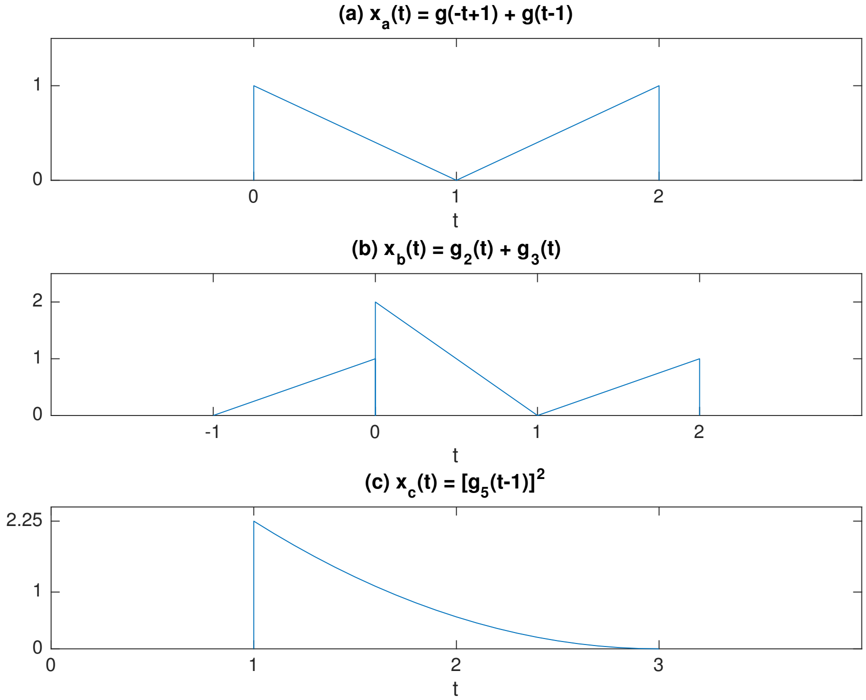

Fig. S2.3-2

(b) $E_g = \int_{-\infty}^{\infty} |g(t)|^2\, dt = \int_6^{15} \left[\frac{1}{6}(t-12)\right]^2\, dt + \int_{15}^{24} \left[-\frac{1}{18}(t-24)\right]^2\, dt = 3.$

Based on the properties derived in Prob. 2.3-6, the energies are
(i) $E_{g(-t)} = 3$
(ii) $E_{g(t+2)} = 3$
(iii) $E_{g(-3t)} = E_g/3 = 1$
(iv) $E_{g(t/3)} = 3E_g = 9$
(v) $E_{g(2t+1)} = E_g/2 = 1.5$
(vi) $E_{g(2t+2)} = E_g/2 = 1.5$

2.3-4 Denote $g(at) = f(t)$. Since $g(t)$ is periodic with period T,

$$\begin{aligned} g(t) &= g(t+T) \\ g(a\ t) &= g(a\ t+T) = g\left(a\big(t+\frac{T}{a}\big)\right) \\ f(t) &= f\left(t+\frac{T}{a}\right) \end{aligned}$$

Therefore, $g(at)$ is periodic with period T/a.

The average power of $g(a\ t)$ is

$$P_{g(a\ t)} = \lim_{T\to\infty} \frac{a}{T} \int_{-T/2a}^{T/2a} g^2(at)\, dt = \lim_{T\to\infty} \frac{a}{T} \int_{-T/2}^{T/2} g^2(z)\, \frac{dz}{a} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} g^2(z)\, dz = P_g$$

Therefore, the average power remains the same.

2.3-5 See Fig. S2.3-5.

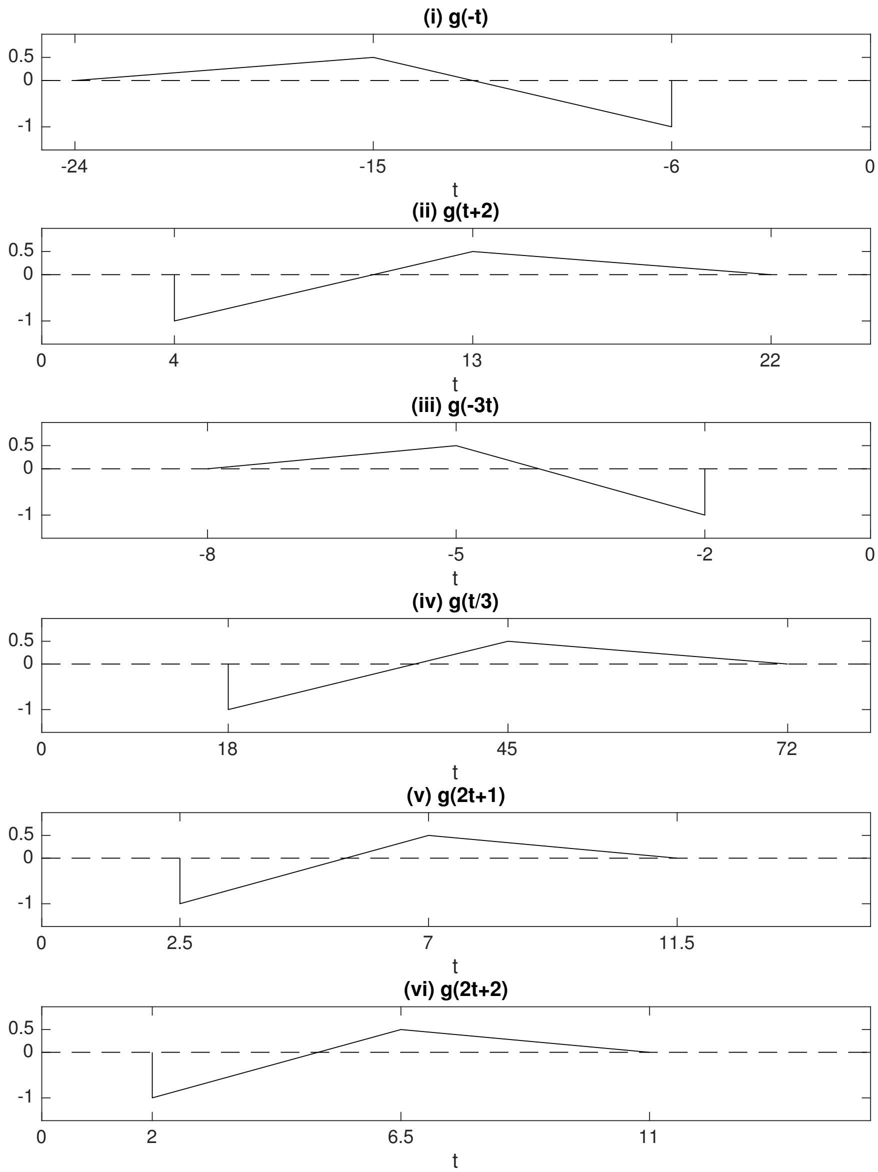
(i) g(-t)
0.5
0
-1
-24
-15
-6
0
t
(ii) g(t+2)
0.5
0
-1
0
4
13
22
t
(iii) g(-3t)
0.5
0
-1
-8
-5
-2
0
t
(iv) g(t/3)
0.5
0
-1
0
18
45
72
t
(v) g(2t+1)
0.5
0
-1
0
2.5
7
11.5
t
(vi) g(2t+2)
0.5
0
-1
0
2
6.5
11
t

Fig. S2.3-3

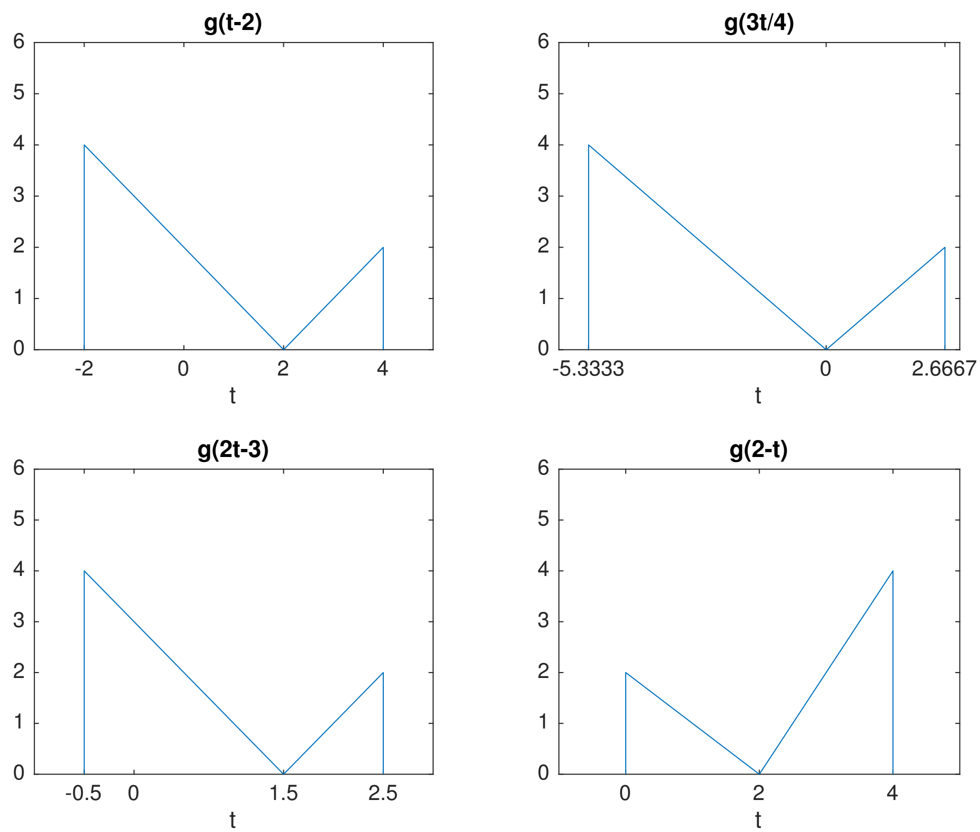

Fig. S2.3-5

2.3-6

$$E_{-g} = \int_{-\infty}^{\infty} [-g(t)]^2\, dt = \int_{-\infty}^{\infty} g^2(t)\, dt = E_g, \quad E_{g(-t)} = \int_{-\infty}^{\infty} [g(-t)]^2\, dt = \int_{-\infty}^{\infty} g^2(x)\, dx = E_g$$

$$E_{g(t-T)} = \int_{-\infty}^{\infty} [g(t-T)]^2\, dt = \int_{-\infty}^{\infty} g^2(x)\, dx = E_g, \quad E_{g(at)} = \int_{-\infty}^{\infty} [g(at)]^2\, dt = \frac{1}{a}\int_{-\infty}^{\infty} g^2(x)\, dx = E_g/a$$

$$E_{g(at-b)} = \int_{-\infty}^{\infty} [g(at-b)]^2\, dt = \frac{1}{a}\int_{-\infty}^{\infty} g^2(x)\, dx = E_g/a, \quad E_{g(t/a)} = \int_{-\infty}^{\infty} [g(t/a)]^2\, dt = a\int_{-\infty}^{\infty} g^2(x)\, dt = aE_g$$

$$E_{ag(t)} = \int_{-\infty}^{\infty} [ag(t)]^2\, dt = a^2 \int_{-\infty}^{\infty} g^2(t)\, dt = a^2 E_g$$

2.4-1 Using the facts that $\phi(t)\delta(t) = \phi(0)\delta(t)$ and $\phi(t)\delta(t-T) = \phi(T)\delta(t-T)$, we have

(a) $\frac{\tan(3\pi/4)}{\pi^2/8+1}\delta(t-\frac{\pi}{4}) = \frac{-8}{\pi^2+8}\delta(t-\frac{\pi}{4})$

(b) $\frac{1-j}{10\pi}\delta(\omega+\pi)$

(c) $-e^{\pi/15}\delta(t+\pi/15)$

(d) $\frac{\sin(1.5\pi)}{1-4}\delta(t-1) = \frac{1}{3}\delta(t-1)$

(e) $\frac{\cos(-3\pi/2)}{1/2} = 0$

(f) $k^2\delta(\omega) - \frac{4\sin^2(k\pi/2)}{\pi^2}\delta(\omega+\pi/2)$ (use L'Hôpital's rule $\lim_{\omega\to 0}\frac{\sin k\omega}{k\omega} = 1$)

2.4-2

(a) $\frac{-8}{\pi^2+8}$

(b) $\frac{1-j}{10\pi}$

(c) $-e^{\pi/15}$

(d) $\frac{1}{3}$

(e) 0

(f) $k^2 - \frac{4\sin^2(k\pi/2)}{\pi^2}$

2.4-3 Letting $at = x$, we obtain (for $a > 0$)

$$\int_{-\infty}^{\infty} \phi(t)\delta(at)\, dt = \frac{1}{a}\int_{-\infty}^{\infty} \phi(\frac{x}{a})\, \delta(x)\, dx = \frac{1}{a}\phi(0)$$

Similarly for $a < 0$, we show that this integral is $-\frac{1}{a}\phi(0)$. Therefore

$$\int_{-\infty}^{\infty} \phi(t)\delta(at)\, dt = \frac{1}{|a|}\phi(0) = \frac{1}{|a|}\int_{-\infty}^{\infty} \phi(t)\delta(t)\, dt$$

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

Therefore,

$$\delta(\omega) = \delta(2\pi f) = \frac{1}{2\pi}\delta(f)$$

.

2.4-4 Using the fact that $\delta(at) = \frac{1}{|a|}\delta(t)$ (see Problem **2.4-3**), and the equality $\int_a^b \phi(t)\delta(t-T)\,dt = \phi(T)$, we get

(a) $g(-3t+a)$

(b) $g(t)$

(c) $e^{j2\omega}$

(d) 0

(e) $e^6/2$

(f) 5

(g) $2g(-4)$

(h) $\frac{\cos(7\pi/3)}{3}$

2.5-1

(a) In this case $E_x = \int_0^1 dt = 1$, and

$$c = \frac{1}{E_x}\int_0^1 g(t)x(t)\,dt = \frac{1}{1}\int_0^1 t\,dt = 0.5$$

(b) Thus, $g(t) \approx 0.5x(t)$, and the error $e(t) = t - 0.5$ over $(0 \le t \le 1)$, and zero outside this interval. Also E_g and E_e (the energy of the error) are

$$E_g = \int_0^1 g^2(t)\,dt = \int_0^1 t^2\,dt = 1/3 \quad \text{and} \quad E_e = \int_0^1 (t-0.5)^2\,dt = 1/12$$

The error $(t-0.5)$ is orthogonal to $x(t)$ because

$$\int_0^1 (t-0.5)(1)\,dt = 0$$

Note that $E_g = c^2E_x + E_e$. To explain these results in terms of vector concepts, we observe from Fig. 2.13 that the error vector $\mathbf{e}$ is orthogonal to the component $c\mathbf{x}$. Because of this orthogonality, the length-square of $\mathbf{g}$ [energy of $g(t)$] is equal to the sum of the square of the lengths of $c \cdot \mathbf{x}$ and $\mathbf{e}$ [sum of the energies of $cx(t)$ and $e(t)$].

2.5-2 In this case $E_g = \int_0^1 g^2(t)\,dt = \int_0^1 t^2\,dt = 1/3$, and

$$c = \frac{1}{E_g}\int_0^1 x(t)g(t)\,dt = 3\int_0^1 t\,dt = 1.5$$

Thus, $x(t) \approx 1.5g(t)$, and the error $e(t) = x(t) - 1.5g(t) = 1 - 1.5t$ over $(0 \le t \le 1)$, and zero outside this interval. Also E_e (the energy of the error) is $E_e = \int_0^1 (1-1.5t)^2\,dt = 1/4$.

2.5-3

$$|\mathbf{e}|^2 = |\mathbf{g}|^2 + c^2|\mathbf{x}|^2 - 2c\mathbf{g}\cdot\mathbf{x}$$

To minimize error, set $\frac{d|\mathbf{e}|^2}{dc} = 0$:

$$2c|\mathbf{x}|^2 - 2\mathbf{g}\cdot\mathbf{x} = 0$$

$$c = \frac{\mathbf{g}\cdot\mathbf{x}}{|\mathbf{x}|^2} = \frac{<\mathbf{g},\mathbf{x}>}{|\mathbf{x}|^2}$$

2.5-4

(a) In this case $E_x = \int_0^1 \sin^2 2\pi t\, dt = 0.5$, and

$$c = \frac{1}{E_x}\int_0^1 g(t)x(t)\,dt = \frac{1}{0.5}\int_0^1 t\sin\,2\pi t\,dt = -1/\pi$$

(b) Thus, $g(t) \approx -(1/\pi)x(t)$, and the error $e(t) = t + (1/\pi)\sin\,2\pi t$ over $(0 \le t \le 1)$, and zero outside this interval.

Also E_g and E_e (the energy of the error) are

$$E_g = \int_0^1 g^2(t)\,dt = \int_0^1 t^2\,dt = 1/3$$

and

$$E_e = \int_0^1 [t + (1/\pi)\sin\,2\pi t]^2\,dt = \frac{1}{3} - \frac{1}{2\pi^2}$$

The error $[t + (1/\pi)\sin\,2\pi t]$ is orthogonal to $x(t)$ because

$$\int_0^1 \sin\,2\pi t[t + (1/\pi)\sin\,2\pi t]\,dt = 0$$

Note that $E_g = c^2E_x + E_e$. To explain these results in terms of vector concept we observe from Fig. 2.13 that the error vector $\mathbf{e}$ is orthogonal to the component $c\mathbf{x}$. Because of this orthogonality, the square of length of $\mathbf{g}$ [energy of $g(t)$] is equal to the sum of squares of the lengths $c\mathbf{x}$ and $\mathbf{f}$ [sum of the energies of $cx(t)$ and $e(t)$].

2.5-5

(a) If $x(t)$ and $y(t)$ are orthogonal, then we can show that the energy of $x(t) \pm y(t)$ is $E_x + E_y$.

$$\begin{aligned}\int |x(t)\,\pm\,y(t)|^2\,dt &= \int_{-\infty}^{\infty}|x(t)|^2\,dt + \int_{-\infty}^{\infty}|y(t)|^2\,dt \pm \int_{-\infty}^{\infty}x(t)y^*(t)\,dt \pm \int_{-\infty}^{\infty}x^*(t)y(t)\,dt \\ &= \int_{-\infty}^{\infty}|x(t)|^2\,dt + \int_{-\infty}^{\infty}|y(t)|^2\,dt\end{aligned}$$

The last result follows from the fact that because of orthogonality, the two integrals of the cross products $x(t)y^*(t)$ and $x^*(t)y(t)$ are zero. Thus the energy of $x(t) + y(t)$ is equal to that of $x(t) - y(t)$ if $x(t)$ and $y(t)$ are orthogonal.

(b) We can use a similar argument to show that the energy of $c_1x(t)+c_2y(t)$ is equal to that of $c_1x(t)-c_2y(t)$ if $x(t)$ and $y(t)$ are orthogonal. This energy is given by $|c_1|^2E_x + |c_2|^2E_y$.

(c) If $z(t) = x(t) \pm y(t)$, then it follows from **part (a)** in the preceding derivation that

$$E_z = E_x + E_y \pm (E_{xy} + E_{yx})$$

2.6-1 We shall use Eq. (2.51) to compute ρ_n for each of the four cases. Let us first compute the energies of all the signals:

$$E_x = \int_0^1 \sin^2 2\pi t\, dt = 0.5$$

In the same way, we find $E_{g_1} = E_{g_2} = E_{g_3} = E_{g_4} = 0.5$.

From Eq. (2.51), the correlation coefficients for four cases are found as follows:

(1) $\frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 \sin 2\pi t \sin 4\pi t\, dt = 0$

(2) $\frac{1}{\sqrt{(0.5)(0.5)}} \int (\sin 2\pi t)(-\sin 2\pi t)\, dt = -1$

(3) $\frac{1}{\sqrt{(0.5)(0.5)}} \int_0^1 0.707 \sin 2\pi t\, dt = 0$

(4) $\frac{1}{\sqrt{(0.5)(0.5)}} \left[\int_0^{0.5} 0.707 \sin 2\pi t\, dt - \int_{0.5}^1 0.707 \sin 2\pi t\, dt\right] = 2.828/\pi$

Signals $x(t)$ and $g_2(t)$ provide the maximum protection against noise.

2.6-2 $E_{g1} = \int_{-\infty}^{\infty} g_1^2(t)dt = \int_0^2 1dt = 2$ and $E_{g2} = \int_{-\infty}^{\infty} g_2^2(t)dt = \int_0^{\infty} e^{-t} = 1$.

Therefore, $\rho = \frac{1}{\sqrt{2\cdot 1}} \int_0^2 e^{-0.5t}dt = \sqrt{2}\left(1 - e^{-1}\right)$.

2.6-3 Given $g(t) = \exp(-2t)\cos(\pi t)u(t)$, the autocorrelation function is

$$\begin{aligned}\psi_g(\tau) &= \int_{-\infty}^{\infty} g(t)g(t+\tau)dt \\ &= \int_{-\infty}^{\infty} \exp(-2t)\cos(\pi t)u(t)\exp(-2(t+\tau))\cos(\pi(t+\tau))u(t+\tau)dt\end{aligned}$$

If $\tau \geq 0$,

$$\begin{aligned}\psi_g(\tau) &= \int_0^{\infty} \exp(-2t)\cos(\pi t)\exp(-2(t+\tau))\cos(\pi(t+\tau))dt \\ &= \int_0^{\infty} \exp(-4t-2\tau)\left[\frac{1}{2}\cos(2\pi t+\pi\tau) + \frac{1}{2}\cos(\pi\tau)\right] dt \\ &= \frac{1}{2}\int_0^{\infty} \exp(-4(t+\frac{\tau}{2}))\cdot\cos(2\pi(t+\frac{\tau}{2}))\, dt + \frac{1}{2}\exp(-2\tau)\cos(\pi\tau)\int_0^{\infty}\exp(-4t)\, dt \\ &= \frac{1}{8}\exp(-2\tau)\cos(\pi\tau) + \frac{1}{8\pi^2+32}\exp(-2\tau)(4\cos(\pi\tau) - 2\pi\sin(\pi\tau)), \quad \tau \geq 0.\end{aligned}$$

Because of symmetry,

$$\psi_g(\tau) = \frac{1}{8}\exp(-2|\tau|)\left[\frac{\pi^2+8}{\pi^2+4}\cos(\pi\tau) - \frac{2\pi}{\pi^2+4}\sin(\pi|\tau|)).\right]$$

2.7-1

(a) $\mathbf{g}_1 = (2, -1)$, $\mathbf{g}_2 = (-1, 2)$, $\mathbf{g}_3 = (0, -1)$, $\mathbf{g}_4 = (1, 2)$, $\mathbf{g}_5 = (2, 1)$, and $\mathbf{g}_6 = (3, 0)$.

(b) Signal pairs $(\mathbf{g}_3, \mathbf{g}_6)$, $(\mathbf{g}_1, \mathbf{g}_4)$ and $(\mathbf{g}_2, \mathbf{g}_5)$ are orthogonal. We can verify this analytically.

$$\begin{aligned}
<\mathbf{g}_3, \mathbf{g}_6> &= (0 \times 3) + (-2 \times 0) = 0, \\
<\mathbf{g}_1, \mathbf{g}_4> &= (2 \times 1) + (-1 \times 2) = 0, \\
<\mathbf{g}_2, \mathbf{g}_5> &= (-1 \times 2) + (2 \times 1) = 0
\end{aligned}$$

We can show that the corresponding signal pairs are also orthogonal.

$$\begin{aligned}
\int_{-\infty}^{\infty} g_3(t) g_6(t)\, dt &= \int_{-\infty}^{\infty} [-x_2(t)][3x_1(t)]\, dt = 0 \\
\int_{-\infty}^{\infty} g_1(t) g_4(t)\, dt &= \int_{-\infty}^{\infty} [2x_1(t) - x_2(t)][x_1(t) + 2x_2(t)]\, dt = 0 \\
\int_{-\infty}^{\infty} g_2(t) g_5(t)\, dt &= \int_{-\infty}^{\infty} [-x_1(t) + 2x_2(t)][2x_1(t) + x_2(t)]\, dt = 0
\end{aligned}$$

In deriving these results, we used the fact that $\int_{-\infty}^{\infty} x_1^2\, dt = \int_{-\infty}^{\infty} x_2^2(t)\, dt = 1$ and $\int_{-\infty}^{\infty} x_1(t) x_2(t)\, dt = 0$.

(c) Because $\int_{-\infty}^{\infty} x_1^2\, dt = \int_{-\infty}^{\infty} x_2^2(t)\, dt = 1$ and $\int_{-\infty}^{\infty} x_1(t) x_2(t)\, dt = 0$, signal energy $E_g = c_1^2 + c_2^2$ for $g(t) = c_1 x_1(t) + c_2 x_2(t)$. Therefore, $E_{g1} = 5$, $E_{g2} = 5$, $E_{g3} = 4$, $E_{g4} = 5$, $E_{g5} = 5$, $E_{g6} = 9$.

2.7-2

(a) Since $x(t)$ and $e(t)$ are mutually orthogonal, we can choose these two functions as the basis functions. After normalizing, we obtain $\psi_1(t) = x(t)/\sqrt{E_x} = x(t) = 1, 0 \le t \le 1$ and $\psi_2(t) = e(t)/\sqrt{E_e} = 2\sqrt{3}\, e(t) = 2\sqrt{3}(t - 0.5), 0 \le t \le 1$.

(b) Through orthogonal decomposition, we know that $g(t) = 0.5\, x(t) + e(t) = 0.5\, \psi_1(t) + \frac{1}{2\sqrt{3}} \psi_2(t)$ and $x(t) = \psi_1(t) + 0\, \psi_2(t)$. Thus, the vector representations are $\mathbf{g} = [\frac{1}{2} \quad \frac{1}{2\sqrt{3}}]$ and $\mathbf{x} = [1 \quad 0]$, respectively.

(c) Similarly, this time $\psi_1(t) = x(t)/\sqrt{E_x} = \sqrt{2}\, x(t) = \sqrt{2} \sin(2\pi t), 0 \le t \le 1$ and $\psi_2(t) = e(t)/\sqrt{E_e} = \frac{1}{\sqrt{1/3 - 1/(2\pi^2)}}(t + \sin(2\pi t)/\pi), 0 \le t \le 1$.

$g(t) = -\frac{1}{\pi}\, x(t) + e(t) = -\frac{1}{\pi\sqrt{2}}\, \psi_1(t) + \sqrt{\frac{1}{3} - \frac{1}{2\pi^2}}\, \psi_2(t)$ and $x(t) = \frac{1}{\sqrt{2}} \psi_1(t) + 0\, \psi_2(t)$. Thus, the vector representations are $\mathbf{g} = [-\frac{1}{\pi\sqrt{2}} \quad \sqrt{\frac{1}{3} - \frac{1}{2\pi^2}}]$ and $\mathbf{x} = [\frac{1}{\sqrt{2}} \quad 0]$, respectively.

2.7-3

(a) We can choose the normalized $x(t), g_1(t), g_3(t)$ as the first three orthonormal bases for the set of signals, since all the correlations between these signals are zeros. Therefore,

$$\begin{aligned}\phi_1(t) &= \sqrt{2}x(t)\\ \phi_2(t) &= \sqrt{2}g_1(t)\\ \phi_3(t) &= \sqrt{2}g_3(t).\end{aligned}$$

Signal $g_2(t)$ is the negative of signal $x(t)$. Therefore, $g_2(t) = -\frac{1}{\sqrt{2}}\phi_1(t)$.

To represent $g_4(t)$, we need an additional basis function $\phi_4(t)$. We can represent $g_4(t)$ as $g_4(t) = c_1\phi_1(t) + c_2\phi_2(t) + c_3\phi_3(t) + c_4\phi_4(t)$, where,

$$\begin{aligned}c_1 &= \int_0^1 g_4(t)\phi_1^*(t)\,dt = \sqrt{2}\int_0^1 g_4(t)x(t)\,dt = \frac{2}{\pi}\\ c_2 &= \int_0^1 g_4(t)\phi_2^*(t)\,dt = \sqrt{2}\int_0^1 g_4(t)g_1(t)\,dt = \sqrt{2}\left[\int_0^{0.5} 0.707\sin 4\pi t\,dt - \int_{0.5}^1 0.707\sin 4\pi t\,dt\right] = 0\\ c_3 &= \int_0^1 g_4(t)\phi_3^*(t)\,dt = \sqrt{2}\int_0^1 g_4(t)g_3(t)\,dt = \sqrt{2}\,(0.707)\int_0^1 g_4(t)\,dt = 0\end{aligned}$$

Therefore,

$$c_4\phi_4(t) = g_4(t) - c_1\phi_1(t) = g_4(t) - \frac{2\sqrt{2}}{\pi}x(t)$$

The energy of the signal is

$$\begin{aligned}E_{c_4\phi_4} &= \int_0^1 \left[g_4(t) - \frac{2\sqrt{2}}{\pi}x(t)\right]^2 dt = \int_0^1 \left[g_4^2(t) + \frac{8}{\pi^2}x^2(t)^2 - \frac{4\sqrt{2}}{\pi}g_4(t)x(t)\right] dt\\ &= \frac{1}{2} + \frac{4}{\pi^2} - \frac{4\sqrt{2}}{\pi}\cdot\frac{\sqrt{2}}{\pi} = \frac{1}{2} + \frac{4}{\pi^2} - \frac{8}{\pi^2} = \frac{1}{2} - \frac{4}{\pi^2} = 0.0946\end{aligned}$$

Therefore,

$$\phi_4(t) = \frac{g_4(t) - \frac{2\sqrt{2}}{\pi}x(t)}{\sqrt{E_{c_4\phi_4}}} = \frac{g_4(t) - \frac{2\sqrt{2}}{\pi}x(t)}{0.3076}$$

(b)

$$\begin{aligned}x_1(t) &= [\ \tfrac{1}{\sqrt{2}}\quad 0\quad 0\quad 0\ ]^T\\ g_1(t) &= [\ 0\quad \tfrac{1}{\sqrt{2}}\quad 0\quad 0\ ]^T\\ g_2(t) &= [\ -\tfrac{1}{\sqrt{2}}\quad 0\quad 0\quad 0\ ]^T\\ g_3(t) &= [\ 0\quad 0\quad \tfrac{1}{\sqrt{2}}\quad 0\ ]^T\\ g_4(t) &= [\ \tfrac{2}{\pi}\quad 0\quad 0\quad 0.3076\ ]^T\end{aligned}$$

2.8-1

(a) $T_0 = 4$, $\omega_0 = \frac{2\pi}{T_0} = \frac{\pi}{2}$. Because of even symmetry, all sine terms are zero.

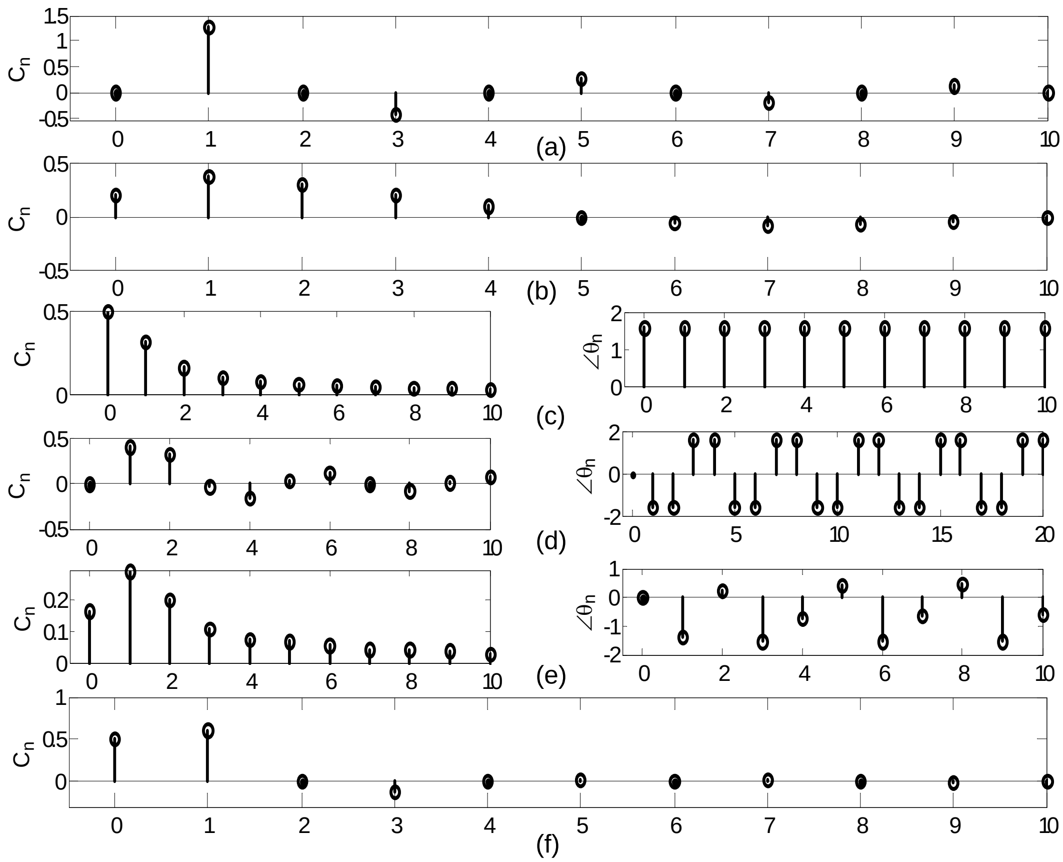

Fig. S2.8-1

$$
\begin{aligned}
g(t) &= a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{2} t\right) \\
a_0 &= 0 \text{ (by inspection of its lack of dc)} \\
a_n &= \frac{4}{4}\left[\int_0^1 \cos\left(\frac{n\pi}{2} t\right)\, dt - \int_1^2 \cos\left(\frac{n\pi}{2} t\right)\, dt\right] \\
&= \frac{4}{n\pi} \sin\frac{n\pi}{2}
\end{aligned}
$$

Therefore, the Fourier series for $g(t)$ is

$$
g(t) = \frac{4}{\pi}\left(\cos\frac{\pi t}{2} - \frac{1}{3}\cos\frac{3\pi t}{2} + \frac{1}{5}\cos\frac{5\pi t}{2} - \frac{1}{7}\cos\frac{7\pi t}{2} + \cdots\right)
$$

Here $b_n = 0$, and we allow C_n to take negative values. Figure S2.8-1(a) shows the plot of C_n which is real-valued.

(b) $T_0 = 10\pi$, $\omega_0 = \frac{2\pi}{T_0} = \frac{1}{5}$. Because of even symmetry, all the sine terms are zero.

$$\begin{aligned}
g(t) &= a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n}{5}t\right) + b_n \sin\left(\frac{n}{5}t\right) \\
a_0 &= \frac{1}{5} \qquad \text{(by inspection)} \\
a_n &= \frac{2}{10\pi}\int_{-\pi}^{\pi} \cos\left(\frac{n}{5}t\right)\, dt \\
&= \frac{1}{5\pi}\left(\frac{5}{n}\right)\sin\left(\frac{n}{5}t\right)\Big|_{-\pi}^{\pi} = \frac{2}{\pi n}\sin\left(\frac{n\pi}{5}\right) \\
b_n &= \frac{2}{10\pi}\int_{-\pi}^{\pi} \sin\left(\frac{n}{5}t\right)\, dt \\
&= 0 \qquad \text{(integrand is an odd function of } t\text{)}
\end{aligned}$$

Here $b_n = 0$, and we allow C_n to take negative values. Note that $C_n = a_n$ for $n = 0, 1, 2, 3, \ldots$. Fig. S2.8-1(b) shows the plot of C_n which is real-valued.

(c) $T_0 = 2\pi$, $\omega_0 = 1$, and

$$g(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos nt + b_n \sin nt$$

with

$$a_0 = 0.5 \quad \textbf{(by inspection of the dc or average)}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \frac{t}{2\pi}\cos nt\, dt = 0, \qquad \text{and} \qquad b_n = \frac{1}{\pi}\int_0^{2\pi}\frac{t}{2\pi}\sin nt\, dt = -\frac{1}{\pi n}$$

and

$$\begin{aligned}
g(t) &= 0.5 - \frac{1}{\pi}\left(\sin t + \frac{1}{2}\sin 2t + \frac{1}{3}\sin 3t + \frac{1}{4}\sin 4t + \cdots\right) \\
&= 0.5 + \frac{1}{\pi}\left[\cos\left(t + \frac{\pi}{2}\right) + \frac{1}{2}\cos\left(2t + \frac{\pi}{2}\right) + \frac{1}{3}\cos\left(3t + \frac{\pi}{2}\right) + \cdots\right]
\end{aligned}$$

The cosine terms vanish because when 0.5 (the dc component) is subtracted from $g(t)$, the remaining function has odd symmetry. Hence, the Fourier series would contain dc and sine terms only. Figure S2.8-1(c) shows the plots of $|C_n|$ and θ_n. Note that C_n is purely imaginary.

(d) $T_0 = \pi$, $\omega_0 = 2$ and $g(t) = \frac{4}{\pi}t$.

$a_0 = 0$ (by inspection)

$a_n = 0 \quad (n > 0)$ (because of odd symmetry)

$$b_n = \frac{4}{\pi}\int_0^{\pi/4} \frac{4}{\pi}t\sin 2nt\, dt = \frac{2}{\pi n}\left(\frac{2}{\pi n}\sin\frac{\pi n}{2} - \cos\frac{\pi n}{2}\right)$$